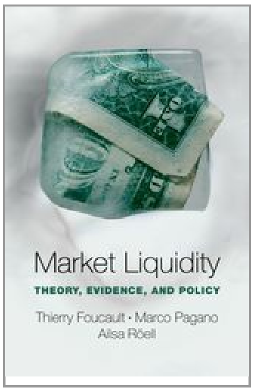
Market Liquidity

THEORY, EVIDENCE, AND POLICY

Thierry Foucault • Marco Pagano
Ailsa Röell



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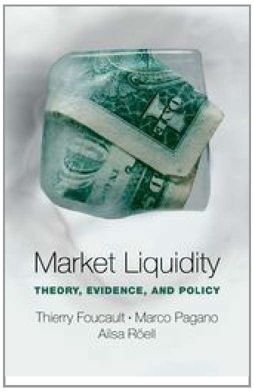

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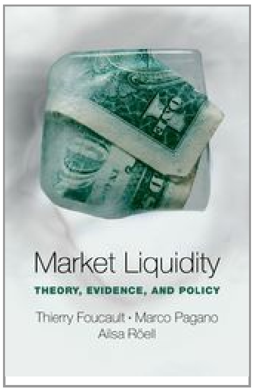
Dedication

(p.v) *To our families* **(p.vi)**



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(p.xii) Preface

Liquid markets enable people to fund investments that require a long-term commitment of wealth, while retaining the opportunity to access that wealth when needed. In this way liquidity facilitates real investment and enhances economic growth. But market liquidity can be elusive at times, as investors discovered to their cost in the recent financial crisis —a forcible reminder of John Maynard Keynes's 1936 warning that "there is no such thing as liquidity of investment for the community as a whole" (p. 155): markets freeze when everyone is seeking liquidity and no one is willing to provide it.

Aim and Structure of the Book

The central topic of this book is the liquidity of security markets: its determinants and its effects. The first part of the book (Chapters 1 through 5) provides the reader with basic modeling and econometric tools needed to understand market microstructure, the area of financial economics that focuses on security market liquidity. This part of the book starts by describing how security markets are organized and how their liquidity can be measured. Then we explain how various market imperfections affect price formation, liquidity, and speed of price discovery. Finally, we show how the interaction of order flow and price movements can be used to assess empirically the relative importance of various determinants of liquidity.

The second part of the book investigates how key features of market design affect the level of liquidity, the speed of price discovery, and gains and losses to market participants: we examine limit order book markets with continuous trading (Chapter 6), and the issues of fragmentation (Chapter 7) and transparency (Chapter 8).

The third part of the book is devoted to interactions between market microstructure, asset pricing, and corporate finance. We first explain how **(p.xiii)** liquidity affects the returns required by investors, and therefore asset prices. Next, we examine how prices in illiquid markets may diverge from underlying long-run values, especially in the context of market freezes and financial crises (Chapter 9). The book concludes by explaining how liquidity affects real investment decisions and corporate policies (Chapter 10).

The book provides an introduction to the field of market microstructure, covering theory, empirical work, and policy issues. It is designed as a textbook for intermediate and graduate-level students in economics and finance, as well as for practitioners with some economics training. The level of technical complexity is kept to a minimum, so that only a very basic knowledge of calculus, statistics, and game theory is necessary to tackle the material. The book does not aim to be a comprehensive survey of the field, but a unified and self-contained treatment of the core concepts and techniques in an area that has greatly developed in the last thirty years. As the book is intended to be a teaching and learning tool, each chapter starts with a list of learning objectives—major points that the student can expect to master by working through the chapter. Most chapters also include boxes that describe business stories or quotes from the financial press that illustrate the real-world relevance of the concepts and results presented. The book also comes with a generous supply of exercises, which vary in complexity and focus: some of them require analytical derivations; others ask for empirical work on small data sets provided on the book's companion web site (which also contains supplemental teaching material for registered educators); see http://www.oup.com/us/marketliquidity/. Our experience is that hands-on practice with the end-of-chapter exercises is the best way to master the material in the book.

How to Use the Book

We can say with some confidence that this is a useful book, having taught from preliminary versions of it over the years. Indeed, we have greatly benefited from the feedback received over more than a decade from undergraduate and graduate students at HEC Paris, Imperial College, Tinbergen Institute, and at the universities of Bologna, Mannheim, Naples, Princeton, Sydney, and Tilburg. The book can be used as the main source of material for a course in market microstructure or as a complement to other books in other areas of finance.

A course that covers the entire book would require thirty to forty lecture hours, depending on the background of the students (plus about ten one-hour exercise sessions). However, the book is designed to allow a "modular" use of its material: by a careful selection of chapters, it can be adapted to either an introductory course in market microstructure pitched at the level of an **(p.xiv)** advanced undergraduate or master class, or a more specialized and advanced course, possibly at the doctoral level. More specifically, here are some examples of typical courses that can be designed by "slicing and dicing" the material in the book:

(i) For a basic course in market microstructure we would recommend including all of Part I (fifteen to twenty lecture hours depending on students' background). The chapters on the institutional setting (Chapter 1) and the basic theory of price determination (Chapter 3) are essential. The theory of market depth (Chapter 4) is highly recommended, although a short course might leave out the sections on imperfect competition. A basic empirical training is provided by the chapters on the measurement of liquidity and on estimating the determinants of liquidity (Chapters 2 and 5). If more time is available, any of the subsequent chapters (from Parts II and III) may be covered: each of them is self-contained, so that they can be chosen in any combination that caters to the interests of the course participants. Each additional chapter would require no less than three lecture hours.
(ii) A master-level course on the architecture of securities markets would start with the basic institutions and theory (Chapters 1, 3, and 4) and then focus on the market design and regulatory issues addressed in Part II (Chapters 6, 7, and 8). Such a course would require fifteen to twenty lecture hours.
(iii) A master-level or Ph.D. course stressing the relevance of market microstructure for asset pricing and corporate finance should include Chapters 3, 4, 9, and 10 (twelve to fifteen hours).
(iv) A suitable complement for a Ph.D. course in asset pricing would include Chapters 3 and 9.
(v) Similarly, to complement a Ph.D. course in corporate finance, we suggest Chapters 3 and 10.

Acknowledgments

This book has been many years in the writing, as our students, colleagues, and family members know only too well. We have accumulated a large debt of gratitude. We would like to thank the colleagues, coauthors, and mentors who inspired and encouraged our work in the area. A partial list includes Viral Acharya, Anat Admati, Alessandro Beber, Bruno Biais, Patrick Bolton, Margaret Bray, Giovanni Cespa, Hans Degryse, Peter Diamond, Andrew Ellul, Laurent Fresard, Alessandro Frino, Thomas Gehrig, Larry Glosten, Charles Goodhart, Oliver Hart, Joel Hasbrouck, Martin Hellwig, Johan Hombert, Charles Jones, Frank de Jong, Ohad Kadan, Eugene Kandel, Mervyn King, Pete Kyle, Albert **(p.xv)** Menkveld, Sophie Moinas, Theo Nijman, Maureen O'Hara, Christine Parlour, Ioanid Rosu, Patrik Sandas, Duane Seppi, Chester Spatt, Ernst-Ludwig von Thadden, Erik Theissen, David Thesmar, Dimitri Vayanos, Paolo Volpin, and Josef Zechner.

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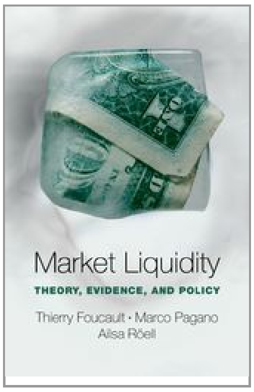

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Introduction

Thierry Foucault
Marco Pagano
Ailsa Röell

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[–] Abstract and Keywords

This introductory chapter begins with an overview of what this book is about. It identifies two key concepts in market microstructure—market liquidity and price discovery—and explains why these are important. It then outlines some puzzling phenomena in securities markets and concludes with a discussion of the three dimensions of liquidity.

Keywords: market microstructure, market liquidity, price discovery, securities markets

Learning Objectives:

- • What is this book about?
- • Two key concepts in market microstructure: market liquidity and price discovery

- • Why do people care about market liquidity and price discovery?
- • Which puzzles can market microstructure address?
- • The three dimensions of liquidity

0.1. What is This Book about?

The way securities are actually traded is far removed from the idealized picture of a frictionless and self-equilibrating market offered by the typical finance textbook. In that idealized version of the trading process, all potential participants are present on the market; these participants convey to the market orders that reflect their demand or supply of securities, and they are not affected by actions of other market participants; and an auctioneer balances the quantities demanded and supplied at a single equilibrium price that reflects a consensus view of the security's "fundamental value." Real-world markets do not work like this, for two main reasons.

First, market players are not all simultaneously present on the market. Such continuous presence would be too costly in time, attention, and access costs. At any given point in time, price formation is delegated to the limited number of market participants who happen to be present. Any temporary imbalance between buy and sell orders for a security will have to be absorbed by whoever **(p.2)** is present, especially by professional intermediaries who specialize in "making the market." Typically, market makers and other investors will absorb order imbalances only if the price is sufficiently attractive. For instance, to absorb a spate of sell orders investors will require the inducement of a sufficiently low price. As a result, the equilibrium price actually struck at any given instant may deviate from the one that would emerge if all investors participated. These price deviations generate profit opportunities, which in turn will draw in more traders. Over time the deviations are ironed out.

Second, even the limited number of participants who are present at any instant in a real-world security market have quite diverse information about the security's fundamentals: some participants are shrewd market professionals with all the latest news and state-of-the-art pricing models at their fingertips; others do not have such up-to-date information but may try to infer it from the behavior of other participants; still others may trade for reasons that are unrelated to information, for instance a need to liquidate their holdings in order to pay their bills. As a result, the order flow is a complex mix of information and noise, and a consensus price only emerges over time, as the trading process evolves and participants interpret the actions of other traders. This is another reason why a security's actual transaction price might deviate from its fundamental value, which would be assessed by a fully informed set of investors.

This book takes these deviations of prices from fundamental values seriously. We explain why and how they emerge in the trading process, and how and why they are eventually eliminated. Fortunately we can draw on a vast body of theoretical insights and empirical findings on security price formation that has been built up in the last thirty years, forming a well-defined field of financial economics known as "market microstructure." As we shall see, the study of market microstructure illuminates two key aspects of real-world markets that are neglected by textbook asset pricing models: liquidity and price discovery.

Liquidity is the degree to which an order can be executed within a short time frame at a price close to the security's consensus value. Conversely, a price that deviates substantially from this consensus value indicates illiquidity: in an illiquid market, buy orders tend to push transaction prices up, while sell orders tend to do the opposite; in extreme cases, the deviation is so great that it is not worthwhile or feasible to trade at all, and the market freezes. In other words, in an illiquid market, the best price at which a security can be bought (ask price) is considerably above the best price at which it can be sold (bid price). And in fact the difference between these two prices—the bid-ask spread—is a common measure of illiquidity. Liquidity differs greatly among securities and over time, one of the aims of this book is to explain why this is so. For instance, the following table shows the bid and ask prices at which different U.S. stocks **(p.3)**

Table 0.1 Bid and Ask Prices Quoted for Selected NYSE/Nasdaq Stocks on September 2, 2010 at 4:20 p.m.

Stock	IBM	Amazon	Barnes & Noble	Borders	Books-A-Million
Best bid price	124.88	135.06	16.02	1.05	5.40
Best ask price	124.89	135.14	16.10	1.06	5.94
$ bid-ask spread	0.01	0.08	0.08	0.01	0.54
% bid-ask spread	0.01%	0.06%	0.50%	0.95%	9.52%

could be sold or bought on the afternoon of September 2, 2010. Clearly, the stock of large companies such as IBM and Amazon is extremely liquid, with bid-ask spreads in the range of a few hundredths of 1 percent. In contrast, for a smaller, less well known internet bookseller like Books-A-Million, the spread is nearly 10 percent—a substantial illiquidity cost for potential investors.

Liquidity also fluctuates significantly over time. During the financial crisis, the average bid-ask spreads for stocks listed on the major exchanges worldwide increased dramatically, from about 3 percent in the first half of 2008 to 6 percent in the six months following the failure of Lehman Brothers in September. The average spread peaked at over 6.5 percent in the period of great uncertainty preceding the announcement of the Citibank rescue on November 23.[1] The connection between uncertainty and illiquidity is underscored by the fact that it was financial stocks whose spreads increased the most sharply by far during those months. The spikes in bid-ask spreads on the stock market coincided with even greater disruptions in the interbank market and the markets for credit default swaps (CDS) and many asset-backed securities. The lack of liquidity was so intense that at some points markets simply seized up. This book examines and explicates the causes of such dramatic changes in market liquidity.

Price discovery is the speed and accuracy with which transaction prices incorporate information available to market participants. Markets sometimes display an astonishing ability to locate information about recent events and extract its implications for underlying stock values. For example, in the wake of the space shuttle Challenger explosion at 11:39 a.m. EST on January 28, 1986, the stock market very quickly determined which of the four potential contracting manufacturers was at fault for the defective parts of the shuttle: within fifteen minutes, there was a sell-induced New-York Stock Exchange (NYSE) trading halt in the shares of only one company, Morton-Thiokol. By the end of the day its shares had fallen by 11.86 percent, while Lockheed, Martin-Marietta, and Rockwell **(p.4)** fell by much less (Maloney and Muhlerin, 2003). By contrast, the general public did not learn of the cause of the crash until two weeks later, on February 11, when Nobel-winning physicist Richard Feynman demonstrated that there were problems with Morton-Thiokol's booster rockets. This episode illustrates the market's ability to create knowledge out of a multitude of individual trades, each of which manages to contribute a small piece of information to the overall picture. In this sense, "securities markets are a vehicle for amalgamating unorganized knowledge" (Maloney and Muhlerin, 2003, p. 474).

This episode also illustrates a general but unintuitive point that will receive considerable attention here, namely, that there is a tension between price discovery and liquidity. When price-relevant information gets to the market by means of trading pressure rather than a public announcement, liquidity suffers. In fact, just as it became apparent that the sell orders of some market participants might be based on superior information about Morton-Thiokol's responsibility for the disaster, the market for its shares became most illiquid: the NYSE specialist dealing in Morton-Thiokol's stock decided to halt trading, to avoid making a market in a situation where he might very easily lose money to informed traders.

0.2. Why should we Care?

Why is market liquidity important? Asset managers and ordinary investors care about liquidity insofar as it affects the return on their investments, simply because illiquid securities cost more to buy, and sell for less. Therefore, illiquid-ity eats into the return. When markets are less than perfectly liquid, investors cannot buy and sell at the same price, and the bid-ask spread is typically wider for large trades. Thus, analysis of the way liquidity arises, builds, or vanishes may be very important in evaluating the portfolio choices of an asset manager or an ordinary investor.

For the same reasons, liquidity is a key concern of all the professionals who specialize in providing securities trading services, such as the trading desks of institutional investors (mutual funds, pension funds, and hedge funds) and retail stock brokers: locating the most liquid trading venue or timing trades so as to minimize trading costs is the key to providing good-quality service.

Beside being a source of costs, the trading process can also be a source of risks for investors. Insofar as liquidity can vary over time in ways that are not perfectly predictable, it can heighten the risk engendered by the unpredictability of assets' fundamentals. So investors will require compensation not only for the expected trading costs associated with illiquidity but also for the additional risks. For both of these reasons, illiquidity affects equilibrium prices, which must discount **(p.5)** not only risky future cash flows generated by the asset, but also the future trading costs that its holders may incur and the associated risks. The need to compensate investors for illiquidity creates a link between the field of market microstructure and that of asset pricing, which we will explore in Chapter 9.

But if illiquidity lowers securities prices, affecting the cost of capital for the issuers, then it will also affect these companies' day-to-day decisions on capital expenditure. The recent financial crisis is a telling example of the linkage between market liquidity, asset prices and economic activity: the drying up of liquidity in several securities markets in 2008 was associated with plunging asset prices and drastic reductions in security issuance and real investment by firms. And although these drops largely represented a correction of previous overpricing and over-investment, the illiquidity of securities markets undoubtedly amplified the effects of the revision in fundamentals. This episode illustrates why policy makers take such a strong interest in the liquidity of securities markets and in how policies and regulations affect it.

Investors, issuers, and policy makers naturally care not just about liquidity but also about the speed of price discovery. This determines the amount of information that at any instant is embodied in the price of a security, and hence how reliable that price is as a reference point for managers' real investment decisions. An informationally efficient price is also useful as a benchmark for evaluating the performance of the firm's management, and for devising equity-and option-based compensation schemes that provide the proper incentives. In short, market microstructure issues prove to be relevant to corporate finance choices, on such matters as capital budgeting and management compensation, as we shall see in Chapter 10.

0.3. Some Puzzles

This book explains a number of puzzling phenomena in securities markets. Let us consider a few specific examples of issues that can be attacked and understood using the analytical tools and empirical methods of market microstructure.

(i) Why does liquidity change over time? As we have seen, securities markets became much more illiquid during the recent financial crisis, especially in the second half of 2008. This also happened at the start of the Great Depression of 1929–30, when the average bid-ask spread on the stocks constituting the Dow Jones index widened from slightly under 0.6 percent before the crisis to over 1.4 percent (figure 0.1). The figure also shows that U.S. stock market liquidity has been basically increasing since World War II. Average bid-ask spreads gradually declined from about 0.6 percent in the 1950s to about 0.2 percent around 2000, with an especially large drop in the 1990s. **(p.6)**

Figure 0.1. Average percent bid-ask spread of Dow Jones stocks (Source: Jones, 2002)

Moreover, liquidity changes systematically in a much more limited time frame as well: within a single trading day, bid-ask spreads tend to feature a U-shaped pattern, higher at the open and at the close than during the rest of the trading day. In addition, for individual stocks liquidity tends to drop in connection with special events, such as takeover battles, or in the wake of other dramatic price-relevant events, as exemplified by the Challenger explosion episode discussed above. The models of security price determination presented in Chapters 3 and 4 offer insight into the reasons for these low-and high-frequency empirical regularities.

(ii) Why do large trades move prices up or down, and why are these price changes subsequently reversed? One of the most widely observed patterns in securities markets is that large orders—known as “block trades”—put temporary pressure on prices: large buys drive them up, and large sells push them down. This was already apparent in the early study by Kraus and Stoll (1972), which analyzes 7,009 block trades in 402 stocks on the NYSE from July 1, 1968 to September 30, 1969. Justified by a detailed analysis of trading by buying and selling parties for a subsample of blocks, the authors identify sell orders as those that are priced on a downtick (i.e., below the previous transaction price), and buy orders as those priced on an uptick. Figure 0.2 shows that the price drop associated with block sales (E_2) is largely (though not entirely) offset by an upswing (E_3) prior to the subsequent market close. The causes of these patterns will be studied in Chapter 3.

(iii) Why is securities trading concentrated? One recurrent feature of financial markets is the agglomeration of securities trading: as practitioners like

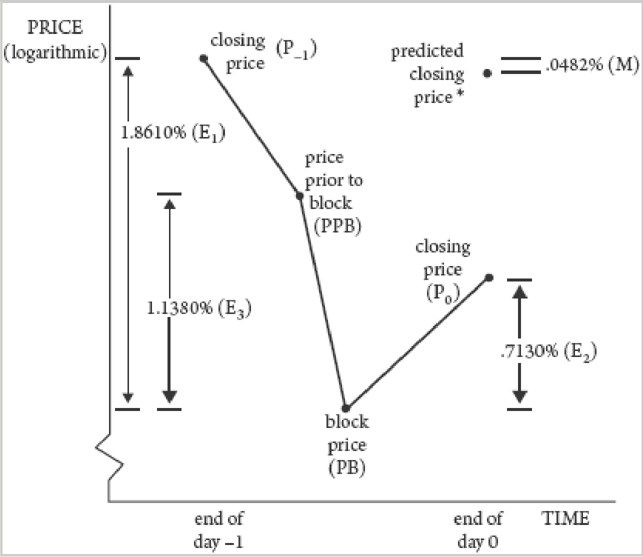


Figure 0.2. Price pattern associated with a block sale (Source: Kraus and Stoll, 1972)

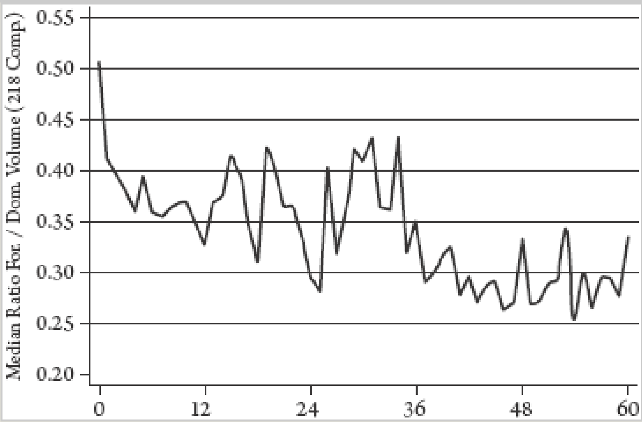


Figure 0.3. Ratio of foreign to domestic trading volume after cross-listing (Source: Halling et al., 2008)

to say, "Liquidity begets liquidity." People like to trade at the same time of day and in the same venue as many other market participants. For instance, after a listed company cross-lists its shares on a foreign exchange, its domestic market appears to exert a "gravitational pull" on the trading volume that initially moved abroad—what practitioners call "flowback." Figure 0.3 shows that the median ratio of foreign market to domestic market volume declines gradually over the sixty months following the cross-listing; the drop is especially marked in the first **(p.8)** year. Chapter 7 inquires into the reasons for this tendency of liquidity to feed upon itself and to persist over time, and its implications for the organization of securities markets.

(iv) Why do some traders willingly disclose their intended trades, and others hide them? Some traders really keep their cards close to the chest, submitting their orders in a way that does not reveal their true size. For example, as Chapter 2 demonstrates, they may submit "hidden orders." Alternatively, they may go off-exchange altogether and trade on "dark pools," where orders are not displayed. On the other hand, some traders opt for "sunshine trading"; that is, they preannounce their trading intentions. Chapter 8 analyzes market transparency, explaining why it harms some traders and benefits others.

(v) Why are there temporary deviations from arbitrage prices? The absence of arbitrage opportunities is a central tenet of asset-pricing theory: assets that generate identical cash flows must command the same market price, so that there is no opportunity for profitable arbitrage trading. Nevertheless, there are instances in which the no-arbitrage condition breaks down for non-negligible periods of time. For instance, Deville and Riva (2007) use intraday transaction data to study why it takes time for the French index options market to return to no-arbitrage values after deviating from put-call parity. Similarly, de Jong, Rosenthal, and Van Dijk (2009) document deviations from theoretical price parity in a sample of twelve dual-listed companies, sometimes known as "Siamese twins." These are pairs of companies incorporated in different countries that contractually agree to operate their businesses as a single enterprise, while retaining their separate legal identities and existing stock exchange listings, as in the case of Royal Dutch/Shell. These companies should trade at the same price, yet from 1980–2002 their prices actually differed so much that simple trading rules could produce abnormal returns of nearly 10 percent per annum in some cases, after adjusting for transaction costs and margin requirements. In Chapter 9 we explore why these deviations can persist, and how they relate to market liquidity.

0.4. The Three Dimensions of Liquidity

0.4.1 Market Liquidity

In this book we use the word *liquidity* to indicate the ability to trade a security quickly at a price close to its consensus value, that is, in the sense of "market liquidity." But readers need to be aware that this is only one of three interrelated dimensions of liquidity. Precisely because of their interrelationship, these three distinct dimensions are often referred to interchangeably in the context of the **(p.9)** same discourse in financial press articles and policy discussions. This can be confusing, so it is useful to clarify what liquidity means outside the context of security trading.

0.4.2 Funding Liquidity

When referring to banks or companies, liquidity is generally taken to mean having sufficient cash or the ability to obtain credit at acceptable terms, to meet obligations without incurring large losses. We can refer to this notion as "funding liquidity." Maintaining adequate liquidity is particularly important for banks, which typically engage in maturity transformation. That is, they use short-term liabilities (bank deposits or repurchase agreements) to fund long-term assets (loans to companies and households). Hence, to be able to satisfy the claims of their depositors or creditors, they must maintain an adequate buffer of cash and short-term assets that can be readily liquidated. Banks have other options for generating liquidity, such as selling loans, borrowing from other banks, or borrowing from a central bank such as the U.S. Federal Reserve or the European Central Bank. However, it could still happen that, say due to a loss of confidence in the bank, depositors may wish to withdraw funds in excess of the bank's cash reserves plus the amount it can raise by selling short-term assets (such as treasury bills or commercial paper) or obtaining overnight credit on the interbank market. Such a "bank run" is described, in fact, as a liquidity crisis, and in the absence of sufficient liquidity provision by the central bank, the distressed bank will be driven into bankruptcy and forced to liquidate its loan portfolio.

Funding liquidity is related to market liquidity in several ways. First, both have value for the same reason: people want to hold assets that can be immediately transformed into consumption, as for instance when the owner suffers a shock (e.g., a health problem or loss of job) or discovers an unforeseen opportunity (e.g., a very cheaply priced house or a very attractive business project). Since it is hard to insure against such individual-specific liquidity shocks, people try to self-insure by holding demand deposits, which they can withdraw without notice in case of need. That is, they prize funding liquidity. By the same token, investors value market liquidity, that is, they prefer assets that can be sold quickly in case of need at prices not far from their fundamental value.

Second, funding liquidity is itself a prerequisite for market liquidity. For instance, market makers often need access to credit to maintain a large enough inventory of the securities in which they are dealing, because they do not have enough equity. Hence, the more abundant and cheaper is the market makers' funding liquidity, the greater is the liquidity of security markets, in the sense that **(p.10)** investors will be able to trade securities in larger amounts at better prices. By the same token, a credit crunch—a drop in funding liquidity—may impair the liquidity of security markets, by forcing market makers to widen their bid-ask spreads and reduce their order size maximum. Chapter 9 analyzes the effect of funding liquidity on price formation in security markets.

Third, the causal relations can also be reversed. That is, market liquidity may be a prerequisite for funding liquidity, because security traders often must post margins (i.e., collateral in the form of cash or securities) to cover the risk that they may not be able to pay for the securities they are buying or deliver those they are selling. This "counterparty risk" can arise if the trader borrows in order to buy the security, or sells short (i.e., without owning it yet). However, margin requirements depend in part on the securities' expected market liquidity: they are typically lower for securities that are expected to be more liquid and less volatile. Hence more liquid markets enable traders to fund their leveraged purchases or short sales more cheaply. This creates a feedback from market liquidity to funding liquidity.

This reciprocal feedback between market and funding liquidity becomes particularly important in times of crisis, when it can lead to liquidity spirals, with market liquidity suddenly drying up for many securities at once, as shown by Brunnermeier and Pedersen (2009). It is important to realize that, however deep and strong the relationship between them, market liquidity and funding liquidity are different notions, and are accordingly affected by different policy actions: market liquidity by security market regulation and funding liquidity by banking regulation, specifically by the role of the central bank as "lender of last resort." This brings us to a third possible meaning of liquidity, that is, the monetary dimension.

0.4.3 Monetary Liquidity

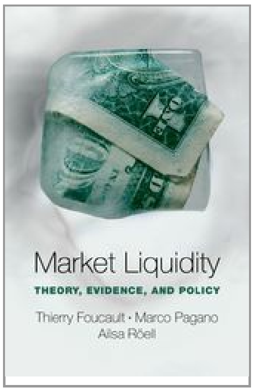
If we rank assets by market liquidity, the most liquid is obviously cash, which by definition is universally accepted in exchange for goods at very stable terms (except in times of hyperinflation). At intermediate levels of liquidity are financial securities such as bonds and stocks, while at the opposite extreme is real estate, which is so heterogeneous that sale typically requires considerable time and effort, or else a large price concession in exchange for quick sale.

This explains why in practice liquidity is often identified with money itself, whether defined as the cash held by households and firms and bank reserves ("monetary base"), or as broader monetary aggregates that also include bank deposits of various types (M1, M2, or M3). Especially in macroeconomics, this notion of "monetary liquidity" is prevalent. This notion of liquidity also **(p.11)** bears some relationship to the previous two: expansion of the money supply by the central bank (say, via open market purchases of bonds or "quantitative easing") increases the supply of funds to banks and thus tends to increase funding liquidity, and with it market liquidity, as we have seen. By the same token, a monetary contraction can be expected to reduce both funding and market liquidity. There is a vast literature that analyzes and documents the link between monetary policy and funding liquidity (see Bernanke and Gertler, 1995). Expansionary monetary policy may increase banks' loan supply either directly (bank lending channel) or indirectly by improving borrowers' net worth and thereby their borrowing capacity (balance-sheet channel). Monetary policy has also been shown to affect the liquidity of securities markets: at times of crisis, monetary expansion is associated with greater liquidity in both stock and bond markets, and bond market liquidity is forecast by money flows to government bond funds (Chordia, Sarkar, and Subrahmanyam, 2005).

Of course, these relationships are neither mechanical nor stable over time, because banks and other financial intermediaries can generate different amounts of funding liquidity in the presence of the same level of money supply. And conversely, they may respond to an expansion of the monetary base by increasing their reserves with the central bank rather than by increasing their lending. **(p.12)**

Notes:

(1.) These numbers are drawn from Beber and Pagano (2013), who analyze daily closing bid and ask prices for 16,491 stocks listed on the exchanges of from 30 countries and present in the Datasteam data base.



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Trading Mechanics and Market Structure

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter discusses the organization of securities markets. A trading mechanism defines the “rules of the game” that market participants must follow: it determines the actions they can take, their information about other market participants' actions, and the protocol for matching buy and sell orders. Trading mechanisms can essentially be viewed as variations of two basic structures: limit order markets and dealer markets. Section 1.2 describes how each mechanism operates, illustrates market structures that combine elements of both, and shows that each prototypical mechanism itself can vary in important ways, such as the degree of transparency and the frequency of trades. Section 1.3 previews some empirical studies that compare limit order and dealer markets or investigate markets with different degrees of transparency. Section 1.4 discusses the evolution of market structure. The final sections provide suggestions for further reading

and exercises.

Keywords: securities markets, trading mechanisms, market structure, limit order markets, dealer markets

Learning Objectives:

- • How securities markets are organized
- • Who sets the rules
- • How the organization of securities markets has changed recently

1.1. Introduction

Securities markets are mechanisms for bringing buyers and sellers together and enabling them to trade. Trading may be prompted by various factors: the need to mitigate risks (hedging), the desire to exploit superior information (speculation), or the urge to rebalance one's portfolio (liquidity shocks). In standard treatments of asset pricing, such as the capital asset pricing model (CAPM), the trading mechanism is not laid out explicitly, on the assumption that it does not matter for securities prices. Yet in reality there is a wide variety of trading mechanisms, and market participants pay close attention to their design. Changes in trading rules are often hotly debated, and market organizers carefully fine-tune these rules to improve the competitiveness of their trading platform. This is because the trading rules affect the efficiency of markets as mechanisms to realize trading gains and discover asset values. They also affect the apportioning of gains among market participants, determining, for instance, the fraction of the gain that is captured by specialized intermediaries.

(p.16) A trading mechanism defines the "rules of the game" that market participants must follow: it determines the actions they can take (e.g., the kinds of orders they can place), their information about other market participants' actions (e.g., whether they observe quotes or orders), and the protocol for matching buy and sell orders (e.g., whether orders are executed at a common price or not). As the possible rules can be put together in a virtually boundless number of combinations, real-world market structures feature great diversity and are constantly evolving, so attempting a complete classification is hopeless. It is more fruitful to focus on two prototype trading mechanisms, namely the limit order market (or auction market) and the dealer market.[1] In fact, all trading mechanisms can be viewed as variations of these two basic structures. In limit order markets the final investors interact directly; their bids and offers are consolidated in a limit order book (LOB) according to price priority, so that higher bids and cheaper offers are more likely to be executed. By contrast, in dealer markets final investors can only trade at the bid and ask quotes posted by specialized intermediaries, called "dealers" or "market makers," and these quotes are not consolidated to enforce price priority. Section 1.2 describes how each mechanism operates, illustrates market structures that combine elements of both, and shows that each prototypical mechanism itself can vary in important ways, such as the degree of transparency and the frequency of trades.

These differences in market design are not inconsequential, and much of this book distills the results of the large body of research on how market design affects trading costs and price discovery. To provide an idea of the impact that the design of security markets can have on their performance, section 1.3 briefly previews some empirical studies that compare limit order and dealer markets or investigate markets with different degrees of transparency.

An obvious question is why in practice markets feature different trading mechanisms. As section 1.4 explains, trading rules are determined by interplay between regulators, intermediaries, issuers, investors, and the managers of trading platforms. The balance between these stakeholders—and hence the actual trading rules—largely depends on the governance and ownership of the platform. For instance, the platform may be managed for profit or not, and the ownership shares of the various stakeholders (intermediaries, issuers, investors) may vary considerably between platforms and over time. And the design of a trading platform must also take into account the possible threat from competing platforms—a concern thatinrecent decades has become more pressing due to a combination of capital market liberalization, changes in security regulation, and technological advances. In particular, digital and communication technology **(p.17)** has radically transformed the trading process, sparking an increasingly lively debate on the impact of new trading technologies on market liquidity, price volatility, and economic efficiency.

1.2. Limit Order Markets and Dealer Markets

Limit order or auction markets are centralized trading mechanisms in which potential participants can show their interest in trading by submitting orders, which then are matched directly by trading platforms. Typical examples are such electronic trading platforms for equities as BATS in the United States or Chi-X in Europe. At the opposite extreme, all trades in dealer markets are intermediated by professional intermediaries that quote ask prices, at which the public can buy securities from them, and bid prices, at which the public can sell to them. That is, buyers and sellers do not trade directly with each other. A good example is the corporate bond market in the United States and in Europe. These are typically over-the-counter (OTC) markets, where brokers must shop around dealers to get the best prices and dealers have no obligation to post continuous two-way quotes.

Actually, many securities markets are hybrid, combining features of these two basic mechanisms. For instance, the European trading platform of NYSE-Euronext is organized as a limit order market, but for some stocks it allows designated dealers to post quotes directly. In other exchanges, auction and dealer mechanisms are used side by side for different securities or for different sets of investors. For instance, the London Stock Exchange (LSE) has different trading mechanisms according to a stock’s trading volume and market capitalization: a hybrid trading platform (SETS) that combines a limit order market with market making for the liquid stocks, and a dealer market (SEAQ) for fixed income securities and less liquid stocks.

Both market structures have a price-setting mechanism that balances the demand and supply for a security. In this sense, they differ from electronic crossing networks (e.g., POSIT), which accumulate buy and sell orders and cross them periodically at the price observed on some other platform (e.g., the NYSE).

1.2.1 Limit Order Markets

As we have seen, a defining feature of the limit order market is that buy and sell orders from final investors are matched directly in a single marketplace, which can be either the floor of an exchange or a virtual trading venue run by **(p.18)**

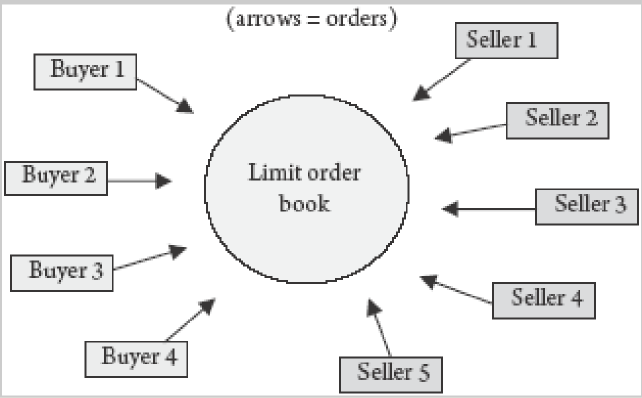


Figure 1.1. Limit order market

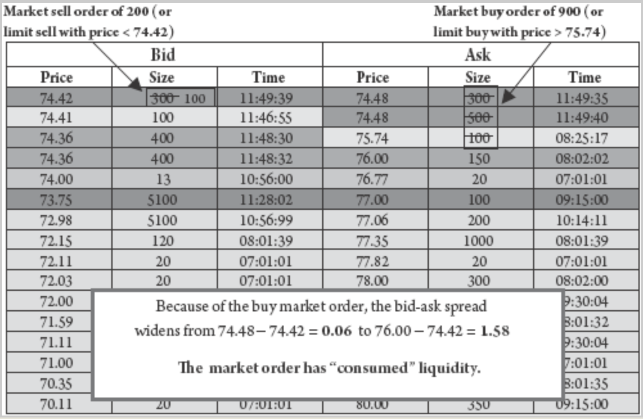
a computer. Orders go into an LOB, which determines the priority with which they will be matched with offsetting orders, according to the rules of the market and the characteristics of the orders themselves. In call (or batch) markets, incoming orders are stored in the LOB and then matched at discrete intervals, such as once per day. In continuous markets, they are matched immediately with orders already present on the LOB, if possible; otherwise they are stored in the LOB to await future execution.

Continuous Limit Order Markets

When submitting orders to a continuous limit order market, investors can design them differently depending on their trading needs. The most basic choice, which determines both speed and price of execution, is between limit and market orders. A buy limit order specifies the maximum price at which the trader is prepared to buy a stated amount of the security; similarly, a sell limit order specifies the minimum price the seller will accept for a given amount. A market order only specifies an amount to buy or sell, not the price: it will therefore be executed at whatever price it can fetch on the market.

Limit orders may not find a counterpart with which they can be matched at the specified price. Market orders, by contrast, are filled immediately if there is any outstanding limit order on the other side of the market. Thus, one difference between limit and market orders is that limit orders do not guarantee immediate execution—indeed, they may never be executed at all—whereas market orders are executed immediately upon submission.

The LOB shown in figure 1.2 illustrates the mechanics of trading in a continuous limit order market. The LOB is a snapshot at a given point in time of all the limit orders awaiting execution. In the LOB, we see the limit orders on the bid and ask sides: buy limit orders (bids) are arranged in decreasing order of price, sell limit orders (asks) in increasing order. The LOB also shows size and time, that is, the number of shares specified and the time the order was **(p.19)**



Bid			Ask		
Price	Size	Time	Price	Size	Time
74.42	~~300~~ 100	11:49:39	74.48	~~300~~	11:49:35
74.41	100	11:46:55	74.48	~~500~~	11:49:40
74.36	400	11:48:30	75.74	~~100~~	08:25:17
74.36	400	11:48:32	76.00	150	08:02:02
74.00	13	10:56:00	76.77	20	07:01:01
73.75	5100	11:28:02	77.00	100	09:15:00
72.98	5100	10:56:99	77.06	200	10:14:11
72.15	120	08:01:39	77.35	1000	08:01:39
72.11	20	07:01:01	77.82	20	07:01:01
72.03	20	07:01:01	78.00	300	08:02:00
72.00					9:30:04
71.59					8:01:32
71.11					9:30:04
71.00					7:01:01
70.35					8:01:35
70.11	20	07:01:01	80.00	350	09:15:00

Figure 1.2. Example of limit order book (LOB)

entered in the book. Depending on the market's trading rules, only a subset of the orders present in the LOB may be visible to market participants. As we shall see later, this is one dimension of market transparency.

Consider an investor who wants to buy nine hundred shares. He has two options. One is to place a buy market order for this amount. In this case, the order is executed immediately against the best limit orders to sell (on the ask side): it will first fill the two limit orders placed at the offer price of $74.48 for eight hundred shares, and then be executed for the remaining hunded shares against the limit order at $75.74, so that its average execution price is $74.62. Note that the order of priority in which limit orders on the book are executed depends on their price: aggressively priced orders are filled before less competitive ones. In other words, execution obeys a price priority rule. If two limit orders have the same price, they are filled according to secondary priority rules such as time of submission or pro-rata allocation (fractional execution proportional to limit order size).

The second option for the investor is to place a buy limit order for nine hundred shares. If he specifies a limit price lower than $74.48, the order is entered in the LOB on the bid side and stored for future execution. The level of the chosen bid price determines the likelihood and speed of execution, as more aggressively priced buy orders are executed first according to price priority.

If the investor instead specifies a limit price equal to or higher than $74.48—that is, if he matches or crosses the best price on the ask side of the limit book—then the order is *marketable*: it can be executed at once, at least partially, **(p.20)** against stored sell limit orders, in this example those at $74.48. If the order specifies a limit price of $74.50, the remaining hundred shares will appear on the bid side of the LOB as a buy limit order at $74.50. Significantly, the transaction price ($74.48) is determined by existing prices on the LOB, not by the price of the incoming marketable limit order.

The treatment of sellers is analogous. For instance, an investor who wants to sell two hundred shares immediately can either place a market sell order for two hundred shares or a marketable limit order with a price of $74.42 or less. If he is more patient, he can improve his execution by placing a sell limit order for two hundred shares at a price above $74.42, on the ask side of the market. But in this case he runs the risk of non-execution.

Hence, the choice between a market and a limit order involves a trade-off between immediate execution at current market prices and a more favorable transaction price at the cost of delayed and uncertain execution. This trade-off is studied in detail in Chapter 6.

Figure 1.2 can also be used to illustrate the notion of illiquidity, which is discussed in Chapter 2. As a thought experiment, consider a “round-trip transaction,” that is, a buy market order followed by an equal-size sell market order. If the market were perfectly liquid, the cost of this round-trip transaction would be zero. Instead, the figure shows that it has a positive cost that increases with its size. If the order size is smaller than three hundred, one buys at $74.48 and resells at $74.42, so that the round-trip cost is $0.06 (i.e., six cents). This cost—the difference between the best bid and the best ask price on the market—is called the “quoted bid-ask spread” and is often used as a measure of illiquidity.

For larger orders, one can compute a similar measure of illiquidity by comparing the average price paid by a buyer placing a large market order and the average price received by a seller for an equally large order. The buy price rises with order size, because the buyer has to “walk up” the schedule of sell limit orders to fill his own buy order. Symmetrically, the sell price is decreasing with the size of the order, as the seller has to “walk down” the schedule of buy limit orders. Thus larger orders are associated with a greater difference between the average execution price for buy and sell market orders—the “weighted average bid-ask spread.”[2] A market in which investors can trade large quantities without substantially moving the price—that is, where the weighted average bid-ask **(p.21)** spread does not increase much with trade size—is said to be “deep.” Therefore, *market depth* is inversely related to the weighted average spread for large trade size. The notion of depth will be made more precise in Chapter 4.

The LOB evolves in real time as market and limit orders are submitted and earlier limit orders are cancelled. For instance, in our example, the submission of a buy market order for nine hundred shares depletes the LOB on the ask side and so widens the bid-ask spread from 0.06 to 1.58. By contrast, if the buyer submits a limit order at $74.45, the bid-ask spread narrows to 0.03. Since market orders widen the spread, they are viewed as consuming liquidity, and traders submitting these orders are called “liquidity demanders” (or liquidity takers). In contrast, those submitting limit orders are called “liquidity suppliers” (or liquidity makers), since aggressive limit orders replenish the LOB.

Box 1.1 Other Types of Order

Limit and market orders are by far the most common kinds of order. But trading platforms often also allow for more complex orders. A *stop order* is an instruction to buy (or sell) only once the price has risen to (fallen to, respectively) a certain level. A stop sell order can be used to limit one's losses on holding a stock if its price nosedives. Moreover, traders can set conditions on cancellation with their orders: *good-until-cancel* orders are valid until they are cancelled, while *good-until* orders are valid until a specified date and *immediate-or-cancel* (or *fill-or-kill*) orders are valid only at the moment they reach the market. Finally, *hidden orders* are limit orders that are stored in the LOB but not displayed to market participants. These orders will be executed in the same way as regular limit orders, but they usually lose time priority against limit orders that are displayed at the same price. A variant of the hidden limit order is the so-called iceberg order, for which a fraction of the actual size is shown to other market participants along with the price. As the order is executed, the hidden size becomes gradually evident to market participants.

Call Limit Order Markets

So far we have considered limit order markets with continuous matching. Another arrangement matches orders at discrete points in time—say, once a day. In this case the limit order market is known as a call (or batch) auction. **(p.22)** Before the call auction is held, market and limit orders gradually accumulate in the LOB unless cancelled. A formal analysis of price formation in call markets is provided in Chapter 4.

In a call auction, the determination of the price at which orders are executed differs from that of the continuous limit order market described in section 1.2.1. In this case, all executable orders are cleared at the same price. For this reason, the call auction is sometimes called a single or uniform price auction.

More precisely, at the time of the call auction, all the buy orders in hand are sorted in decreasing order of limit price, with buy market orders treated as at the highest possible price. This determines the cumulative quantity that traders are prepared to buy at each possible price. Symmetrically, the sell orders are sorted by increasing limit price, with sell market orders treated as at the lowest possible price. This determines the cumulative quantity that would be sold at each possible price. The resulting demand and supply functions are the two stepwise schedules shown in figure 1.3.

The price set in the auction—the market-clearing or equilibrium price—is determined by the point where these two stepwise schedules intersect. At this price, orders from all buyers with a bid higher than the clearing price and all sellers with a price below that clearing price are fully executed. Limit orders with a price just equal to the clearing price (the marginal traders) may be partially executed. For instance, in figure 1.3, the total demand at the clearing price exceeds the total supply, so the marginal buyer will get only partial execution. Buy limit orders below the clearing price or sell orders above it are not filled. It is easy to see that the clearing price maximizes the (voluntary) trading volume, as it leaves no trading opportunity unexploited.

In the past, many exchanges in continental Europe were essentially call markets. Walras (1874) was inspired by the mechanism of the Paris Bourse

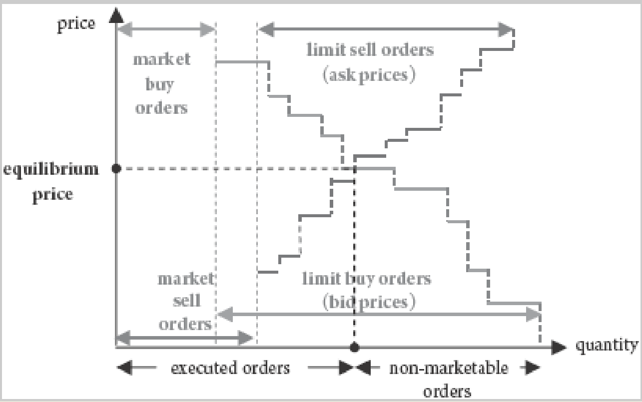


Figure 1.3. Call auction

(p.23)

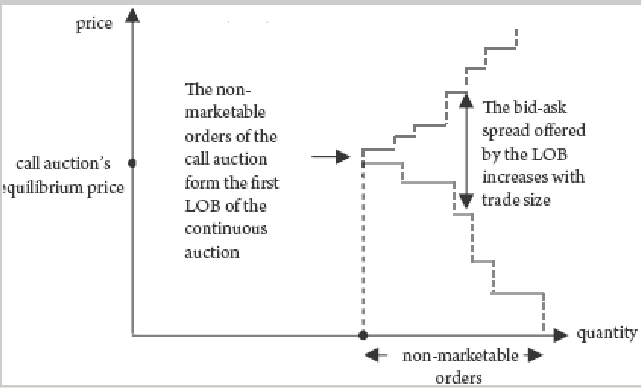


Figure 1.4. Forming the initial limit order book of the trading day

call auction when he formalized the process by which supply and demand are balanced in competitive markets. Today, with the advent of computerized trading, the call mechanism serves mainly to determine the opening price before the start of continuous limit order trading on trading platforms such as the NYSE-Euronext, LSE, Italian Stock Exchange, and Madrid Stock Exchange. In this case, the limit orders unfilled at the opening call auction form the initial LOB for the continuous session. Figure 1.4 shows this initial LOB in the case of the call auction displayed in figure 1.3 and illustrates that the bid-ask spread on the LOB increases with trade size, as noted. Some markets also use the call auction to set the closing price at the end of the trading day.

Call auctions are used as the only trading mechanism for stocks that are traded infrequently. In this way, market organizers make sure that there is sufficient interest on both sides of the market. They increase the likelihood of finding a counterpart for each side while reducing the risk that the clearing price will be distorted by a temporary imbalance between supply and demand. For instance, on the LSE, SETSqx is a trading platform that runs four electronic auctions a day (alongside a dealer market) for securities that are less liquid than those traded on SETS.

1.2.2 Dealer Markets

In dealer markets, the final investors do not trade directly with each other, but must contact a dealer, find out his price, and trade at this price, or else try another dealer. So in a dealer market there is a sharp distinction between liquidity suppliers (the dealers) and liquidity demanders (final investors), whereas in a limit order market each participant chooses whether to provide or to demand liquidity. **(p.24)**

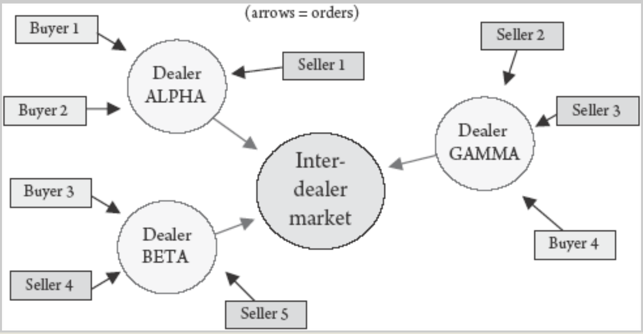


Figure 1.5. Dealer market

2	ALPHA ZETA 326–329 GAMMA EPSILON IOTA			3
Market Maker Identity Code	Bid Price	Offer Price	Quote Size	Time of Latest Quote Update
ALPHA	326	330	75 × 75	08:53
BETA	324	330	75 × 75	09:14
GAMMA	325	329	75 × 75	09:16
DELTA	323	332	75 × 75	08:53
EPSILON	325	329	25 × 25	09:36
ZETA	326	330	75 × 75	11:30
ETA	325	330	75 × 75	09:45
THETA	325	330	75 × 75	09:23
IOTA	324	329	75 × 75	10:27
KAPPA	323	330	75 × 75	09:45
LAMBDA	325	330	75 × 75	08:53

Figure 1.6. Dealers bid and ask quotes

Figures 1.5 and 1.6 illustrate the trading process in a dealer market. As an example, suppose that Seller 4 wants to sell sixty shares and that he first contacts dealer Beta. As shown in figure 1.6, Beta is willing to buy at $324 and sell at $330. Seller 4 can then decide either to sell at $324 or to seek another dealer. In the first case, Beta fills Seller 4’s order by buying the security and adding it to his inventory. This exposes Beta to the risk of a sudden fall in the price of the security, and hence a loss on the value of his inventories. To avoid this “inventory risk,” Beta can either rebalance his position by trading with a customer who wants to buy the security (for instance, Buyer 3 in the figure) or he can contact other dealers to sell them part or all of his position. As we will **(p.25)** see in Chapter 3, the management of inventory risk is a major determinant of bid and ask prices.

Thus, we can distinguish two different segments in dealer markets: the retail segment, in which dealers serve final investors, and a wholesale segment (the “interdealer market”), in which dealers trade with each other to share inventory risk. Examples of interdealer markets are such trading platforms as EBS and Reuters D2000/3000 (in the foreign exchange market). The volume of trade on the interdealer market is typically much larger than on the retail market, as each trade with a given client trickles down to other dealers until it is passed on to final investors on the opposite side of the market. For instance, a 2001 survey of the Bank of International Settlements found that interdealer trading accounts for about 80 percent of all foreign exchange market volume.

In a dealer market, unlike a limit order market, there is no enforcement of price priority: in our example, Seller 4 trades with Beta even though he could obtain a better price from Alpha. This is because quotes are not necessarily displayed to final investors, who must find the best price by contacting dealers by phone or messaging systems. This search is costly; it takes time and effort.

However, when information on quotes is publicly available, market participants can identify the dealers who post the best bid and ask prices. In some dealer markets, such as Nasdaq and the SEAQ trading platform of the LSE, the dealers’ quotes are displayed on screens providing real-time information similar to that in figure 1.6. In this example, no single dealer quotes a bid-ask spread of less than 4, but the spread resulting from the consolidation of the quotes (sometimes called the “inside spread” or “market touch” in the United Kingdom) is 329 − 326 = 3, as is shown at the top of the panel (which also tells us which, and how many, market makers quote the best price on each side of the market). Thus, the market as a whole offers more liquidity than any individual dealer.

Many dealer markets, however, offer far less detail on dealers’ quotes. For instance, no real-time information is available in OTC markets such as the U.S. corporate bond market. In currency markets, the Reuters and Bloomberg screens do give information on quotes, but it is only indicative; the quotes do not commit dealers to actually trade at those prices.

Dealers’ quotes are typically valid only for a limited number of shares. So a large order may be executed by splitting it among several dealers. Suppose that a seller wishes to sell three hundred shares given the dealers’ quotes in figure 1.6. He can execute this order by selling seventy-five shares each to dealers Alpha and Zeta, who post the best bid price, and then another seventy-five each to Gamma and Lambda at the next best bid price. Effectively, the investor is walking down the demand curve resulting from the aggregation of dealers’ bid quotes. Similarly, a buyer with a large order will walk up the aggregate supply curve resulting from the dealers’ ask quotes. These aggregate demand **(p.26)**

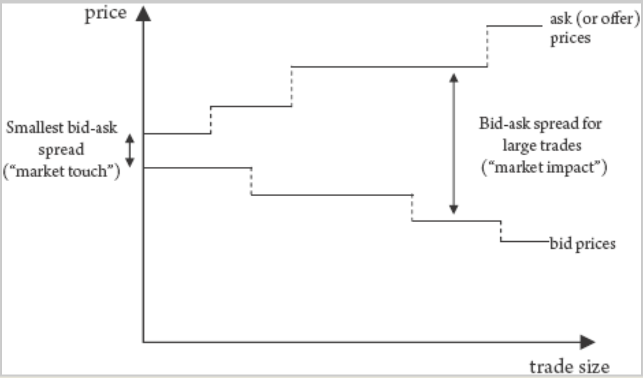


Figure 1.7. Dealer market quotes for various trade sizes

and supply curves are shown in figure 1.7. Therefore, as in a limit order market, in a dealer market one can also define a weighted-average bid-ask spread that is also increasing in trade size.

Unlike limit order markets, dealer markets often enable traders to bargain over price and quantity. For instance, instead of searching for a better price, Seller 4 in our example could ask Beta for a better price than 324. If Beta agrees, then Seller 4 gets what is called *a price improvement.* These are common in some dealer markets (e.g., the LSE) and result in trades at prices within the quoted bid-ask spread. Moreover, by design, dealer markets allow dealers to establish long-term relationships with their clients. They may then offer different prices to different clients. For instance, in the U.S. market for municipal bonds ("munis"), dealers offer better prices to institutional than to retail investors, because institutions trade larger amounts, trade more frequently, and have greater bargaining power.

Moreover, bargaining may also speed up execution. For instance, an investor with a large order can ask the dealer to quote the price at which he is willing to take the entire order. Typically, this may be worse than the price the trader would obtain by splitting the order among several dealers over time, but it guarantees immediate execution of the full order. This is important to some traders, such as arbitrageurs who must take long and short positions simultaneously in different markets. Thus speed of execution constitutes an additional dimension of market liquidity, sometimes no less important than the cost of trading itself (i.e., the bid-ask spread).[3]

(p.27) Sometimes dealers enter *preferencing* arrangements with brokers. In this case, a broker commits to route his orders to a specific dealer, and the dealer commits to execute them at the best quoted price in the market or even to improve systematically upon these prices. A related practice is *payment for order flow*: dealers offering rebates to brokers who route specific categories of orders (e.g., those below a given size) to them.

In some markets, such as the SEAQ trading platform of the LSE, dealers undertake special market-making obligations. For instance, they commit to continuous firm bid and ask prices for up to a specified trade size. In this case, they must execute all incoming orders up to the threshold size at the price quoted. Dealers with such obligations are called “designated market makers,” although often this term refers to any type of dealer.

Bear in mind that dealers differ from brokers. Brokers (such as Charles Schwab in the United States) only execute buy or sell orders of final investors (indifferently in dealer or limit order markets), but they do not act as counterparties for these orders. Dealers, instead, are the counterparties to final investors and so take inventory risk. Some securities firms (e.g., Goldman Sachs or Merrill Lynch) offer both services to their clients, and are accordingly known as broker-dealers.

A feature that brokers and dealers have in common is that both help final investors to carry out their trades. For this reason, brokers and dealers are often collectively referred to as the “sell side” of the securities industry, whereas final investors (households, institutional investors, firms, and government) are called the “buy side” (since they buy trading services from the sell side).[4] This distinction is important since the two sides often have opposing views on how trading should be organized: as we shall see, a change in trading organization often affects the distribution of trading gains between investors (the buy side) and intermediaries (the sell side).

1.2.3 Hybrid Markets

Many actual securities markets are hybrids comprising both a limit order platform and a dealer segment, or having a design that mixes features of the two types of market.

(p.28) For instance, the NYSE has a mix of three different trading mechanisms that operate simultaneously for each stock:

1. An open-outcry market where floor brokers trading on behalf of other investors or on their own account bargain bilaterally.[5]
2. A dealer market with one market maker, the “specialist,” for each stock; there were seven specialist firms in 2009.
3. An electronic LOB for each stock that allows investors to bypass the specialist and floor traders.

Coordination of the prices in these trading mechanisms is ensured by the NYSE’s priority rules: when the specialist receives a market order, he must execute it against the limit orders in the book or else improve upon their prices. Other hybrid markets are traditional quote-driven markets—such as Nasdaq and the LSE—that have recently added a limit order trading facility so that orders can be routed to the LOB rather than to dealers.

A different form of hybridization is that of MTS, an interdealer trading platform for European government bonds in which only some dealers (the “primary dealers”) can post limit orders, and the other dealers can only submit market orders. As the platform uses price and time priority to execute limit orders, it is a limit order market. But, since only primary dealers can post limit orders, MTS may also be viewed as a dealer market. Moreover, for each bond some primary dealers serve as designated market makers;

that is, they are obliged to post firm bid and ask prices continuously, for at least five hours per day, for a minimum quantity and with a maximum bid-ask spread that depends on the bond's maturity and liquidity.

1.2.4 Market Transparency

A market's degree of transparency is determined by the amount of trading information available to participants. This information matters to traders, as it enables them to sharpen their estimates of securities' values and devise better trading strategies. For instance, as section 1.2.2 explained, in a dealer market an investor can generally get better terms if he observes all dealers' quotes, and can thus save on the cost of searching for the best price. Here we briefly discuss this **(p.29)** important dimension of market structure, leaving a more in-depth analysis to Chapter 8.

Transparency varies considerably from market to market, regardless of whether they are limit order or dealer markets. Indeed, the choice of transparency is often very controversial as it affects how trading gains are distributed between the sell side and the buy side. Intuitively, the demand for brokerage services and dealers' market power are greater in opaque markets, since it is harder for final investors to identify all trading opportunities. But when platforms do provide a good deal of information on the trading process, they may charge significant fees for it. Thus, transparency depends not only on the availability but also the cost of information.

In general, electronic limit order markets tend to be very transparent. Data on the best orders and their limit prices are displayed in real time, and—as trades are executed directly by the system—realized transaction prices and quantities can be published immediately. Even here, however, transparency is a matter of degree. The market may display only the best two limit prices, or the best five or ten prices in the LOB; the market may also show the quantities and the identities of the brokers placing the orders, or the times at which the orders are submitted.[6] Moreover, even in such a market, traders may have the option of not entirely disclosing their trading intentions but may post hidden orders, as section 1.2.1 explains.

Transparency can also vary across dealer markets, depending on whether quotes are displayed centrally through a single screen and whether they are firm or merely indicative. In the latter case, customers still have to contact each dealer directly to verify the actual prices at which a trade will be executed. Since dealer markets are fragmented (deals being struck with individual dealers), it is not easy to ensure that the best quotes are centrally displayed in real time. For example, some dealers may quote prices only on their own proprietary systems. It is even harder to ensure that completed trades and their prices are published promptly, as this requires dealers to report their trades equally promptly to a central market authority. In practice, dealers often oppose prompt publication of trades or try to circumvent rules requiring such publication.

The lowest degree of transparency is found in so-called dark pools of liquidity, trading platforms that are not accessible to all comers but only to financial **(p.30)** institutions that wish to trade large blocks of securities anonymously. These institutions are attracted to dark pools precisely because they can avoid disclosing the size of trades and traders' identities, which attenuates and defers the price change caused by large-volume sales or purchases. Some dark pools are platforms created by independent companies; others are operated by brokers to allow their clients to trade anonymously; still others are special segments created by public exchanges to grant their clients the benefits of anonymity and opacity. When dark pools are available, the portion of the order flow that they intermediate is not visible to those who trade on the open market, whether it be a limit order or a dealer market. Hence their opacity also makes the market more fragmented: in fact, market opacity and fragmentation are closely connected.

The degree of transparency also varies by type of security. The market for major listed stocks is traditionally the most transparent. In the United States, since the 1975 amendments to the Securities Exchange Act, consolidated market information has been made available (at a cost) through designated securities information processors (SIPs) that aggregate both quote information and trade reports. Over the years, the information provided has been expanded considerably. At the other extreme are markets in thinly traded securities such as municipal bonds, which are traded over-the-counter, typically by broker-dealers who fill most of their customers' orders in a principal capacity and trade among themselves in an interdealer market to manage their inventories. No firm prices are quoted, and the prices at which trades are executed are made known only for the most frequently traded issues, and even then with a one-day lag. In between these two extremes, of course, there are many intermediate degrees of transparency.

Computerized trading facilitates data storage and dissemination, but it does not necessarily entail greater transparency in the trading process. Actually, the physical interactions of traders in open-outcry or "floor" markets permit the transmission of a host of informal cues that escape even state-of-the-art electronic systems. If traders are physically present in the same room, their every word and gesture are visible, yielding insights into their trading strategies and the urgency of their trading intentions.[7] Moreover, electronic communication technologies have facilitated the dispersal of trading across multiple venues, making it hard to piece together an accurate view of overall market conditions.

(p.31) 1.3. Does Market Structure Matter?

The two main roles of a securities market are to provide trading services for investors who wish to alter their portfolios, and to determine prices that can guide the allocation of capital by investors and firms. That is, a market is efficient if it enables investors to trade quickly and cheaply (i.e., if it is liquid) and if it incorporates new information quickly and accurately into prices. Trading rules affect market efficiency on both accounts: indeed, much of this book is devoted to explaining why this is so, and distilling the empirical findings of a vast literature. This section previews a few empirical studies on how market design affects liquidity and price discovery.

Some studies compare trading costs (bid-ask spreads) in limit order markets and dealer markets for matched samples of stocks (i.e., stocks with similar characteristics). Typically they find that trading costs are higher in dealer markets, especially for small orders. For instance, Huang and Stoll (1996) report that, by several measures, trading costs for a sample of Nasdaq stocks are twice as high as for a matched sample of NYSE stocks. One possible explanation is that concentrating trading in a single marketplace improves liquidity. Chapter 7 explores why the consolidation of trading can increase market liquidity.

Changes in trading rules that foster competition among liquidity providers also result in a more liquid market. In early 1997, the Securities and Exchange Commission (SEC) introduced rules that exposed Nasdaq market-makers to competition from the general public. The Limit Order Display Rule forced dealers to execute or display any customers' limit orders better than their own. This regulatory change (together with the "quote rule" that required dealers trading in multiple venues to make their best quotes available to the public) increased competitive pressures on Nasdaq and led to an immediate and substantial reduction in Nasdaq trading costs (Barclay, Christie, Harris, Kandel, and Schultz 1999).

Changes in market transparency also affect liquidity. For instance, Boehmer, Saar, and Liu (2005) analyze how measures of market liquidity changed when the NYSE started releasing information on limit orders in 2002. They document that this increase in transparency was followed by a drop in the price impact of market orders—a measure of illiquidity alternative to the weighted average bid-ask spread. Similarly, Edwards, Harris, and Piwowar (2007) examine an increase in the transparency of the U.S. corporate bond market. Since July 2002, the completed transaction prices of corporate bonds have been shown to market participants, albeit with a fifteen minute delay. Since this change, bid-ask spreads have declined significantly.

Market structure also affects the quality of price discovery. Green, Li, and Schuerhoff (2010) find that in the United States the prices of Treasury issues **(p.32)** react to macroeconomic news much faster than the prices of munis. As the market for Treasuries is more liquid and active, the authors suggest that this is responsible for the different speed of price discovery. In turn, munis' lack of liquidity largely stems from the lack of transparency for their market, as documented by Green, Hollifield, and Schuerhoff (2007). Many other studies have explored the impact of market transparency on liquidity and price discovery, as the detailed analysis in Chapter 8 shows.

By affecting liquidity and price discovery, market structure also affects the cost of capital. In Chapter 9, we will see that changes that increase liquidity raise stock prices, as investors require a lower rate of return to invest in more liquid stocks. Similarly, Chapter 10 shows how more efficient price discovery is conducive to better investment decisions by companies.

1.4. Evolution of Market Structure

We have seen that trading platforms are organized in a wide variety of ways. Why do actual trading mechanisms differ so much? This depends largely on who decides the trading rules, which is to say that it depends on the governance of trading platforms. In the past, such governance was partly the result of historical accident, which created different “initial conditions” in the various countries, as section 1.4.1 explains. In recent decades, however, the governance and organization of trading platforms, especially in equity markets, have tended to converge on one basic model. Section 1.4.2, argues that this convergence reflects the heightened competition in the securities industry, itself the product of the combined pressure of three forces: liberalization of international capital markets, securities regulation overhaul, and advances in information and communication technologies. But technological innovation has transformed security trading over and above its tendency to reinforce competition between trading platforms. Section 1.4.3 describes the enormous impact of technological progress on the automation of order generation, routing, and execution, and the increasingly lively debate on the pros and cons of this evolution.

1.4.1 Who Makes the Rules?

The design of trading rules is the outcome of the interplay between government regulation and self-regulation by the trading platforms, such as stock exchanges or trading networks. For instance, in the United States much of securities regulation is designed in broad outline by the SEC, then implemented and **(p.33)** specified in detail by the self-regulating organizations that govern the markets, such as the NYSE and Nasdaq.

The scope of self-regulation and the way it has been used have varied widely over time and across countries. From their inception, stock exchanges have differed very significantly in governance and organization. Many emerged from informal trading. For example, what is widely regarded as the first modern exchange, the Amsterdam Stock Exchange, emerged at the start of the seventeenth century, when trading in the transferable shares of the Dutch East India Company started immediately upon issue in 1602. Trading was at first concentrated outdoors around the Nieuwe Brug in central Amsterdam (in foul weather, the nearby St. Olof’s chapel was used), moving in 1611 to a purpose-built centralized merchants’ exchange, created to facilitate public trading not just in shares but also in other financial instruments, goods, and insurance (see Petram 2011, for a detailed description of seventeenth-century Dutch share trading). Similarly, the LSE developed from informal trading that took place around coffee shops in the city, where in 1698 John Casting first published lists of stock prices, entitled “The Course of the Exchange and Other Things.” Not until 1801 did the exchange turn into an official, regulated stock exchange. The precursor to the NYSE was created in 1792 by a group of stock brokers who signed the Buttonwood Agreement under a buttonwood tree on Wall Street.

Many continental European exchanges, however, were created at the initiative of government authorities and were generally regulated and managed by public agencies, such as local chambers of commerce. The Paris Stock Exchange was established by an order of the Royal Council of State in 1724. By the middle of the nineteenth century, trading was conducted by government-appointed *agents de change* who shouted out prices on the floor and could not trade for their own account: they were strictly brokers, not dealers. In several cases, the exchanges were founded for the express purpose of forming a market specifically for state-issued bonds, not shares or even corporate bonds. This was the case of the Vienna Stock Exchange, founded in 1771 under Empress Maria Theresa, and the Milan Stock Exchange, established in 1808 by a Napoleonic decree and used to trade only government bonds for its first half-century.

Thus, the constituencies that created stock exchanges were quite different, and the differences in governance were persistent. As late as the 1980s, most exchanges were still governed much as they had been at the turn of the twentieth century. These differences in governance shaped the rules of the exchanges, in an illustration of the more general principle that the structure of securities markets is affected by the interests of their controlling constituencies. Depending on the relative power and importance of domestic and foreign intermediaries, **(p.34)** institutional investors, and issuers in the share ownership structure, different exchanges pursue different policies. For instance, an exchange that is strictly controlled by a cartel of domestic intermediaries will be more inclined to impede remote access for foreign investors, and more generally will be reluctant to revise the trading system to the detriment of local intermediaries, as in the case of the LSE's protracted resistance to the introduction of an LOB and the NYSE members' opposition to ending floor trading.

More generally, the interests of intermediaries often conflict with those of issuers and investors: the former tend to favor trading rules that protect their rents at the expense of the latter. Chapters 7 and 8 show that such conflicts are a common feature in policy debates about market fragmentation and transparency.

By the same token, changes in the institutional form and governance of exchanges can impact the structure of their markets. Such changes have been frequent in recent years, as most exchanges have been transformed from essentially private clubs (such as the LSE) or semi-public entities (such as the Paris Bourse) into publicly held corporations, in which the largest stakes are usually held by major financial intermediaries such as banks, hedge funds, and brokerage houses. The LSE, Deutsche Börse, Euronext, Nasdaq, Hong Kong Stock Exchange, and Toronto Stock Exchange all went public in 2000, while several other exchanges in continental Europe had done so in the late 1990s. The NYSE followed suit in 2006.

1.4.2 Competition between Exchanges

As just noted, from sharply divergent initial conditions, stock exchanges have converged on a similar status as publicly listed companies, through either the privatization of the government-controlled exchanges or the demutualization of those owned and controlled by groups of intermediaries. As for-profit firms, their choices regarding market structure are naturally more and more geared to maximizing shareholders' profits.[8]

To understand how this profit orientation altered their choices, note that trading revenues account only for a fraction of the total profits of an exchange. Two other important sources of revenue are listing fees (paid by issuing firms **(p.35)**

Table 1.1 Breakdown of Stock Exchange Revenues (Percent)

Source of Revenue/Area	Americas	Europe	Asia
Listing fees	25	7	12
Trading fees	34	42	47
Other services	39	45	35

Source: Annual Report of the World Federation of Exchanges, 2005. “Other services” comprise the sale of market data, trading technologies, and clearing and settlement services.

who list their shares on the exchange), and the sale of market data, such as real-time quotes. For instance, in 2003 the sale of market data generated $386 million for U.S. equity markets, while the cost of their dissemination was just $38 million. The sale of market data is important for European exchanges as well: in 2005 the sale of market information accounted for 33 percent of the LSE’s annual revenue and 10 percent of Euronext’s. Moreover, in some cases (such as Deutsche Börse), exchanges also sell clearing and settlement services. Table 1.1 reports the breakdown of revenues among the various services sold by stock exchanges in different geographical areas (the revenue from the sale of data is included in “other services”).

The various services sold by exchanges are complements. Hence, decisions on market structure are made not only considering the impact on trading volume but also with an eye to their effect on listing and data sale revenues.[9] For instance, an exchange may be willing to charge low listing fees in order to attract many issuers and so earn large trading revenues. Or one that operates its own clearing and settlement system may charge low trading fees in order to capture trading flows and charge large clearing and settlement fees.

In recent years, competition has been increasing in all of these business lines. The change in governance has been driven largely by this evolution: the exchanges became publicly listed companies so as to gain the flexibility needed to compete with alternative venues, both at home and abroad. The intensification of competition between trading platforms is the result of three forces:

> (i) The removal of barriers to international capital flows has increased firms’ propensity for initial public offerings (IPOs) on foreign markets or cross-listing of their shares, and prompted increased **(p.36)** cross-border trading by investors. For instance, investors can currently trade many French blue-chip stocks not only on Euronext Paris but also on foreign markets: multilateral trading facilities (MTFs), such as Chi-X, or exchanges where the stocks are cross-listed (such as Deutsche Börse and the LSE).[10] In this environment, stock exchanges increasingly compete for listings and cross-listings. One sign of this is the recent decline in the share of global IPOs taking place in the United States (see Zingales, 2007).

(ii) Changes in the securities market regulations have also played a role. In the United States, the increased fragmentation resulting from the proliferation of electronic communication networks (ECNs) provided major impetus for creating a new regulatory framework, Regulation National Market System (Reg NMS), in 2005. A major goal of Reg NMS is to organize and facilitate competition between trading platforms. For instance, its order protection rules oblige trading platforms to re-route incoming market orders to the platform that posts the best price at the moment the order is received. Clearly, this rule heightens the competition for order flow among platforms, and among the liquidity suppliers that operate on each platform. Chapter 6 analyzes the effects of this rule in greater detail. In Europe, the Markets in Financial Instruments Directive (MiFID), which went into effect in 2007, also increased competition for order flow between trading platforms. In particular, it abolished the "concentration rule" by which member states could oblige investors to route their orders to the national market. As a consequence, it spurred the creation of many new platforms, such as Chi-X (launched in 2007), Project Turquoise, NASDAQ OMX Europe, Bats Europe (all launched in 2008) and NYSE Arca Europe (launched in 2009). All these platforms operate Europe-wide electronic LOBs.
(iii) Technological advances in information and communication technologies for securities trading have greatly facilitated entry into the market for the provision of trading services. The cost of setting up an electronic platform such as a LOB is now extremely low, and many platforms have come into being since the turn of the 1990s, known as ECNs or Alternative Trading Systems (ATSs) in the United States, and MTFs in Europe. Such platforms include Island (subsequently **(p.37)** renamed INET and bought by Nasdaq), Archipelago (now part of NYSE), Instinet and BATS. Moreover, with smart order-routing technologies (SORs) brokers can easily split orders across markets to get the best prices. Such technologies have lowered the cost of searching for the best price across trading platforms and so made it easier for exchanges to attract trading business by offering narrow bid-ask spreads and low fees.

Faced with such competition (whether actual or merely potential), the incumbent markets have reacted by merging to achieve economies of scale and capitalize on liquidity externalities (see Chapter 7). In 2000 the Paris Bourse merged with the Amsterdam Stock Exchange and the Brussels Stock Exchange (subsequently joined by the Lisbon Stock Exchange), to form Euronext. In 2007, Euronext in turn merged with the NYSE, enabling these exchanges to cut overhead costs and build up volume and liquidity.

Competition has also forced trading platforms to slash fees and pass some of the cost savings from mergers on to users. For instance, in 2003, the LSE announced that it would introduce a trading platform for Dutch stocks, EuroSETS. This was spurred by the Dutch brokerage community as a way to lower Euronext trading fees, and in fact Euronext slashed its fees by 50 percent in early 2004 (Foucault and Menkveld 2008).

Finally, competition prompted the incumbents to overhaul trading systems. For instance, until recently the NYSE still relied heavily on a seriously outdated floor-based system, totally unable to match the execution speed of electronic trading. That trading system gave an informational advantage to members with a seat on the floor; they thus had a vested interest in retaining floor trading. But as competition from ECNs eroded the NYSE's market share, the position became untenable. In 2006, the NYSE acquired a rival electronic order market, Archipelago, renamed it NYSE Arca, and made it the core of its trading system. By 2009, Arca was processing four-fifths of the NYSE's total trading volume. Floor trading is essentially being phased out.

1.4.3 Automation

As we have seen, technology has intensified competition between trading platforms. But the effects of information processing and communication technology on securities trading go well beyond this: over the last half-century technology has completely reshaped the trading process. In the past, securities were bought and sold on trading floors where brokers were in charge of matching the buy and sell orders they received from their clients. But securities exchanges progressively replaced or complemented their trading floors with computerized trading **(p.38)** systems. Today, most exchanges use electronic trading (mostly LOBs), and orders are now routed via high speed fiber optic lines to computers that match them according to predetermined trading rules. For instance, Jain (2005) finds that 101 of 120 countries examined have electronic trading, and 85 no longer have floor trading.

For the NYSE, this evolution began with the DOT system in 1976, which allowed electronic submission of market orders of up to one hundred shares. Upon reaching the trading floor, the electronic DOT orders were manually executed by the specialist. But the NYSE did not become a fully electronic exchange until 2006 with the introduction of the NYSE Hybrid system. By contrast, such exchanges as the Toronto Stock Exchange and the Paris Bourse went over to fully electronic LOB markets quite early (1977 and 1986, respectively). Their trading systems were the blueprint for others.

This evolution has a number of significant consequences. First, it increases the number of quotes and the amount of trade data that exchanges can make public and the speed at which they do so. In fact, trading platforms compete in the speed of disclosure of information on trades and quote updates. The time that elapses between, say, a quote update and the release of this information to market participants—called *latency*—is by now measured in milliseconds.

Second, professional traders (proprietary trading desks or hedge funds) have exploited the automated trading process to develop electronic trading strategies, a practice often referred to as *algorithmic trading* or *high-frequency trading*.[11] These strategies rely on sophisticated computer programs to generate, route, and execute orders, and their success often depends on extremely rapid transmission of orders (and of their cancellations) to the trading platforms and reception of timely data from them.[12] These diverse strategies can be classified in four broad types: (i) passive market making, (ii) arbitrage, (iii) directional trading, and (iv) order splitting (see SEC 2010). Algorithmic trades now account for a large fraction of trading volume, at least in equities markets.[13]

(p.39) Passive market making denotes the submission of non-marketable buy and sell limit orders. These orders are traditionally viewed as providing liquidity to market participants, very much as dealers do. Firms that do this (such as GETCO, Optiver, ATD, etc.) are therefore functionally comparable to market makers; they attempt to close out each trading day with no inventory exposure. However, in some respects they differ from traditional market makers: in their highly automated routines for the submission and revision of their quotes (based on inventory exposure, price signals from other markets, etc.); and in their not trading directly with clients but rather operating only on LOB markets, competing with other limit order traders.

Computerized trading also enables traders to exploit price discrepancies between related securities immediately. An obvious example is the case of a stock traded on two platforms, say the French stock Alcatel, which trades on both Chi-X and NYSE-Euronext. If the bid price for Alcatel on Chi-X exceeds the ask price on NYSE-Euronext, one can make a profit by buying it on NYSE-Euronext and selling it on Chi-X. Speed is of the essence, though, as the profit will be locked in only if the two transactions are virtually simultaneous. Such straightforward arbitrage opportunities are rare and fleeting, precisely because participants take advantage of them in a split second. More generally, algorithmic trading can enable one to detect and benefit from transient deviations between the prices of related securities (e.g., derivatives and their underlying securities or exchange rates that should be tied by triangular arbitrage) before other market participants.

Directional traders exploit information not yet reflected in prices. Their informational advantage can be very short-lived. For instance, consider a news release that leads investors to mark up the value of a firm. Investors who can place buy market orders almost instantaneously can profit by picking off “stale” sell limit orders whose prices do not yet reflect the new value.[14] Again, speed is of the essence, since investors who trade on such information can make a profit only by moving before traders on the other side cancel their limit orders. Directional traders also benefit from technologies that assist them in searching and processing information (e.g., scanning the Internet for certain keywords about stocks).

Lastly, algorithmic trading is also used by traders who need to accumulate or liquidate large positions. They often use smart order-routing technologies to **(p.40)** break up large orders optimally in space (between trading platforms) or time, so as to minimize total trading costs. Smart routers very quickly identify where the best quotes are posted and also optimize the placement of the order in continuous time, depending on changing market conditions (as described by Bertsimas, and Lo 1998 or Huberman, and Stanzl 2007).

This evolution has triggered hot debate on the impact of algorithmic trading on liquidity, price discovery, volatility, and risk. On the one hand, automation can reduce the costs borne by liquidity providers (see Chapter 3). For instance, dealers’ ability to quickly refresh their quotes on the basis of new information reduces their exposure to the risk of being picked off by informed traders. Automation also helps them to manage inventory risk by, say, taking a position in one market and hedging it almost instantaneously in another. Thus high-frequency market makers may enhance market liquidity. Hendershott, Jones, and Menkveld (2011) provide empirical support for this conjecture, showing that the rise of algorithmic trading on the NYSE coincided with a narrowing of effective bid-ask spreads, mainly because algorithmic trading seems to reduce the adverse selection component of the spread (that is, the compensation required by dealers for the risk of trading with better informed investors; see Chapter 3). Moreover, computerized trading should attenuate price inefficiencies and improve price discovery, since it helps dealers adjust quotes more quickly and makes arbitrageurs speedier in correcting mispricing. Hendershott and Riordan (2009) accordingly find that algorithmic trading incorporates more information into prices than human trading and that the quotes posted by algorithmic traders have more information content.

On the other hand, algorithmic trading has prompted concerns. First, it might be nothing but a way for the algorithmic traders to make profits at the expense of slower traders, including long-term retail and institutional investors (just as informed investors profit at the expense of the less informed; see Chapter 3). In this case, the technological investment in this activity would be socially useless, consuming resources without increasing overall gains from trade. This view was vividly expressed by the Nobel prize winner Paul Krugman in 2009:

> High-frequency trading probably degrades the stock market's function, because it's a kind of tax on investors who lack access to those superfast computers—which means that the money Goldman spends on those computers has a negative effect on national wealth. As the great Stanford economist Kenneth Arrow put it in 1973, speculation based on private information imposes a "double social loss:" it uses up resources and undermines markets."[15]

(p.41) A second concern is the impact of algorithmic trading on volatility and systemic risk. There are several grounds for this worry. Recent years have witnessed a number of "fat finger" mistakes—cases in which a trader mistakenly executed a much larger trade than intended. Also, the trading strategies of algorithmic traders tend to resemble one another, so algorithmic trades can be highly correlated. Simultaneous movements into and out of specific securities may lead to sharp variations in liquidity supply and demand, increasing price volatility.[16] Finally, as algorithmic traders can carry information swiftly from one asset class to another, they are likely to increase co-variation in stock returns and liquidity.

Lastly, ultra-fast trading increases the possibility of technological bugs with dramatic consequences. Peaks of algorithmic trading activity in reaction to the same event may strain the capacity of trading systems and cause severe market disruptions. For instance, algorithmic trading is often associated with huge numbers of order submissions that are subsequently cancelled. A flurry of order cancellations on a platform can delay the speed at which it reports trade information. In turn, this delay distorts the information sent to other participants and may create arbitrage opportunities for investors who are aware of the delay.[17] This suggests that some market participants may deliberately swamp platforms with messages (quotes and cancellations) solely in order to manipulate the tape (the quote and trade information reported to other participants), a stratagem known as “quote stuffing.” The potentially destabilizing role of algorithmic trading has recently been under the spotlight in connection with the “Flash Crash” on U.S. equities markets in May 2010 (see box 1.2).

Box 1.2 The Flash Crash of May 6, 2010

On May 6, 2010, the Dow Jones Industrial Average experienced its second-largest intra-day point swing ever recorded, falling by 9 percent in a matter of minutes before rebounding by 5 percent by the end of the day. The stocks making up the index lost about $1 trillion in market value in the half hour between 2:30 p.m. and 3:00 p.m., before bouncing back to a level not too far from the initial level. Some individual stocks underwent even more dramatic swings. For instance, Accenture shares fell by over 99 percent, from $40 to $0.01, while Sotheby’s shares rose three-thousand-fold, from $34 to $99,999.99. What was the cause of the flash crash? Specifically, can we consider it an instance of the destabilizing role of algorithmic trading? This question is still under debate; the full chain of events is not yet well understood.

A joint report by the Commodity Futures Trading Commission (CFTC) and the SEC (CFTC-SEC 2010) found that the crash was triggered by a single very large sell order in the E-mini S&P 500 index: a large mutual fund sold an unusually large number of futures; first the order exhausted available buyers, and then it precipitated additional sales by high-frequency traders, which spread the crash quickly from the futures market to the stock market.

Specifically, the initial order was to sell seventy-five thousand E-mini futures contracts on the S&P 500 index traded on the CME–the largest change in a trader’s position since January 2010.[18] The trader in charge of executing this order decided to split it up in order to attenuate the impact on prices. He adopted the “constant participation rate” strategy of splitting the order in such a way that each “child” order represents a fixed fraction of the total trading volume over a given period of time, say a minute. That is, it calls for larger trades if overall volume increases. This is a problem when the “parent” order is relatively large (like the seventy-five thousand futures contract order), as the sub-orders themselves can give a false impression of large volume, accelerating the main order’s execution.

In such a situation, a large sell order may trigger a very sharp price drop, as it quickly exhausts the market’s liquidity (by hitting lower and lower quotes), unless the trader makes his strategy contingent on the execution price received (trading less as the price impact increases). This snowball effect is apparently what triggered the flash crash (although the exact cause is still much discussed).

(p.42) Figure 1.8 (drawn from the CFTC-SEC report on the flash crash) shows the evolution of the depth of the LOBs of the E-mini futures contract on the S&P **(p.43)**

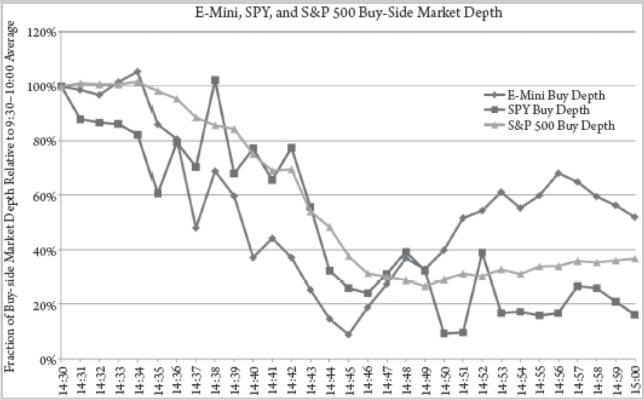


Figure 1.8. Market liquidity of the E-mini futures on the S&P500, the SPY ETF, and the constituent stocks of the S&P500 from 2:30 p.m. to 3 p.m. on May 6, 2010

(Source: CFTC-SEC, 2010)

500 index, the SPY Exchange-traded fund and the stocks in the S&P 500 index (averaged across all its individual constituents). Depth is measured as a fraction of the depth available at 1:30 p.m. (the start of the crash). The evaporation of liquidity in all these securities is evident: by 2:45 p.m., the liquidity available in each market was down to just 20 percent of its level at 1:30. The LOBs for these securities then started replenishing as traders submitted new limit orders. This injection of new limit orders was a factor in the upturn in prices and liquidity observed towards the end of the crash. These dynamics after a large sell order (a drop in price and liquidity followed by an upturn) is an extreme manifestation of a more general phenomenon in illiquid markets: buy and sell market orders trigger transient price movements followed by reversals (see figure 0.2 in the Introduction). Chapters 3 and 4 explore the reason for such reversals.

The pathological nature of the flash crash lies not only in the extraordinary magnitude of the collapse in price and liquidity and the subsequent recovery, but also in the fact that it was propagated so widely and so quickly in different markets. The reason for this is not completely clear, but algorithmic trading is likely to have been a factor, as directional trading programs take account of the information conveyed by price movements in any security more quickly than in the past. So even though they were not at the origin of the crash, high-frequency traders are likely to have exacerbated the resulting price decline and spread it across markets.

(p.44) The automated trading made possible by technological innovation makes markets more interlinked. While in principle this should contribute to better and faster price discovery, insofar as prices are noisy signals, it also increases the likelihood that traders in one security may react to price movements in another security that are actually triggered by uninformative trades.

1.5. Further Reading

Harris (2003) provides a detailed description of the trading mechanisms and the organization of the securities industry in the United States, while Stoll (2006) discusses the spread of electronic trading in U.S. equity markets. Lyons (2001) describes the structure and functioning of currency markets and offers a theoretical and empirical analysis of their operation. Pagano and Röell (1990, 1993) describe how European trading systems started changing in the 1980s under the pressure of deregulation, competition, and technological innovation; more up-to-date descriptions of this process are given in Demarchi and Foucault (2000) and Pagano and Steil (1996). Lee (1998) analyzes in depth the governance of exchanges and their sources of revenue. Mizrach and Neely (2007) describe the structure of the U.S. Treasury bond market. Pagano and von Thadden (2004) and Dunne, Moore, and Portes (2006) describe and analyze the European government bond market. Biais, Declerck, Dow, Portes, and von Thadden (2006) analyze the European corporate bond market, with special attention to its transparency. Buti, Rindi, and Werner (2011) examine data on dark pool activity for a large cross-section of U.S. stocks in 2009, while Buti, Rindi, and Werner (2010) provide a theoretical analysis of the effects of dark pools on the liquidity of the public market's LOB. The concept release on equities market structure by the SEC (2010) describes recent changes in trading technologies and the concomitant growth in so-called high-frequency strategies and discusses the potential costs and benefits for securities markets (see also Angel, Harris, and Spatt 2010). Recent empirical studies on the effects of algorithmic trading include Foucault, and Menkveld (2008); Hendershott, Jones, and Menkveld (2011); Chaboud, Chiquoine, Hjalmarsson, and Vega (2009); and Hendershott, and Riordan (2009).

1.6. Exercises

1. Call auction

Graph total market demand and supply curves in {price, quantity} space for a call auction market where the following orders are submitted to a central auctioneer:

> **(p.45)** Limit orders to buy: 100 shares at \$3.00, 200 shares at \$4.00, 200 shares at \$3.50, and 500 shares at \$2.50.
>
> Limit orders to sell: 500 shares at \$5.00, 600 shares at \$3.00, and 500 shares at \$4.00.
>
> Market orders to buy: a total of 500 shares.
>
> Market orders to sell: a total of 200 shares.
>
> What is the market clearing price? What quantity of stock is traded? Are all orders that are executable at the market clearing price fully filled?

2. Continuous order-driven market

Now suppose that the above orders arrive on the market over time, in the order of arrival that is listed above (that is, at time $t = 1$ the limit order to buy 100 at \$3.00 is submitted, at time $t = 2$ the limit order for 200 at $4.00, and so on, continuing until time $t = 9$, when the market order to sell 200 arrives). Track the state of the LOB (show it after each new order has arrived and any transactions are triggered, for $t = 0,\ldots,9$, in the trading screen format of figure 1.2) and the time, price and quantity of any transactions that take place. Record the dollar bid-ask spread, that is, the difference between the lowest ask and the highest bid, in the continuous market as it evolves from $t = 5$ onwards.

3. Comparison: efficiency and market presence

Consider again the two markets described in questions 1 and 2. Assume that the limit order prices are equal to the order placer's valuation for the block of shares submitted in the order, and think of market orders as placed by agents whose valuation is well outside (above for buyers, below for sellers) the relevant range of trading prices. Which market is Pareto efficient, in the sense that at the end of the trading day there is no pair of agents who could both benefit by trading with each other (i.e., after $t = 9$ in the continuous order-driven market)? Intuitively, why?

Notes:

(1.) These two basic market structures are also known respectively as "order-driven" and "quote-driven" markets.

(2.) For instance, in the LOB depicted in figure 1.2, only eight hundred shares can be bought at $74.48: a buyer looking for one thousand shares would also have to buy one hundred shares at $75.74 and one hundred more at $76.00. As a result, the average price paid per share would be $(74.48 \times 0.8) + (75.74 \times 0.1) + (76.00 \times 0.1) = 74.76$. Conversely, one thousand shares can be sold at an average price of $(74.42 \times 0.3) + (74.41 \times 0.1) + (74.36 \times 0.6) = 74.38$. Therefore, the weighted average bid-ask spread for one thousand shares is $74.76 - 74.38 = 0.38$, rather than 0.06. As a percentage of the mid-quote, the bid-askspread rises from 0.08 percent to 0.5 percent.

(3.) The importance of speed of execution for investors partly explains why the introduction of the continuous dealer market SEAQInternational in London following the 1986 "Big Bang" reforms attracted so much trading in continental European shares. The desire to offer comparably rapid execution also induced continental European exchanges to replace their batch auction system with continuous electronic LOBs (Pagano and Röell 1990).

(4.) It is worth emphasizing that these terms do not denote buyers or sellers of securities but of trading services, so that the buy side includes both buyers and sellers of securities.

(5.) In an open-outcry market, brokers physically meet on a floor to trade. For instance, a broker announces that he is willing to buy one hundred shares (a market order) and other brokers respond by shouting bids (limit orders) at which they are willing to execute the order. These markets mainly trade derivatives such as futures and options; with the development of electronic trading platforms they are tending to vanish.

(6.) The transparency of LOB markets may also change over time. For instance, until 2002 only the best bid and ask prices in the LOB for the stocks listed on the NYSE were disseminated to market participants. Since then, all limit orders have been displayed through the exchange's "OpenBook" system. Another example is the Paris Bourse, which terminated disclosure of the identities of the brokers submitting limit orders in 2001.

(7.) Coval and Shumway (2001) show that changes in the sound level on the floor of the Chicago Board of Trade's 30-year Treasury bond futures forecast subsequent changes in the cost of transacting and volatility. Their interpretation is that the sound level reflects how anxious market participants are to trade at current prices.

(8.) This organizational form does not prevent deviations from the sole pursuit of profit maximization, where there are private benefits of control. In particular, if the stock exchange is controlled by sell-side firms, these may distort choices regarding market organization so as to serve their own interests. For instance, if their in-house systems are competing with the exchange's platform in the provision of trading services, they may oppose changes in trading organization that increase the market share of the platform.

(9.) For instance, Cespa and Foucault (2012) show that an exchange can optimally sacrifice revenues from information sales (at the expense of informational efficiency) in order to increase its trading revenues.

(10.) The number of companies with a cross-listing has increased steadily. For instance, Pagano, Röell, and Zechner (2002) show that the number of European and U.S. companies with a cross-listing increased from 461 in 1986 to 521 in 1997.

(11.) High frequency trading is one form of algorithmic trading that uses computers to place and modify orders very quickly (sometimes in less than a few milliseconds) in reaction to market events (changes in order books) or news.

(12.) Investment in algorithmic trading by proprietary trading desks is often compared to an arms race, as the purpose is not speed per se but outrunning competitors. A brochure describing IBM products for algorithmic traders reads: "The ability to reduce latency (the time it takes to react to changes in the market) […] to an absolute minimum. Speed is an advantage […] because usually the first mover gets the best price" (*Tackling latency: the algorithmic arms race*, IBM 2008).

(13.) Exact figures are difficult to obtain, but anedoctal evidence suggests that algorithmic trading accounts for at least half of the trading volume. In an interesting study, Hendershott and Riordan (2009) use data from the Deutsche Börse, where investors must declare whether they use algorithms or not, as the fee schedule for the two types of trader differs. This enables the researchers to distinguish orders from algorithmic and from human traders. They find that algorithmic traders account for 50 percent of liquidity supply (volume of limit orders) and 52 percent of liquidity demand.

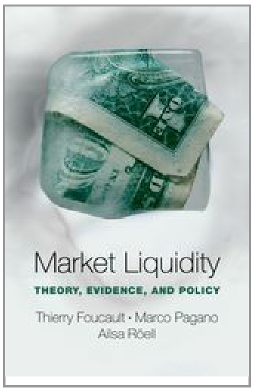
(14.) Chapter 6 analyzes the effects of the risk of being picked off for limit order traders on market liquidity.

(15.) P. Krugman, “Rewarding Bad Actors”, *New York Times*, August 2, 2009.

(16.) However, empirical studies such as Chaboud, Chiquoine, Hjalmarsson and Vega (2009) and Hendershott and Riordan (2009) do not find evidence of a relation between algorithmic trading and volatility.

(17.) See “SEC probes canceled trades,” *Wall Street Journal*, September 1, 2010. The article reports the case of Procter & Gamble stock on April 28. On that day, a burst of limit orders was transmitted to the NYSE shortly before 11:48 a.m. and cancelled almost immediately, triggering a several-second outage of the NYSE reporting system, sufficient to create an apparent arbitrage opportunity between the prices of Procter & Gamble on the NYSE and the competing BATS platform.

(18.) See Commodity and Futures Trading Commission & Securities and Exchange Commission, “Findings Regarding the Market Events of May 6, 2010,” *Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues*, September 30, 2010.



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Measuring Liquidity

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter addresses liquidity measurement. Liquidity has several dimensions, such as trading costs, the depth available to customers placing large orders, speed of execution, protection against execution risk, and so on. It is of paramount importance to practitioners, since illiquidity affects portfolio returns. Illiquidity is often gauged by the cost of trading, which has both an explicit and an implicit component. Explicit costs include broker commissions, transaction taxes, platforms' trading fees, and clearing and settlement fees. Implicit trading costs are those arising from the illiquidity of the market. The most direct way to measure implicit trading costs is to look at market quotes and do a “what if” experiment: what would it cost to make a round-trip transaction, that is, to buy and instantly resell a given amount of securities? Different spread measures give different answers. Section 2.2 discusses the following spread measures: the quoted

spread, the effective spread, and the realized spread. Section 2.3 presents additional measures of implicit trading costs: the volume-weighted average price; the estimated price impact of orders; measures of illiquidity based on non-trading; and Roll's measure of illiquidity, which is based on the serial covariance of transaction price changes. Section 2.4 considers the notion of implementation shortfall, a measure of execution quality that considers not only the price impact of orders but also the opportunity cost of delayed or partial execution. The final sections provide suggestions for further reading and exercises.

Keywords: liquidity measurement, illiquidity, implicit trading costs, spread measures, volume-weighted average price, implementation shortfall

Learning Objectives:

- • The components of trading costs
- • How implicit trading costs can be measured from quote data
- • What to do when only transaction data are available
- • How to take the time dimension in order execution into account

2.1. Introduction

If the structure of a securities market is compared to a car design, measuring market liquidity can be likened to assessing the car’s driving performance. Several different aspects of performance need to be considered (for cars, fuel efficiency, speed, safety, etc.), and liquidity has several dimensions as well: trading costs, the depth available to customers placing large orders, speed of execution, protection against execution risk, and so on. Liquidity measurement is of paramount importance to practitioners, since illiquidity affects portfolio returns, as Chapter 9 explains. Investors and intermediaries accordingly want to design trading strategies that minimize the effects of illiquidity on their investment performance. Liquidity measurement is also important for researchers and regulators who need to understand the relationship between market structure and performance, just as engineers are keen to understand the relationships between various aspects of a car’s design and its road performance.

(p.47) Illiquidity is often gauged by the cost of trading, which has both an explicit and an implicit component. Explicit costs include broker commissions, transaction taxes, platforms’ trading fees, and clearing and settlement fees. As they are charged explicitly to final investors, these are easy to measure. Implicit trading costs are those that arise from the illiquidity of the market; they are measured by the gap between execution price and some benchmark used to proxy for the price that would be obtained in a perfectly liquid market. Commonly, where the data are available, this benchmark is the midquote (the average of the best bid and ask prices) at the time the order is placed or executed.

In practice, professional traders pay close attention to trading costs, and some brokers periodically report summary measures of both components based on the orders executed. For instance, the Investment Technology Group (ITG) brokerage firm releases these measures by country for each quarter, as shown in table 2.1.

The last column in table 2.1 reports the total trading costs for orders executed by ITG in Europe (excluding the United Kingdom) for each quarter (trading costs are expressed in basis points, i.e., as hundredths of a percent of the value traded).[1] The first three columns break this total down into its components. Implicit trading costs consist of delay costs (second column) and impact costs (third column). For a buy order, the delay cost is the difference between the midquotes at order execution and submission times; the impact cost is the difference between execution price and midquote at execution time. Explicit trading costs (commissions and fees) are shown in the fourth column as "Comm Costs."

Several interesting facts emerge. First, explicit trading costs have declined steadily since 2003, partly because of increased competition between intermediaries and improvement in trading technologies (see Chapter 1). Second, the implicit costs are significant relative to the explicit ones; delay costs in particular are relatively high, because on average market prices move against the order. Finally, implicit trading costs declined between 2003 and 2007 but rose sharply thereafter, as the financial crisis unfolded and liquidity dried up.

Just as someone choosing a car seeks comparative indicators of its performance, traders are interested in measures of implicit trading costs. For instance, a broker who needs to buy five thousand shares of Microsoft will compare its price on Nasdaq (where the stock is listed) to its price on several other platforms **(p.48)**

Table 2.1 Trading Costs (in BPS) European Equity Markets (Excluding United Kingdom)

Quarter	Delay Costs	Impact Costs	Comm. Costs	Total
2003				
Q1	61	16	17	94
Q2	31	9	15	55
Q3	42	10	15	67
Q4	45	10	15	70
2004				
Q1	41	8	16	65
Q2	34	10	16	60
Q3	43	7	15	65
Q4	32	5	15	51
2005				
Q1	44	4	15	64
Q2	38	4	15	57

Q3	35	10	15	57
Q4	36	9	14	58
2006				
Q1	35	8	13	57
Q2	36	0	12	58
Q3	39	7	12	59
Q4	36	6	11	48
2007				
Q1	35	7	11	54
Q2	37	6	11	54
Q3	34	8	10	52
Q4	58	10	11	79

Source: ITG Global Cost Review, 2008, available at http://www.itg.com/category/knowledge/reports

where it is traded (BATS, Archipelago, etc.). He must then decide how many shares to buy in each market; whether to split the order over time; and whether to use market orders, limit orders, or a combination of the two.

In this situation, information on implicit trading costs is useful: for instance, the broker may want to trade more on the platform with lower implicit trading costs. Estimating this cost requires detailed data on past orders (such as the time of their submission, to measure delay cost) and on quotes (e.g., the full LOB in each trading venue). When such data are available, traders can assess market liquidity via simple measures of implicit trading costs, such as the quoted bid-ask spread, the effective bid-ask spread, and the realized bid-ask spread, which **(p.49)** are presented in section 2.2. In some cases, regulators oblige trading platforms to publish periodic data on some of these implicit cost measures.[2]

These measures of trading costs require data on bid and ask quotes, but these are sometimes simply unavailable. Even so, implicit trading costs can be measured using time series of recent transaction prices and possibly trading volume. In section 2.3 we present these additional measures of implicit trading costs: the volume-weighted average price (VWAP); the estimated price impact of orders; measures of illiquidity based on non-trading; and Roll's measure of illiquidity, which is based on the serial covariance of transaction price changes.

All these measures, however, neglect the time dimension of implicit trading costs: if traders are willing to trickle their orders into the market gradually, they may get a better average price, because with time new counterparties may emerge and post new orders to replace those that have been exhausted-a dynamic dimension of liquidity known as market resiliency. But a gradual execution strategy exposes traders to the risk of incomplete execution. Section 2.4 discusses the notion of implementation shortfall, a measure of execution quality that considers not only the price impact of orders but also the opportunity cost of delayed or partial execution. Traders frequently use this measure to devise a strategy for minimizing their costs on large orders.

2.2. Measures of the Spread

As explained in section 1.2.1 of Chapter 1, the most direct way to measure implicit trading costs is to look at market quotes and do a “what if” experiment: what would it cost to make a round-trip transaction, that is, to buy and instantly resell a given amount of securities? Different spread measures give different answers.

2.2.1 The Quoted Spread

The obvious, intuitive measure of the cost of a *small* round-trip transaction is the difference between the best ask quote a and the best bid quote b, that is, the *quoted bid-ask spread* $S = a - b$. If this spread is normalized by the midprice $m = (a + b)/2$, one obtains the relative quoted spread:

(2.1)

$$s \equiv \frac{S}{m} = \frac{a-b}{m}.$$

(p.50) The quoted spread is a good measure of trading costs for orders that are so small that they can be entirely filled at the best quotes, which in the United States are known as the Best Bid and Offer (BBO). The quoted spread for small trades is the most widely reported measure of illiquidity; this is what people mean when they refer to “the” bid-ask spread.

As we saw in Chapter 1, the spread on larger orders can be gauged in a similar way, computing a weighted average bid-ask spread from the quotes posted. For instance, for the buy and sell limit orders posted at a given point in time, suppose that the average execution price for a buy market order of size q is $\bar{a}(q)$, and the average execution price for a sell market order of size q is $\bar{b}(q)$. The weighted-average bid-ask spread for an order of size q is thus $S(q) = \bar{a}(q) - \bar{b}(q)$, and the relative weighted average bid-ask spread is:

(2.2)

$$s(q) \equiv \frac{\bar{a}(q) - \bar{b}(q)}{m}.$$

Clearly, when q is so small that the entire offer can be filled at the BBO, this reduces to the spread s in equation (2.1). As the quantity offered at each price is limited, $s(q)$ increases with the trade size q. The deeper the market, the milder the increase in the spread $s(q)$ associated with larger trade sizes q.

The drawback to this method is that it requires data on limit orders or quotes posted in the market at various points in time. Moreover, for trade sizes large enough to exhaust the liquidity offered at the BBO, the LOB at price points beyond the BBO is required. As this information is not always readily available, practitioners and researchers often measure implicit trading costs based on transaction prices alone, or on transaction prices together with the average of the BBO. In the rest of this section, we describe two such measures: the effective spread and the realized spread.

2.2.2 The Effective Spread

The quoted spread reflects the liquidity available at a given point in time for a hypothetical transaction. Instead, one could measure trading costs using the prices actually obtained by investors. This gauge is the *effective half-spread*, defined as the difference between the price at which a market order executes and the midquote on the market the instant before. Suppose that a market buy order for one thousand shares arrives and executes at an average price of 75.50, while the prevailing midquote is 75.45. The effective half-spread on to this order is thus 75.50 − 75.45 = 0.05 or, as a percentage of the midquote, 0.05/75.45 = 0.067%. Formally, the absolute effective half-spread is defined as:

(2.3)

$$S_e \equiv d\,(p - m)\,,$$

(p.51) where d is the order direction indicator (1 for buyer-initiated and −1 for seller-initiated trades) and m is the midquote on the market prior to a transaction executed at price p. In relative terms, the effective spread is

(2.4)

$$S_e \equiv d \cdot \frac{p - m}{m}\,.$$

The effective spread can be seen as a measure of a transaction's impact on the price, since it measures the deviation of the actual execution price from the midprice prevailing just before the transaction. This impact (sometimes called "slippage") is positive precisely because the liquidity of the market is limited. Thus, the effective spread averaged across a large number of transactions is a way to gauge market liquidity. The effective spread is likely to increase with the size of the transaction in that (as Chapter 1 explains) larger market orders execute at less favorable prices. So it is useful to sort transactions into size classes before averaging, and so get a series of estimates of the effective spread, for various sizes.

As it is based on actual transactions, the effective spread captures any price improvement that market orders may receive when they execute against hidden orders or when they receive a price improvement in dealer markets, as explained in Chapter 1. However, being based on past prices, the effective spread is a retrospective measure of liquidity, in contrast with the quoted spread that relies on prices at which traders can actually trade at a given point in time.

As shown by equation (2.3), to calculate the effective spread one needs transaction prices, the midprice prior to each transaction, and an order direction indicator (buy or sell initiation).

In practice, performing this calculation can be difficult for at least two reasons. First, in theory the effective spread for an order that is split over time should be measured by comparing the average price over the entire order with the market midquote at the time of the first part of the transaction. For an investor assessing the quality of execution of a completed trade, it is a simple matter; one can readily determine the average price—even if the order is executed at multiple price points and/or involves some use of limit orders—and compare it to the initial market midprice. And analyzing such data can help evaluate the effectiveness of trading strategies: whether splitting orders and trickling them into the market improves the average price, and how slowly large orders should be worked. For an econometrician or a regulator analyzing liquidity, however, the splitting of transactions can make it nearly impossible to reconstruct total orders from transaction records. Thus the analysis of market liquidity by such a non-participant may be restricted to the effective spread for smaller transactions placed by an identifiable subset of traders, such as market order placers.

(p.52) Second, some data sets do not tell whether transactions stem from buy or sell market orders. When this information is not available, one must devise a way to “sign” transactions, that is, to decide for each trade whether $d = 1$ or $d = -1$. The various methods used to classify order direction are not free from classification errors, which introduce noise in the estimation of effective spreads, as explained in box 2.1.

Box 2.1 How to Classify buy and sell Orders

Quote and transaction data generally include transactions at prices both outside and inside the prevailing bid-ask quote interval. This is puzzling, as one would expect a small buy or sell order always to be executed right at the quoted ask or bid.

Trades may be priced strictly inside the quotes for a variety of reasons. Effective liquidity may be better than quoted liquidity, for example, if a broker matches two offsetting customer orders outside the main market and records the trade at a price within the spread, or if a broker-dealer gives a price improvement on the quoted price to a client (see Chapter 1). In other cases, though, a transaction price inside the quotes simply reflects a misalignment in the recorded sequence of quotes and transaction times. Similarly, trades priced outside the quotes may be either genuinely costly trades that are too large to be filled at the best quotes or simply trades whose time of execution has been misrecorded.

In practice, the electronic system for recording quote changes is generally faster than the transaction recording process, which may require some manual input. Thus, if we see a transaction at a price of 98.88 at a moment when the best posted bid is 98.85 and the best ask is 98.92, the transaction may actually have occurred a few seconds earlier, when the best quotes were still 98.88 and 98.92; indeed, the best bid quote's fall from 98.88 to 98.85 may have been triggered precisely by a seller-initiated transaction that exhausted the best bid. Lee and Ready (1991) find that delays of up to about five seconds in transaction reports are commonplace.

Under these circumstances, it may be hard to determine the sign of market orders. Researchers commonly use the Lee-Ready algorithm to resolve this conundrum, classifying a transactions as "buyer-initiated" if its price is closer to the prevailing ask quote than to the bid, and "seller-initiated" if the converse. Any transaction priced exactly at the midquote is classified as a buy if the price is higher than the previous transaction price (that is, on an "uptick"), a sell if lower. A possible improvement on this algorithm could be to first seek a matching quote-depth change in the five seconds preceding it that seems to have been triggered by the transaction itself. In the above example, if the trade priced at 98.88 is for two hundred shares and the bid quote of 98.88 that disappeared a few seconds earlier was for two hundred shares, then it would make sense to realign the transaction time with that of the quote change. Only if no such realignment seems appropriate would the Lee-Ready procedure then be applied.

But it should be clear that the Lee-Ready algorithm and any of its refinements cannot classify trade initiation with perfect accuracy and so may bias the measured transaction cost. Odders-White (2000) uses the TORQ data to investigate the accuracy of the Lee-Ready algorithm, finding that it correctly classifies 85 percent of the transactions in her sample but systematically misclassifies trades at the midpoint of the bid-ask spread, small transactions, and transactions in large-capitalization or frequently traded stocks.

(p.53) 2.2.3 The Realized Spread

The quoted spread and the effective spread implicitly adopt the viewpoint of liquidity demanders. That is, they gauge the extra cost sustained by a trader submitting a market order relative to an ideal environment in which trades are made at the midprice. It is tempting to conclude that this extra cost is a gain for liquidity suppliers, the counterparties to all trades by liquidity demanders, but this is not actually the case, because buy and sell orders may exert lasting pressure on prices, to the detriment of liquidity suppliers (e.g., dealers) who absorbed such orders into their inventories.

To illustrate, suppose that a dealer buys seventy-five shares at $326 when the best bid and ask prices are respectively $326 and 327, respectively. If the dealer unwinds his position immediately at the ask price, he earns a profit of $1 on each share. But if the best bid and ask prices decline to, say, 325.5 and 326.5 respectively, he earns a zero average profit (assuming that he is equally likely to unwind his inventory at the best ask or the best bid price). This example shows that quoted or effective bid-ask spreads are likely to overestimate liquidity providers' gains (and therefore liquidity demanders' trading costs) if after a trade prices move in the direction of the trade.[3]

(p.54) One way to cope with this problem is to compute the *realized half-spread*, that is, the difference between the transaction price and the midprice at some time, Δ, after the transaction (say five or ten minutes later), where the interval Δ should be long enough to ensure that market quotes have adjusted to reflect the price impact of the transaction. Let p_t be the price of the transaction at time t, d_t the direction of the market order triggering it, and m_t the midprice at time t. The realized half-spread for this transaction is then given by

(2.5)

$$S_r = d_t\left(p_t - m_{t+\Delta}\right) = d_t\left(p_t - m_t\right) - d_t\left(m_{t+\Delta} - m_t\right).$$

This can be seen as a measure of the profit earned by the liquidity supplier on the transaction at time t if he unwinds his position at the midprice at $t + \Delta$. Using the definition of the effective spread in equation (2.3) in expression (2.5), one can rewrite the average realized bid-ask spread as

(2.6)

$$\mathrm{E}\left(S_r\right) = \mathrm{E}\left(S_{\mathrm{e}}\right) - \mathrm{E}\left(d_t\left(m_{t+\Delta} - m_t\right)\right).$$

This expression shows that the average realized spread is smaller than the average effective spread if $E(d_t\,(m_{t+\Delta} - m_t)) > 0$, that is, if the change in the midprice following a transaction is positively correlated with its direction. Interestingly, if the effective spread is small enough, liquidity providers would lose money on average, as $E(S_e) < E(d_t\,(m_{t+A} - m_t))$. Since in the long run this would drive them out of the market, the effective spread cannot be too low: it must at least compensate them for the adverse price movement following a trade.

The value of the realized spread is sensitive to the choice of the reference post-trade market price (i.e., to $m_t + \Delta$). In practice, market participants need time to respond to the information content of transactions with fresh quotes and limit orders. Thus, the choice of Δ depends on how quickly market participants adjust their quotes after a transaction. In transparent and active markets adjustment is generally fast, so a modest value of Δ is appropriate; too high a value introduces unnecessary noise. In more opaque markets, information on trades takes more time to be disseminated, and higher values of Δ should be chosen.

Quoted, effective, and realized spreads are used routinely by market participants to evaluate market liquidity. Since 2000, U.S. trading platforms have been required to report monthly statistics (known as Dash-5 reports) on the quality of order execution, pursuant to SEC rule 605. These statistics include measures of market liquidity such as the average effective bid-ask, the average realized spread, and the speed of execution for orders of various sizes (small, medium, and large) and types (limit and market orders). **(p.55)**

SEC Rule 11AC1-5: Disclosure of Order Execution Information
Market Center: NASDAQ
Issue: MSFT - Microsoft Corporation - Common Stock
Report Type: Comprehensive - November 2005
Execution System: NASDAQ market Center NASDAQ-Listed Trading
Order Type: Market Orders

Order Size	Number of Orders	Number of Shares	Cancelled Shares	MC Exec. Shares	Other MC Exec. Shares	0–9 Seconds	10–29 Seconds	30–59 Seconds	60–299 Seconds	5–30 Seconds
100–499	180	34,424		32,024	2,400	34,424				
500–1,999	136	125,521	500	113,021	12,000	125,021				
2,000–4,999	36	103,499		95,137	8,362	103,499				
5,000–9,999	14	98,224		97,934	290	98,224				

Order Size	Avg. Realized Spread	Avg. Effective Spread	Price Improved Exec. Shares	Avg.Price Improvement Amount	Price Improved Avg. Exec. Time	At Quote Exec. Shares	At Quote Avg. Exec Time	Outside of Quote Exec. Shares	Avg. Outside of Quote Amount	Outside of Quote Avg. Exec. Time
100–499	$0.0066	$0.0076	600	$0.0100	0.1	32,374	0.1	1,450	$0.0100	2.5
500–1,999	$0.0019	$0.0067	1,100	$0.0100	0	120,821	0.2	3,100	$0.0100	2.5
2,000–4,999	$0.0012	$0.0077				97,999	0.1	5,500	$0.0100	0.6
5,000–9,999	$0.0159	$0.0187				98,224	0.4			

Figure 2.1. Execution quality statistics provided by NASDAQ 2005

Figure 2.1 is an example of the information contained in a Dash5 report for Microsoft (listed on Nasdaq) in 2005.[4] It gives the average realized and effective spread on Microsoft shares for various trade sizes. As expected, realized spreads are smaller than the corresponding effective spreads (for all except the largest trades).[5]

2.3. Other Measures of Implicit Trading Costs

The measures of trading costs considered so far require knowledge of bid and ask quotes at the time of execution (for the quoted spread), immediately before (for the effective spread) or shortly after (for the realized spread). Even when quote data are simply unavailable, however—as is often the case—implicit trading costs can be measured using transaction data only, as shown below.

2.3.1 Volume-weighted Average Price

When quote data are not available, the midquote cannot be used as the benchmark for the execution price. One can then use easily observable alternative benchmarks, such as the day's opening or closing price. But often averages of transaction prices over the day are used: a popular benchmark in trading cost **(p.56)** analysis is the volume-weighted average price (VWAP) for all transactions in the stock over some relevant interval, normally the entire trading day (though one may also take a shorter interval around the time at which an order was placed). Whatever the benchmark time interval T chosen, we have:

(2.7)

$$VWAP = \frac{\$\text{ volume of trading}}{\#\text{ of shares traded}} = \sum_{t \in T} w_t p_t, \quad \text{where } w_t = \frac{|q_t|}{\sum_{t \in t} |q_t|},$$

and q_t and p_t are the size and price of the t^{th} trade.

Investors evaluate their broker's performance in getting a good price for their order by comparing their own price with the day's VWAP. The difference is a mixture of the effective and the realized spreads, as it involves comparison with both pre- and post-transaction prices.

In practice, several problems arise with the use of VWAP as a benchmark for trading costs. First, the average may depend on the order one is interested in itself, if it accounts for a significant proportion of trading volume. In the extreme case, in which execution of a single order (in multiple fills) generates all the day's trades, then a measure of trading cost based on VWAP is automatically zero.

Second, VWAP-based measures of trading costs can be gamed by brokers. Consider, for instance, a broker in charge of executing a large buy market order. Typically, he splits the order and executes it in multiple fills, to avoid having too large an impact on prices. If his performance is judged by the VWAP, the broker has an incentive to trickle the order into the market extremely slowly to ensure that his average execution price is as close as possible to the VWAP. This could lead him to delay trade execution excessively, unduly increasing the risk of non-execution and the client's opportunity cost of not trading. As we will see in section 2.4, clients can assess the opportunity cost of delay by applying the broader notion of implementation shortfall.

2.3.2 Measures Based on Price Impact

Other measures of transaction costs are based on the extent to which an order generates an adverse reaction in the market price. As we saw in section 2.2.3, the midprice tends to rise when buy orders arrive, to an extent that is positively correlated with their size. Symmetrically, it tends to fall in the wake of sell orders. If the midprice change is proportional to the buying or selling pressure, the relationship can be expressed as follows:

(2.8)

$$\Delta m_t = \lambda q_t + \varepsilon_t,$$

where Δm_t is the change in the midprice over a fixed time interval (a half-hour, say, or a day) and q_t is the order imbalance, that is, the total value of buy less sell **(p.57)** market orders executed in the same interval. For instance, suppose that in a one-hour interval, the dollar value of all executed buy market orders is $10,000, and that of sell orders, $8,000. The order imbalance is +2,000. This differs from trading volume, which in this case is eighteen thousand. Computation of the order imbalance again requires signing market orders. If data on the direction of buy and sell market orders is not available, one can use the Lee-Ready algorithm (box 2.1).

The intuitive meaning of equation 2.8 is that the net demand during the chosen interval from traders placing market orders puts pressure on the price that is gauged by the coefficient λ. Chapter 3 provides the theoretical underpinning for this equation. The reciprocal of λ can be seen as a measure of market depth (see Chapter 1) in that a lower value of λ means prices are less sensitive to order imbalance. In practice, people often estimate this price impact measure λ by running a regression of the change in the midquote (Δm_t) on the order imbalance (q_t).[6] A closely related approach is the price impact regression presented in Chapter 5.

Stoll (2000) applies a modified version of this approach to daily observations for stocks listed on the NYSE/AMEX and Nasdaq, including a lagged term (q_{t-1}) as an additional explanatory variable to capture a possible reversal of the price impact the following day. He finds that the value of λ is positive for 98 percent of the stocks and significantly different from zero for 63 percent. The measure varies considerably across stocks: a one-percentage-point increase in order imbalance has a price impact of 0.75 percent for lowest-capitalization NYSE/AMEX stocks and 0.52 percent for the highest-capitalization ones. So, as one would expect, larger stocks seem to have deeper markets, where the price better withstands order imbalances.

Measuring order imbalances is sometimes difficult, as it requires data on the signed flow of buy and sell market orders. An alternative is to gauge the sensitivity of returns to trading volume (see Hasbrouck 2007, p. 93). While trading volume and order imbalance are certainly distinct concepts, they are likely to be correlated (days with larger order imbalances may well be the days with high trading volume). Therefore, one can estimate a regression of $|\Delta m_t|$ (the absolute value of price changes) on the trading volume Vol_t (the monetary value of the total amount traded) over the same interval (e.g., a day or a month). In this modified version, the slope can be interpreted as a measure of the price change associated with one additional unit of trading volume.

Conceptually this measure is related to the “illiquidity ratio” proposed by Amihud (2002), that is the ratio I_t of the absolute return for a stock ($|r_t|$) to **(p.58)** trading volume over a given period (e.g., a day):

(2.9)

$$I_t = \frac{|r_t|}{Vol_t}.$$

This is also known as the Amihud ratio.[7] The inverse of this measure,

(2.10)

$$L_t = \frac{Vol_t}{|r_t|},$$

also frequently used, is known as the Amivest liquidity ratio. A low value of this ratio is a sign of market illiquidity.

2.3.3 Non-trading Measures

Often the level of trading volume in a security or its turnover rate (volume divided by the market capitalization of the security) is used as an indicator of its liquidity—the idea being that a market where many buyers and many sellers congregate will offer better trading opportunities. But there is reason to doubt that these are good measures of liquidity, because volume tends to increase when new information reaches the market, which is also a time of high volatility and concomitantly wide bid-ask spreads (as we shall see in Chapter 3). So, turnover may increase at a time when trading costs are high. For instance, Fleming (2003) finds that in the market for U.S. Treasury notes trading volume is a weak proxy for liquidity as measured by bid-ask spreads. He also finds that a trading frequency measure (namely, the number of trades executed in a specified interval) is a similarly poor proxy.

However, the frequency with which no trading at all occurs can be a useful proxy for illiquidity in some very thin markets, including many emerging markets. In such markets there are days or even weeks without transactions are concluded and the exchange simply reports the “stale price” of the last actual trade. Moreover, in these markets real-time quote data are generally unavailable. In a limit order market, this can happen because of the lack of counterparties; in a dealer market, because dealers’ bid-ask spreads are perceived as prohibitively costly or unrepresentative of actual trading opportunities.

As no trading is associated with no change in price, Lesmond, Ogden, and Trzcinka (1999) and Bekaert, Harvey, and Lundblad (2007) propose to measure illiquidity as the fraction of no-trade days, defined as days with zero daily returns. The advantage of this measure is that it requires only a time series of daily returns, and no transaction volume data. Bekaert, Harvey, and Lundblad **(p.59)** (2007) and Lesmond (2005) reinforce the measure’s credibility, showing that it is positively correlated with bid-ask spreads for the limited periods when overlapping data are available, and negatively correlated with trading volume. They confirm this finding by comparing this illiquidity measure with more standard liquidity measures using U.S. data.[8]

Nevertheless, this measure also has a drawback: the maintained assumption that there is no trading whenever prices do not move is not always valid. For instance, on any given day the price of a security may stay level because there is no news about that security, even though trading does take place. Indeed it is precisely in a very liquid market that trades can take place without moving the price! To the extent that such cases occur, identifying non-trading with zero returns leads to an overestimate of the illiquidity of the corresponding market.

2.3.4 Measures Based on Return Covariance

Roll (1984) sets out an ingenious method for measuring the bid-ask spread based on transaction prices alone. The idea is that orders will sometimes, at random, hit the ask and the bid price, so that transaction prices bounce back and forth between them, straddling the midquote m_t. The transitory deviations around the midprice are called bid-ask bounce. Intuitively, this bounce engenders negative serial correlation in transaction-to-transaction returns.

To see this, suppose that a buy market order arrives at time t, followed by a sell market order at date $t + 1$. The first trade is at the ask price, the second at the bid price. Thus the return between dates t and $t + 1$ is negative. One accordingly expects a positive return between $t + 1$ and $t + 2$, the time of the subsequent transaction. Either the next market order is again a sell order executed at the bid price (in which case the return from t +1 to $t + 2$ is zero) or it is a buy market order executed at the ask price (in which case the return from $t + 1$ to $t + 2$ is positive). A similar argument shows that after a positive return, one would expect a negative return.

Roll (1984) exploits this intuition to construct an estimator of the bid-ask spread s based entirely on the serial covariance of returns. This estimator is called Roll's measure and is derived below. It is a function of the autocovariance of returns. The estimator depends on specific assumptions regarding the order **(p.60)** arrival process. After presenting Roll's measure, we shall discuss how it should be adjusted when these assumptions do not hold.

Roll's Measure

To derive Roll's measure, suppose that the security's fundamental value, as captured by the midquote, follows a random walk:

(2.11)

$$m_t = m_{t-1} + \varepsilon_t,$$

where ε_t is mean-zero white noise ($E(\varepsilon_t) = 0$ for all t, and $E(\varepsilon_t \varepsilon_s) = 0$ for all $t \neq s$). This variable represents the change in the value of the stock due to new information emerging between time $t - 1$ and t. As $E(\varepsilon_t) = 0$, the expected fundamental return is zero ($E(m_t - m_{t-1}) = 0$), which is reasonable for small time intervals (a day or less). In any case this assumption can easily be relaxed (see below).

Now suppose that the bid-ask spread S is constant over time. The ask and bid prices in the market are:

(2.12)

$$a_t = m_t + \frac{S}{2} \text{ and}$$

(2.13)

$$b_t = m_t - \frac{S}{2}.$$

Transactions are at either the bid or the ask price depending on whether a buy or a sell market order arrives. Thus, the price of the t^{th} transaction can be written:

(2.14)

$$p_t = m_t + \frac{S}{2} d_t,$$

where d_t (= 1 or − 1) indicates whether a transaction is buyer or seller initiated (see section 2.2.1). Therefore, using equations (2.11) and (2.14), the dollar transaction-to-transaction return is:

(2.15)

$$p_t - p_{t-1} = m_t + \frac{S}{2} d_t - \left(m_{t-1} + \frac{S}{2} d_{t-1} \right) = \frac{S}{2} d_t - \frac{S}{2} d_{t-1} + \varepsilon_t.$$

To compute Roll's estimator for the bid-ask spread, we need several additional assumptions on the order arrival process, namely:

a. **Balanced order flow** Market orders are equally likely to be buy or sell orders: $\Pr(d_t = 1) = \Pr(d_t = -1) = 1/2$ (so that $E(d_t) = 0$ for all t).
b. **No autocorrelation in orders.** Buy and sell market orders are serially uncorrelated, i.e., $E(d_t d_s) = 0$ for $t \neq s$.
c. **No effect on the midquote.** Market orders are assumed to carry no news, meaning that they are uncorrelated with current and future innovations in fundamentals: $E(d_t \varepsilon_t) = E(d_t \varepsilon_{t+1}) = 0$ for all t.
(p.61) d. **Constant (zero) expected return.** The fundamental value follows a random walk, so that $E(m_t - m_{t-1}) = E(\varepsilon_t)$ is constant and equal to zero for all t.

This set of assumptions regarding the order arrival process and the dynamics of price and quotes constitutes Roll's model. As shown in subsequent chapters, this model is an important element in many market microstructure models.

Under the assumptions of Roll's model:

$$E\left(p_t - p_{t-1}\right) = 0.$$

Therefore, using equation (2.15),

(2.16)

$$\operatorname{cov}\left(p_{t+1} - p_t, p_t - p_{t-1}\right) = \frac{S^2}{4} E\left[\left(d_{t+1} - d_t + \varepsilon_{t+1}\right)\left(d_t - d_{t-1} + \varepsilon_t\right)\right]$$

(2.17)

$$= \frac{S^2}{4} E\left[d_{t+1} d_t - d_t^2 - d_{t+1} d_{t-1} + d_t d_{t-1}\right]$$
$$= -\frac{S^2}{4},$$

where we have used the assumptions on the order arrival process and the fact that $E\left(d_t^2\right) = 1$: as d_t can be only +1 or −1, its square is always 1. As expected, equation (2.17) implies that the bid-ask spread induces a negative correlation in price changes ($\operatorname{cov}(p_{t+1} - p_t, p_t - p_{t-1}) \langle 0$ if $S \rangle 0$). Moreover, it yields Roll's estimate of the absolute value of the bid-ask spread:
(2.18)

$$S_R = 2\sqrt{-\text{cov}\left(\Delta p_{t+1}, \Delta p_t\right)},$$

also known as Roll's measure.

Stoll (2000) reports estimates of the (half) Roll's measure for all stocks listed on the NYSE and Nasdaq (see Stoll 2000, p. 1493, Table III). For each stock, he calculates the average daily serial covariance in trade-to-trade price changes over sixty-one trading days. He then reports an estimate of $\sqrt{-\text{cov}\left(\Delta p_{t+1}, \Delta p_t\right)}$ by decile of capitalization. Overall, for the NYSE, the half-Roll's measure is 3.81 cents and 11.5 cents for Nasdaq. Obviously this difference is due to differences in the characteristics of the stocks listed on the two exchanges, but even after controlling for market capitalization, for instance, it persists. Stoll (2000), p. 1491, Table II also reports the average values of the quoted and the effective spreads for his sample of stocks. On average, the half-quoted spread on the NYSE is 7.9 cents against 12.6 cents on Nasdaq; the half-effective spread, 5.6 against 10.7 cents. The effective spread is smaller than the quoted spread in both markets, since dealers in each sometimes offer price improvement to their clients, which results in trades inside the bid-ask spread. Roll's measure **(p.62)** underestimates the quoted spread, and for the NYSE it also underestimates the effective spread.

Extensions

As noted, Roll's measure depends on a series of assumptions concerning the process of order arrival and on stock returns, without which it will not yield an unbiased estimate of the bid-ask spread. We now examine why and in which direction the failure of these assumptions biases Roll's measure. Let us relax them one at a time.

Assumption (a): ***balanced order flow.***

In reality, market buy and sell orders are not necessarily of equal probability. For instance, at the end of the trading day agents may be more anxious to close out short positions, in which case buyer-initiated transactions would be more prevalent. To account for this, suppose that $\Pr(d_t = 1) = \eta$. When η is different from $\frac{1}{2}$, the order flow is unbalanced: there are more buy orders than sell orders if $\eta \rangle \frac{1}{2}$, conversely if $\eta \langle \frac{1}{2}$. In the appendix to this chapter we show that in this case the autocovariance of price changes is

(2.19)

$$\text{cov}\left(\Delta p_{t+1}, \Delta p_t\right) = \frac{S^2}{4}\text{E}\left[\left(d_{t+1} - d_t\right)\left(d_t - d_{t-1}\right)\right] = -\eta\left(1-\eta\right)S^2$$

Therefore, an unbiased estimator of the bid-ask spread is

(2.20)

$$S_a = \sqrt{-\frac{\text{cov}(\Delta p_{t+1}, \Delta p_t)}{\eta(1-\eta)}},$$

which implies

$$S_R = \left(2\sqrt{\eta(1-\eta)}\right) S_a.$$

Thus, unless the order flow is perfectly balanced $\left(\eta = \frac{1}{2}\right)$, Roll's spread estimator is biased, underestimated by a factor of $2\sqrt{\eta(1-\eta)}$. To see intuitively why, note that if almost all transactions are buyer initiated (say, $\eta = 0.99$), the bid-ask bounce is almost nil, as almost all trades are at the ask price. Thus, the covariance in the change in prices is close to zero and Roll's estimate is also close to zero, even though the actual bid-ask spread may be large. Thus, one must adjust the estimate of the covariance (here by a factor $\eta(1-\eta)^{-1} \simeq 101.01$) to obtain a correct spread estimate.

Assumption (b): ***non-autocorrelated orders.***

Roll's method is also biased if there is serial correlation in the trade direction d_t. Such correlation may occur in practice if traders slice their orders and execute them piecemeal over time. For example, suppose the direction of market orders is first-order autocorrelated:

$$\Pr(d_{t+1} = d_t) = \delta$$

(p.63) We can then show (see appendix) that

(2.21)

$$\text{cov}(\Delta p_{t+1}, \Delta p_t) = \frac{S^2}{4}\text{E}\left[(d_{t+1} - d_t)(d_t - d_{t-1})\right] = -(1-\delta)^2 S^2.$$

Hence, an unbiased estimate of the bid-ask spread is given by

(2.22)

$$S_b = \frac{1}{1-\delta}\sqrt{-\text{cov}(\Delta p_{t+1}, \Delta p_t)}.$$

Thus, unless $\delta = \frac{1}{2}$ (the order flow is serially uncorrelated), Roll's estimator of the bid-ask spread underestimates the bid-ask spread by a factor of $2(1-\delta)$. Choi, Salandro, and Shastri (1988), using the Lee-Ready algorithm to classify transactions, estimate δ to be about 0.7, which implies positive serial correlation in the direction of trade. Then Roll's original measure would again underestimate the true spread by a factor of $2 \times 0.3 = 0.6$.

Assumption (c): ***no effect of orders on the midquote.***

Roll assumes that changes in the value of the security (ε_t) and the direction of market orders (d_t) are independent. This assumption is problematic if some traders submitting market orders have information on the stock's future payoff. In this case, as will be seen in Chapter 3, the direction of market orders does carry information, and market participants' estimate of the value the security moves in the direction of the order. Specifically, liquidity suppliers permanently revise their value estimate upward after buy orders and downward after sell orders, which implies that ε_t and d_t are positively correlated. This attenuates the bid-ask bounce, which means that Roll's estimator again underestimates the true spread. In this case, obtaining a more accurate estimator requires deeper understanding of the trading process in the presence of asymmetric information. The method will be presented in Chapter 5, section 5.2.

Assumption (d): ***non-varying fundamental expected return.***

One of the assumptions used to derive Roll's measure is that the expected return is constant and indeed equal to zero: $E(m_t - m_{t-1}) = 0$. However, this assumption may fail: expected returns for a security can vary over time, and if the variations are serially correlated this may be yet another source of bias in Roll's measure. To see this, suppose that

(2.23)

$$m_t = m_{t-1} + \bar{r}_t + \varepsilon_t,$$

where the term $\bar{r}_t$ is the expected component of the return from time t to $t + 1$ (that is, $\bar{r}_t \equiv E(m_t - m_{t-1})$) and ε_t is the unexpected component. We index $\bar{r}_t$ by t because it can be time-varying. Then, using equations (2.14) and (2.23), we obtain

(2.24)

$$\Delta p_t = \bar{r}_t + \frac{S}{2}\left(d_t - d_{t-1}\right) + \varepsilon_t.$$

(p.64) Hence,

(2.25)

$$\text{cov}\left(\Delta p_{t+1}, \Delta p_t\right) = \text{cov}(\bar{r}_{t+1}, \bar{r}_t) - \frac{S^2}{4}.$$

Thus, the covariance of change in prices now depend on two factors: (1) the bid-ask bounce $\left(-\frac{s^2}{4}\right)$, whose effect is to make change in prices negatively correlated, and (2) the covariance in expected returns ($\text{cov}(\bar{r}_{t+1}, \bar{r}_t)$). If the latter is positive, then Roll's estimate is biased again. By equation (2.25), an unbiased estimator of the bid-ask spread is

(2.26)

$$S_d = 2\sqrt{-\operatorname{cov}\left(\Delta p_{t+1}, \Delta p_t\right) + \operatorname{cov}(\bar{r}_{t+1}, \bar{r}_t)}.$$

If expected returns are positively autocorrelated ($\operatorname{cov}(\bar{r}_{t+1}, \bar{r}_t) > 0$), S_d is larger than the Roll's estimator, which therefore again underestimates the true spread. George, Kaul, and Nimalendran (1991) discuss various ways of correcting for this problem through suitable time-varying estimators of $\bar{r}_t$.

The serial correlation in expected returns may explain why in some samples we find a positive covariance of price changes: in equation (2.25) this happens if $\operatorname{cov}(\bar{r}_{t+1}, \bar{r}_t)$ is positive and large enough to swamp the bid-ask bounce effect. This was reported even in Roll's original 1984 study, which implements his estimator using daily returns (based on closing prices) and finds that roughly half of the cases show positive serial correlation.[9] Then according to equation (2.3.4), the spread estimate would be the square root of a negative number—a nonsensical outcome! Harris (1990) shows that positive autocovariances are more likely for low values of the spread. Therefore, in such cases a rough remedy would be to impute a value of zero to the estimated spread. In general, Roll's estimator works better the shorter the time interval over which the returns are measured, so that bid-ask bounce is larger relative to other determinants of returns.

To sum up, the bid-ask spread induces negative serial correlation in returns, that is, it is a source of price reversals. The idea that illiquidity is a source of reversals is quite general, as Chapter 3 demonstrates. So, estimators for the bid-ask spread can be obtained by measuring the covariance of subsequent returns. These returns can be measured at high frequency (e.g., from trade to trade) or at lower frequency (e.g., from daily close to close).[10] But the accuracy of these estimators depends on the properties of the order arrival and price processes, and they must be fine-tuned accordingly. For instance, exercise 4 asks the reader **(p.65)** to derive an estimator of the bid-ask spread in Roll's model when some trades occur at the midprice, as sometimes happens in reality.[11]

2.4. Implementation Shortfall

All the measures of trading costs presented so far are static in that they do not take the time dimension of execution quality into account. In practice, however, large orders from institutions are very often split among several brokers over time (to maintain secrecy about the real size of the desired trade), and the brokers themselves may trickle the orders slowly into the market to get better prices. Thus, there can be a delay between the moment in which a portfolio manager makes his investment decision and the time he starts implementing it. This delay can be costly if it is systematically associated with an adverse price movement. Moreover, the portfolio manager also faces the risk of a large adverse price movement before the order is entirely filled, perhaps obliging him to cancel part of his order, which results in an opportunity cost.

For these reasons, portfolio managers often use a more encompassing measure of execution quality, namely the "implementation shortfall," first proposed by Perold (1988). This gauge factors in not only the price impact of trades but also the opportunity cost of delayed or unexecuted orders. The basic idea is to benchmark the actual performance of a portfolio against a hypothetical "paper portfolio" in which all rebalancing trades are made instantaneously and fully at midprices. In these conditions, the difference between actual performance and that of the portfolio's paper benchmark is precisely due to all the trading frictions (delays, price impact, and partial execution) encountered by the portfolio manager in rebalancing his portfolio. Thus, the concept of implementation shortfall enables us to disentangle, in the overall return of a portfolio, the portion deriving from the investment strategy of the manager and that depending on the implementation of these ideas, which is the responsibility of the trading desks hired for this purpose.

To understand how the measure of implementation shortfall is constructed and used, suppose that a portfolio manager decides to buy q shares of a stock on day 0 and passes the order to a broker. He plans to evaluate his position at some point in the future, say at date t (his "investment horizon"). In the absence of trading costs (i.e., in a perfectly liquid market), the portfolio manager would buy the shares at price m_0 and evaluate them at time t at price m_t. On paper, his **(p.66)** dollar return is therefore

(2.27)

$$R_p = q(m_t - m_0).$$

In reality, however, the portfolio manager will typically buy the stock at a higher price than m_0 because the market is illiquid. And as we have seen, part of the order may go unexecuted. For instance, the manager may decide to stop buying if the price goes too high. If $m_t > m_0$, this results in an opportunity cost. To account for these costs, suppose that only a fraction κ of the manager's buy order is eventually filled, at average execution price $\bar{p}$. Thus, the actual return is

(2.28)

$$R_a = \kappa q(m_t - \bar{p}).$$

The implementation shortfall, IS, is the difference between the portfolio's theoretical, paper return and the actual return:[12]

(2.29)

$$IS \equiv q(m_t - m_0) - \kappa q(m_t - \bar{p}) = \underbrace{\kappa q(\bar{p} - m_0)}_{\text{execution cost}} + \underbrace{(1-\kappa)q(m_t - m_0)}_{\text{opportunity cost}}.$$

Expression (2.29) shows that the implementation shortfall consists of two components: execution cost and opportunity cost. The first term should be familiar by now: it captures the cost of the actual execution at the average price $\bar{p}$ rather than the benchmark price m_0 (i.e., the effective spread). The second component represents the opportunity cost of the forgone returns on the unexecuted portion of the order, $(1 - \kappa)q$. For example, imagine that a client submits a buy order for ten thousand shares when the midquote is \$100, and evaluates the performance of his broker five days later, when the broker has only bought three thousand shares at an average price of \$101. If the current price is

103, then the implementation shortfall is

$$3{,}000 \times (101-100) + 7{,}000 \times (103-100) = 3{,}000 + 7{,}000 \times 3 = 24{,}000.$$

To put this figure in perspective, consider that the initial value of the "paper" portfolio in this example is qm_0 = 1,000,000, so that the implementation shortfall is 2.4 percent of its value.

As observed earlier, there may be a delay between the moment when the portfolio manager makes the investment decision and the moment of implementation. The execution cost component of the implementation shortfall can itself be divided in two components to reflect the cost of this delay. To see this let τ be the date at which the buy order begins to be executed. Then, the execution cost component can be written

(2.30)

$$\kappa q(\bar{p} - m_0) = \kappa q(\bar{p} - m_\tau) + kq(m_\tau - m_0),$$

(p.67) where the second element $\kappa q(m_\tau - m_0)$ is the delay portion of the execution component of the implementation shortfall. Table 2.1 shows that in practice the delay portion accounts for a significant fraction of total execution cost.

For any given order, one or all components of the implementation shortfall may be negative, as when the broker manages to fill the order at a better average price than the initial price of the paper portfolio. This may happen when the price of the stock goes down after a buy order is issued, or alternatively if the broker is able to use limit rather than market orders. The opportunity cost is negative if the value of the stock declines over the investment horizon ($m_t < m_0$). In hindsight, failing to buy the security turns out to be the better choice and the opportunity cost is negative.

In practice, the amount of the implementation shortfall is not negligible in relation to portfolio returns.[13] For this reason, institutional investors increasingly focus on trading cost management in the effort to reduce this cost, often with the help of consulting services from brokerage houses, which now commonly provide so-called trading cost analysis (TCA) services to portfolio managers.[14] Typically, TCA inquires into the sources of the implementation shortfall on a portfolio manager's orders.

As with the other measures of trading costs, the average implementation shortfall over a large number of orders is a more meaningful gauge than its value for any given order, in that the average filters out the impact of random price variations. The average implementation shortfall is

$$\mathrm{E}(IS) \equiv \kappa \mathrm{E}(q(\bar{p} - m_0)) + (1-\kappa)\mathrm{E}(q(m_t - m_0)).$$

The opportunity cost component will be positive on average if the direction and size of the orders placed (q) is positively correlated with price changes over the portfolio manager's investment horizon ($m_t - m_0$) so that $E(q(m_t - m_0)) > 0$. In practice, this positive correlation may exist for several reasons: one's manager may trade based on the same signal as other market participants, the order may exert pressure on prices, or some traders may "front-run" the order (i.e., buy the security to resell it to the buyer at a higher price). Further, brokers often face a trade-off between the execution and the opportunity cost components. By trading patiently (as with limit orders), a broker can generally get a better price and so reduce the execution cost component. But as this strategy increases the risk of non-execution $(1 - \kappa)$, it results in a larger opportunity cost component on average.

(p.68) The optimal strategy given this trade-off depends on the broker's forecast of the dynamics of the midprice and market liquidity. Again consider a broker with a large buy order. Obviously, if he expects the price to increase, he should increase the execution speed. But, as Chapter 1 shows, buy market orders deplete liquidity on the sell side. In general, after a while, liquidity suppliers will post new quotes and the market will become more liquid. Resiliency, the speed at which liquidity returns to normal after a trade (e.g., how soon a limit order is replenished after the arrival of a large market order), is another dimension of market liquidity. If the resiliency of the market is high, the broker can accelerate the execution of his large order; if resiliency is low, he should trade more slowly. In highly resilient markets, therefore, brokers will be able to achieve both lower execution costs and lower opportunity costs.

2.5. Hands-on Estimation of Transaction Costs

We have presented many measures of transaction costs. The best way to appreciate similarities and differences is for the reader to estimate a few of them and see how their values depend on the characteristics of the stock and how they change in the course of the trading day. For this, we provide data from real-world securities markets, as training grounds for hands-on estimation. In the exercises of this chapter, the reader is prompted to use these data, some of which are available at http://www.oup.com/us/marketliquidity/.

2.6. Further Reading

Scholars have proposed several other measures of illi quidity, in addition to those presented here. The "effective tick" proposed by Holden (2006) is based on the idea that price observations tend to cluster at certain ticks, specifically at rounder increments (price clustering), only due to the bid-ask spread. The measure of the spread is obtained in two steps: first, one estimates the probability of each bid-ask spread from the observed frequencies of prices at the various ticks; second, these probabilities are used to compute the probability-weighted average of each effective spread size.

Another proposed estimator of the effective spread, the LOT measure Lesmond, Ogden, and Trzcinka (1999), is based on the idea that if no trading occurs on zero-return days, informed traders must have faced transaction costs exceeding the price change that their information implies. Therefore, the authors propose a maximum likelihood estimator for the transaction cost corresponding to the observed no-trading price interval.

Hasbrouck (2002) sets out two alternative methods for estimating Roll's measure. He notes that in Roll's model both the spread, S, and the variance **(p.69)** of the change in the value of the security, $\mathrm{var}(\varepsilon_t)$, are unknown parameters. They can be estimated either with the classical method-of-moments or with Bayesian techniques, which involve the specification of a prior. An additional argument for the Bayesian approach in this context is its ability to accommodate latent (unobserved) data, which in Roll's model include bids, asks, and trade direction indicators, and which are suppressed in the GMM estimation. Since Hasbrouck implements this Bayesian estimation via an iterative procedure known as the Gibbs sampler, the resulting estimate of the spread is known as "Gibbs estimate."

Hasbrouck (2009) compares the Gibbs estimate based on daily closing prices from CRSP with the effective cost of trading based on single-transaction trade and quote data from the TAQ database over the period 1993–2005, and finds that they are highly correlated. This suggests that less data-intensive estimates obtained at the daily frequency are a good approximation of the more precise estimates that can be obtained from intradaily-frequency data.

Goyenko, Holden, Lundblad and Trzcinka (2006) effect a similar comparison for a larger set of measures of effective spread and price impact, investigating how closely the measures computed at monthly and annual frequencies and based on daily data are correlated with the corresponding estimates based on intra-daily data from TAQ and rule 605 data. Again, their exercise is designed to identify the measures that, when computed on daily data, best approximate the behavior of their more data-intensive counterparts.

Determining what trading strategies are optimal to minimize implementation shortfall has become an important issue for practitioners: Bertsimas and Lo (1998) lay out the foundations for this type of analysis; a systematic treatment can be found in Kissell and Gantz (2003).

2.7. Appendix

We derive the expressions for the various estimators of the bid-ask spread in section 2.3.4. Let $\Delta d_{t+1} \equiv d_{t+1} - d_t$. Recall that

(2.31)

$$\begin{aligned}\mathrm{cov}(\Delta p_{t+1}, \Delta p_t) &= \mathrm{cov}\left[\left(\frac{S}{2} d_{t+1} - \frac{S}{2} d_t + \varepsilon_{t+1}\right), \left(\frac{S}{2} d_t - \frac{S}{2} d_{t-1} + \varepsilon_t\right)\right] \\ &= \mathrm{cov}\left(\frac{S}{2} \Delta d_{t+1} + \varepsilon_{t+1}, \frac{S}{2} \Delta d_t + \varepsilon_t\right) \\ &= \left(\frac{S}{2}\right)^2 \mathrm{cov}(\Delta d_{t+1}, \Delta d_t) + \left(\frac{S}{2}\right) \mathrm{cov}(\Delta d_{t+1}, \varepsilon_t) \\ &\quad + \left(\frac{S}{2}\right) \mathrm{cov}(\varepsilon_{t+1}, \Delta d_t) + \mathrm{cov}(\varepsilon_{t+1}, \varepsilon_t)\end{aligned}$$

(p.70) a) Relaxing the assumption of balanced order flow

Consider the case in which the order flow is unbalanced, that is, $\Pr(d_t) = \eta$. Other assumptions are as in Roll's baseline model. In this case, using equation (2.32), we have:

$$\operatorname{cov}\left(\Delta p_{t+1}, \Delta p_t\right) = \left(\frac{S}{2}\right)^2 \operatorname{cov}(\Delta d_{t+1}, \Delta d_t).$$

By definition

(2.32)

$$\operatorname{cov}(\Delta d_{t+1}, \Delta d_t) = \mathrm{E}\left(\Delta d_{t+1}\, \Delta d_t\right) - \mathrm{E}\left(\Delta d_{t+1}\right)\mathrm{E}\left(\Delta d_t\right).$$

To calculate cov(Δd_{t+1}, Δd_t), consider table 2.2, which shows the various possible realizations for (d_{t-1}, d_t, d_{t+1}) and their probabilities. The table thus allows us to calculate E(Δd_{t+1} Δd_t) and E(Δd_{t+1})E(Δd_t). Hence

$$\begin{aligned} \mathrm{E}\left(\Delta d_{t+1}, \Delta d_t\right) &= \eta^2\,(1-\eta)\,(-2)\,(+2) + \eta(1-\eta)^2\,(+2)\,(-2) \\ &= \eta\,(1-\eta)\,(\eta + 1 - \eta)\,(-4) = -4\eta\,(1-\eta), \end{aligned}$$

and

$$\mathrm{E}\left(\Delta d_t\right) = \eta\,(1-\eta)\,(-2) + \eta\,(1-\eta)\,(+2) = 0,$$

so that

$$\operatorname{cov}\left(\Delta d_{t+1}, \Delta d_t\right) = \mathrm{E}\left(\Delta d_{t+1}\, \Delta d_t\right) = -4\eta\,(1-\eta).$$

Substituting this expression into (2.31) and using the assumptions that the order flow and change in asset values are uncorrelated, we obtain

Table 2.2 Probability, Value, and Change of Trade Direction, if Pr(d_t) = η

Pr(d_{t-1})	η				1 − η			
$d_t - 1$	+1				−1			
Pr(d_t)	η		1 − η		λ		1 − η	
(d_t)	+1		−1		+1		−1	
Δd_t	0		−2		+2		0	
Pr(d_{t+1})	η	1 − η	η	1 − η	η	1 − η	η	1 − η
(d_{t+1})	+1	−1	+1	−1	+1	−1	+1	−1
Δd_{t+1}	0	−2	+2	0	0	−2	+2	0

(p.71)

$$\operatorname{cov}\left(\Delta p_{t+1}, \Delta p_t\right) = -4\eta\,(1-\eta)\,\frac{S^2}{4} = -\eta\,(1-\eta)\,S^2,$$

which is equation (2.19).

b) Relaxing the assumption of uncorrelated order flow

Now consider the case in which the direction of orders is first-order auto-correlated, with constant transition probability, $\Pr(d_{t+1} = d_t) = \delta$, while other assumptions are as in the baseline Roll model. To analyze this case, we have to modify the previous table as follows:

From table 2.3, we obtain

$$\begin{aligned} \mathrm{E}\left(\Delta d_{t+1}\,\Delta d_t\right) &= \frac{1}{2}(1-\delta)^2(-2)(+2) + \frac{1}{2}(1-\delta)^2(+2)(-2) \\ &= (1-\delta)^2(-4) = -4(1-\delta)^2 \end{aligned}$$

and

$$\mathrm{E}\left(\Delta d_t\right) = \frac{1}{2}(1-\delta)(-2) + \frac{1}{2}(1-\delta)(+2) = 0.$$

Hence, using equation (2.32),

$$\mathrm{cov}\left(\Delta d_{t+1}, \Delta d_t\right) = \mathrm{E}\left(\Delta d_{t+1}\,\Delta d_t\right) = -4(1-\delta)^2.$$

Substituting this expression into (2.31) yields

$$\mathrm{cov}\left(\Delta p_{t+1}, \Delta p_t\right) = -4(1-\delta)^2\frac{S^2}{4} = -1(1-\delta)^2 S^2,$$

which is equation (2.21).

Table 2.3 Probability, Value, and Change of Trade Direction, if Pr ($d_{t+1} = d_t$) = δ

$\Pr(d_{t-1})$	$\frac{1}{2}$				$\frac{1}{2}$			
$d_t - 1$	+1				−1			
$\Pr(d_t)$	δ		1 − δ		1 − δ		δ	
(d_t)	+1		−1		+1		−1	
Δd_t	0		−2		+2		0	
$\Pr(d_{t+1})$	δ	1 − δ	1 − δ	δ	δ	1 − δ	1 − δ	δ
(d_{t+1})	+1	−1	+1	−1	+1	−1	+1	−1
Δd_{t+1}	0	−2	+2	0	0	−2	+2	0

(p.72) 2.8. Exercises

1. Quoted spread for different trade sizes.

Consider the following LOB:

Bid			Ask		
Price	Size	Time	Price	Size	Time
74.42	300	11:49:39	74.48	300	11:49:35
74.41	100	11:46:55	74.48	500	11:49:40
74.36	400	11:48:30	75.74	100	08:25:17
74.36	400	11:48:32	76.00	150	08:02:02
74.00	13	10:56:00	76.77	20	07:01:01
73.75	5,100	11:28:02	77.00	100	09:15:00
72.98	5,100	10:57:39	77.06	200	10:14:11
72.15	120	08:01:39	77.35	1,000	08:01:39
72.00	20	07:01:01	77.82	20	07:01:01
72.03	20	07:01:01	78.00	300	08:02:00
72.00	100	07:46:19	78.38	1,000	09:30:04
71.59	50	08:02:02	78.60	375	08:01:32
71.11	20	07:01:01	78.64	500	09:30:04
71.00	10	09:30:36	78.87	20	07:01:01
70.35	200	08:00:54	78.95	200	08:01:35
70.11	20	07:01:01	80.00	350	09:15:00

From the data in this table, compute the weighted average quoted spread (in absolute and relative terms) for 100, 500, 1,000 and 2,000 shares. Which side of the LOB is deeper for transactions of 2,000 shares or more?

2. Measures of the bid-ask spread.

Your fund is considering trading 10-year bonds issued by the Austrian government, and you see that at 9:30 a.m. their lowest ask price is 102.31 and their highest bid price is 99.50. Five seconds later a buy order for a block of €10 billion is executed at 102.76. At 10:30 a.m. you check the market again and see that the lowest ask price is 102.55 and the highest bid price is 100.02.

a. Compute the absolute and the relative quoted spread at 9:30 and 10:30.
b. Compute the absolute and the relative effective ask-side half-spread at 9:30.
c. Compare the quoted half-spread with the effective ask-side half-spread (both in absolute and in relative terms) at 9:30. What explains the difference between them?
d. Compute the absolute realized spread in the 9:30–10:30 interval.
(p.73) **e.** Compare the realized spread computed under point d with the absolute effective spread at 9:30 computed under point b. What explains the difference between them?

3. Implicit bid-ask spread in call auction.

In a call auction there is no bid-ask spread, as all trades clear at a single price. However, there is an implicit spread, insofar as the order flow exerts price pressure: the difference between the hypothetical prices that would clear the market if one tried to buy and sell more shares. Specifically, the implicit bid-ask spread for a transaction of size q can be defined as the difference in market clearing price arising from an extra market order of size q to buy and an extra order of size q to sell. Using the data in exercise 1 of Chapter 1, compute the bid-ask spread for transaction sizes q = 50, 150, 250, and 350.

4. Roll's estimator and price improvements.

Consider Roll's model presented in section 2.3.4. All the assumptions are unchanged, but we do assume that the transaction occurring at time t occurs either at the ask or bid price with probability λ, or at the midprice with probability $1 - \lambda$. Thus, $1 - \lambda$ can be seen as the fraction of trades that receive a price improvement or are crossed by brokers at the midprice. Propose an estimator of the quoted bid-ask spread S for this case.

5. Empirical measurement of quoted spreads.

The data for this exercise are contained in an Excel file, Ch2_AGF_data.xls available on the companion website for the book: a record of one day's transactions in the shares of a French company, AGF, on the Paris Bourse. The data comprise:

- • time of the transaction
- • size of the transaction
- • (average) transaction price
- • best bid price immediately before the transaction
- • best ask price immediately before the transaction
- • direction of trade initiation (−1 for transactions below the midprice, +1 above the midprice)

a. For each transaction, compute the absolute spread S in euro (€); compute the relative spread, s and the log spread: ln(ask)—ln(bid); and compare the average of these three measures. Then compute the average absolute spread for each of the 17 half-hour time periods of the trading day and plot a graph of your results showing the intraday evolution of the spread. What kind of pattern over the day would you expect, a priori?
(p.74) **b.** Compute and compare the average effective trading cost or "half-spread" in absolute terms, relative terms and logs.
c. Compute the VWAP for the day. Then calculate the VWAP for buyer-and seller-initiated transactions for the whole day and compare your results with the VWAP benchmark. Repeat, again separately for buyer-and seller-initiated transactions, for transactions divided into three time periods: 9:00 a.m–12:00 a.m., 12:00–3:00, and 3:00–5:30 p.m.

d. Compute Roll's estimate of the bid-ask spread both in euro (€) and in relative terms (using the logarithm of the prices). Then repeat the computations clock time rather than transaction time: take the last transaction in every 15-minute time interval. Compare your results with those previously obtained: what explains the difference in Roll's measure?
e. Split the trading day into 15-minute intervals: 9:00–9:15 a.m., 9:15–9:30 a.m.,…, 5:15–5:30 p.m. For each interval, compute the midprice change (from the last transaction of the previous interval to the last one of the current interval; for the first interval take the midprice at 9:06:04 as the initial midprice) and the cumulative signed order flow over the interval expressed as a fraction of the day's total (unsigned) order flow. Perform a regression analysis on the 34 data points you obtain in this way, to estimate the price impact parameter λ. Is the estimated parameter significantly different from zero? What is the impact on the midprice of a 1% relative order flow increase?

6. Inferring trade direction.

A data set containing a 1 day time series of quote and trade data for Krispy Kreme, listed on the NYSE, is provided in the Excel file Ch2_KrispyKreme_raw_data.xls available on the companion website for the book. This data set contains both transaction data and quote revisions, but no trade direction indicator. (The variable "type" is equal to 1 for transaction data and 0 for quote data.)

a. For each quote revision, compute the absolute spread S in dollars ($), the relative spread s, and the log spread. Then compute the average absolute spread for each of the 14 half-hour time periods of the trading day and plot a graph of your results showing the intraday evolution of the spread.
b. Use the Lee-Ready trade classification algorithm to establish trade direction. Then generate a data set containing only transaction data (for each transaction consider the last bid price and the last ask price before the transaction takes place) of the same form as the AGF data set.

7. Further empirical transaction cost measurement.

Using the data set generated at point b of exercise 6 (or alternatively the data in the Excel **(p.75)** file Ch2_KrispyKreme_data.xls, which also contains the trade direction indicator):

a. Compute and compare the average effective trading cost or "half-spread" in absolute terms, relative terms, and logs.
b. Compute the VWAP for the day. Then calculate VWAPs for buyer-and seller-initiated transactions for the whole day and compare your results with the VWAP benchmark. Repeat, again separately for buyer-and seller-initiated transactions, for transactions divided into 3 time periods: 9:30 a.m-12:00 a.m., 12:00–2:00 p.m., and 2:00–4:00 p.m.
c. Compute Roll's estimate of the bid-ask spread both in dollar ($) and in relative

terms (using the logarithm of the prices); then repeat the computations using not transaction time but clock time: take the last transaction in every 15-minute time interval.

d. Split the trading day into 15-minute intervals: 9:30 a.m-9:45 a.m., 10:00–10:15 a.m.,…, 3:45–4:00 p.m. For each interval, compute the midprice change (from the last transaction of the previous interval to the last one of the current interval; for the first interval take the midprice at 9:30:02 as the initial midprice) and the cumulative signed order flow over the interval expressed as a fraction of the day's total (unsigned) order flow. Perform a regression analysis on the 26 data points you obtain in this way, to estimate the price impact parameter λ. Is the estimated parameter significantly different from zero? What is the impact on the midprice of a 1% relative order flow change?

8. Implementation shortfall.

Suppose that at time 0 your brokerage firm receives an order from a client who would like to buy a number q of shares in company XYZ, planning to hold them until some future time t. The midprice of XYZ is $m0$ when you receive the order, and the client expects its midprice to be m_t at time t. Although in a perfectly liquid market the client would like the entire purchase to go through at time 0, he realizes that this may not be in his best interest in the rather illiquid market for XYZ shares. So the client's mandate to your firm is "choose the fraction k of the total purchase q that is to be bought between time 0 and time t so as minimize the implementation shortfall." You know that the average price $\bar{p}$ at which you can buy shares for this client between time 0 and t is affected by how much you buy in that interval, according to the function

$$\bar{p} = m_0 + \lambda kq,$$

where λ is a price pressure parameter.

(p.76)

a. Write the expression for the implementation shortfall in this specific case (using the above expression for $\bar{p}$).

b. Determine the value of k that minimizes the implementation shortfall.

c. How does the optimal k found under b respond to changes in $m_t - m_0$, in q and in λ? What are the intuitive explanations for these comparative statics?

Notes:

(1.) One basis point (bp) is 0.01 percent. Thus, a total trading cost of 90bps means that for a trade of $100, the investor pays a total trading cost of 90 cents. This may seem a small amount but it is not if one considers that the trading volume in securities markets is often large. For instance, in June 2009, the total value traded on the LSE was £138 billion (source: LSE website). In this case, a 90bps trading cost implies a total illiquidity cost of about £1.24 billion.

(2.) Since 2000, the SEC has required platforms that trade stocks listed on Nasdaq or the

NYSE to report monthly statistics (known as Dash-5 reports) on their execution quality. These statistics include the average effective bid-ask spread, the average realized spread, and the speed of execution for orders of various sizes and types.

(3.) One reason for this is that some market orders may be placed by investors with advance information (see Chapters 3 and 4).

(4.) The information was drawn from the Nasdaq website in 2005. At present Nasdaq provides the raw data from which these measures can be computed, not the measures themselves.

(5.) The SEC requires realized spreads to be calculated by setting Δ at five minutes after the order is received.

(6.) Another approach is to regress the percentage change in the midquote on the order imbalance scaled by trading volume (i.e., the sum of buy and sell orders).

(7.) Goyenko et al. (2009) show empirically that the Amihud ratio is a good proxy for price impact, as it is highly correlated with high—frequency measures of price impact.

(8.) In their study, Bekaert, Harvey, and Lundblad (2007) also propose and construct another measure of illiquidity based on no-trade days. Their measure gives greater weight to no-trade days on which there are larger changes in the index of the market on which the stock is listed. The idea is that the change in the index should lead to a change in the price of individual stocks. When no such change is observed for a given stock, this is more likely to be due to absence of trading in this stock, possibly because of prohibitive trading costs.

(9.) In contrast, Stoll (2000) finds that for more than 99 percent of the stocks listed on Nasdaq and the NYSE, the covariance of trade-to-trade returns is negative, as Roll's model implies.

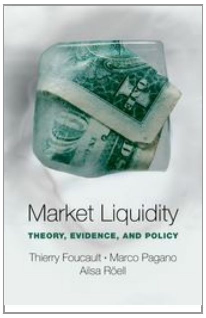
(10.) Nowhere do we impose that date t and date $t + 1$ correspond to successive transactions.

(11.) Another bias in Roll's estimator arises when there is no trade on a given day, in which case some data bases (such as CRSP) report the midquote instead of a transaction price. If these days are retained in the sample, the estimated cost will generally be biased downwards, because the midquote has no bid-ask bounce.

(12.) The implementation shortfall is computed symmetrically for a sell order, but in that case q is negative (q being the signed order size).

(13.) For instance, Leinweber (1995) reports that the paper return of the Value Line Portfolio (based on the recommendations of the Value Line newsletter) was 26.2 percent over the period 1971–1991 while the actual return was 16.1 percent, implying an implementation shortfall of 10.1 percent.

(14.) For examples of services associated with TCA, see the Investment Technology Group (ITG) website at http://www.itg.com/offerings/.



Market Liquidity: Theory, Evidence, and Policy
Thierry Foucault, Marco Pagano, and Ailsa Roell

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Order Flow, Liquidity, and Securities Price Dynamics

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter provides a framework for understanding intraday stock price variations. The framework also helps to explain the factors that determine variables such as the order flow and the bid-ask spread. It also clarifies how price volatility, spreads, and order flow are interrelated, and lays the ground work for a more detailed, subsequent analysis of market design issues. The discussions cover price dynamics and the Efficient Market Hypothesis and with informative order flow; price discovery; price dynamics with inventory risk. The final sections provide suggestions for further reading and exercises.

Keywords: stock price, price variations, price volatility, order flow, bid-ask spread, Efficient Market Hypothesis, inventory risk

Learning Objectives:

- • Why and how orders move prices
- • Why there is a bid-ask spread
- • How prices are formed when orders convey information
- • The determinants of market illiquidity
- • What inventory risk is and how it affects prices

3.1. Introduction

In principle, asset prices change in response to news about fundamentals, namely, future cash flows and discount factors. But security prices fluctuate continuously, even within extremely short time spans and in the absence of price-relevant news. These movements are responses to incoming orders to buy or sell stocks.

Market microstructure theory explains how these intraday price movements are related to the order flow. It identifies illiquidity as one cause of these short run fluctuations, which can be either transient or longer lasting depending on the motives underlying orders and the way market participants interpret them. If orders are seen as reflecting news about fundamentals, the price movements tend to be long lasting; otherwise, they tend to be reversed quickly.

The prevalence of very short-term price movements and their responsiveness to orders is illustrated in table 3.1, which reports trade and quote data for the **(p.78)**

Table 3.1. Sample Trading Session for Agf on Euronext, March 26, 2001

Time	Trade size	Price	Direction	Bid	Ask
t	$(1 q_t 1)$	(p_t)	(d_t)	(b_t)	(a_t)
90604	20	66.70	−1	66.90	67.00
90611	25	66.64	−1	66.65	66.70
90626	18	66.60	−1	66.60	66.65
90718	273	66.42	−1	66.50	66.55
90736	27	66.55	+1	66.15	66.55
91803	100	66.25	−1	66.25	66.35
91937	267	66.20	−1	66.20	66.30
92308	12	66.15	−1	66.15	66.25
92331	157	66.15	−1	66.15	66.20
92338	30	66.10	−1	66.10	66.15
92626	1,000	66.20	+1	66.00	66.20
93010	1	66.20	+1	66.05	66.20
93054	24	66.16	+1	66.05	66.20
93440	90	66.05	−1	66.05	66.20
93539	6	66.05	−1	66.05	66.20
93610	1,000	66.00	−1	66.00	66.20
93614	15	66.20	+1	66.00	66.20
93956	75	66.05	−1	66.05	66.10

first transactions of a typical trading day (March 26, 2001) for the stock of a large French insurance company, AGF.[1] This stock is traded on Euronext's electronic limit order market. For each transaction between 9:05 a.m. and 9:40 a.m., the table displays: (i) the time of the trade, t; (ii) its absolute size, $|q_t|$; (iii) the price, p_t, at which it is executed; (iv) the highest bid quote, b_t, and the lowest ask quote a_t on the limit order book immediately before the trade; and (v) the direction, d_t, of the trade, defined as $+1$ or -1 depending on whether the party initiating the trade (i.e., demanding liquidity) is buyer or seller. The (signed) *order flow* is thus the absolute trade size multiplied by its direction: $q_t = |q_t| \cdot d_t$.

Clearly, the price varies from one transaction to the next.[2] There is considerable movement even though the time interval is just thirty-five minutes, and **(p.79)** this pattern is in no way specific to this particular stock or period. This chapter serves precisely to provide a framework for understanding such intraday stock price variations. This framework also helps to explain the factors that determine the other variables appearing in table 3.1, in particular the order flow and the bid-ask spread. It will also clarify how price volatility, spreads, and order flow are interrelated. Lastly, it will lay the ground work for a more detailed, subsequent analysis of market design issues.

We start with the Efficient Market Hypothesis (EMH), a pillar of modern finance theory, according to which security price changes are induced by the arrival of new information and should follow a random walk. In section 3.2 we briefly review the EMH to observe that, while useful as a starting point, it does not exactly address the way information gets embodied in asset prices; in particular, it does not assign any explicit role to the variables reported in table 3.1, such as the bid and ask quotes and the order flow. This is problematic, as price changes do appear to relate to these variables. Specifically, when markets are not perfectly liquid (that is, when the spread is not zero), buy market orders push prices upward, and sell market orders push them downward. The larger the order, the sharper is the price movement, and price volatility is related to the order flow.

These empirical patterns can be reconciled with the EMH by recognizing that the order flow itself provides new information. A series of buy orders is a signal that informed traders may be buying because the stock is undervalued, just as a series of sell orders suggests it may be overvalued. Accordingly, liquidity suppliers should revise their expectations concerning the stock's value based on the observed order flow, and price changes should be partly determined by the order flow.

Section 3.3 sets out a trading model that formalizes this intuition. The model yields several insights. For instance, it explains the existence of the bid-ask spread and relates price changes to the order flow. In this model, the bid-ask spread is a compensation required by liquidity suppliers to offset their potential losses on trades with better informed investors. Thus, the asymmetry of information between the suppliers and the demanders of liquidity determines the market's degree of liquidity.

But in reality, liquidity is also determined by other factors. In particular, liquidity suppliers need to cover the cost of executing orders and maintaining a presence in the market (so-called order-processing costs). Moreover, imperfect competition among them generates rents, which take the form of bid-ask spreads greater than order-processing costs. In section 3.4, we enrich **(p.80)** the model to include both order-processing costs and the rents due to imperfect competition among liquidity suppliers.

Another reason for the existence of a bid-ask spread is the fact that dealers are risk averse. As prices fluctuate during the trading day, liquidity suppliers are exposed to variations in the value of their positions ("inventory risk"), because these positions cannot necessarily be unwound immediately. As a dealer accumulates a long position in the stock, he is exposed to the risk of a price drop; conversely, a short position entails the danger of a rise in the price. If the dealer is risk averse, his quotes should depend directly on his exposure to inventory risk. Section 3.5 analyzes the impact of inventory risk and dealers' risk aversion on bid and ask quotes.

Overall, the models presented in this chapter describe three kinds of cost for liquidity suppliers: (i) the cost of trading with better informed investors (adverse selection costs), (ii) the real cost of processing orders (order-processing costs), and (iii) the cost of holding risky assets (inventory holding costs). Since liquidity suppliers need a larger bid-ask spread to compensate for these costs, each cost category will contribute to market illiquidity. In addition, oligopolistic liquidity suppliers may charge a markup that also fosters illiquidity. Understanding the relative importance of these determinants of liquidity is important for policy-making and market design. For instance, if order-processing costs predominate, then a change in the trading system might be called for. But if adverse selection costs are the main factor, improvements in disclosure and greater parity between traders should be considered.

Thus, it is important to distinguish between the various costs borne by liquidity suppliers. The models presented in this chapter suggest a way to assess their relative importance, because they show that different costs carry different implications for short-term price movements after the execution of a market order. Even though the immediate impact of a market order is the same in all cases (a buy pushes the price up; a sell, down), the subsequent price dynamics differ with the source of market illiquidity, as explained here (summarized in Section 3.6). Specifically, if the bid-ask spread is compensation for adverse selection costs, then the price impact of a market order is permanent, as it leads liquidity suppliers to revise their estimate of the security's value. But if the spread is compensation for order-processing costs or inventory risk, then the price impact of a market order is transient and should dissipate over time; that is, these kinds of cost induce reversals in returns. Moreover, the speed of the reversal differs depending on the relative importance of inventory holding vis-à-vis order-processing costs.

Casual observation of price movements during the day suggests that the immediate price impact of market orders actually has both a permanent and a transient component: in the real world, all kinds of costs matter. Chapter 5 **(p.81)** describes various empirical techniques to exploit the implications of the models presented here for assessing the relative importance of adverse-selection costs, order-processing costs, and inventory holding costs.

The analysis in this chapter is based on the simplifying assumption that all orders are for a fixed number of shares, which we normalize to 1 so that $q_t = dt \in \{-1,+1\}$.[3] Trades occur when investors ("liquidity demanders") submit orders to buy or sell; the trades are registered in the sequence in which they occur, just as in table 3.1. Thus, p_t is the price at which the t^{th} trade takes place, and d_t indicates whether the investor initiating the t^{th} trade is a buyer or a seller. Liquidity is supplied by a pool of traders, whom we call "dealers" or "market makers," but who could also be limit order traders: in this framework, the precise nature of those providing liquidity is irrelevant. Dealers post their bid and ask quotes, at which they are willing to execute sell or buy orders. We denote by b_t^i the bid price posted by dealer i at time t (i.e., what dealer i bids for the stock). Similarly, we denote by a_t^i the ask (or offer) price posted by dealer i at time t (i.e., the price that he asks for the security or at which he offers it for sale). We denote the best ask price by a_t and the best bid price by b_t:

$$a_t = \min_i \left\{a_t^i\right\} \quad \text{and} \quad b_t = \max_i \left\{b_t^i\right\}.$$

Investors trade with the dealer that posts the best price: they buy at the lowest ask price a_t and sell at the highest bid price b_t. So the transaction price at time t is either a_t or b_t, depending on the direction of the order that triggers it.

3.2. Price Dynamics and the Efficient Market Hypothesis

Some football clubs are listed on stock exchanges. Not surprisingly, their stock price is very sensitive to their on-field performance: a good showing brings more fans and television coverage, swelling expected revenues. So the outcome of matches affects the club's stock market value. Most matches take place on week-ends. On Monday, the stocks of winners rise typically and those of losers fall (Palomino et al., 2009). This illustrates a general principle: new information is a source of price changes. For any security, new information leads investors to revise their estimates of future cash flows. The price adjusts to reflect investors' updated valuation.

The speed of this adjustment is an important issue. For instance, imagine that Ajax Amsterdam meets Juventus in the UEFA Champions League, and Ajax wins in an upset. Assume the closing price for Ajax on the last trading day before **(p.82)** the match was €7. In view of Ajax's victory, investors revise their estimate of its stock value (i.e., the discounted value of its expected future dividends) to €7.20. How does its stock price evolve on the trading day after the match? One scenario could be that it will gradually approach its new fundamental level of €7.20. For instance, a first trade takes place at $p1$ =€7.10, a second at $p2$ =€7.15, and so on.

The EMH, however, holds that the adjustment should be instantaneous.[4] According to the EMH, at any point in time, trades are made at a price that is equal to the best possible estimate of the value of the asset, incorporating all available information. We shall refer to this as the "fundamental value" v of the security, which can be thought of as the value once trading is over. Formally, the EMH states that

(3.1)

$$pt = \mu_t \equiv \mathrm{E}\left(v|\Omega_t\right),$$

where μ_t is the market makers' estimate of the security's value v as of time t, and Ω_t denotes the information available to them at that time. The notation $\mathrm{E}(v|\Omega_t)$ indicates the expected value of v conditional on information Ω_t. The absence of a discount factor is justified by the shortness of the interval (intraday).

The conditional expectation μ_t can change only if new information reaches the market. Let the random variable $\varepsilon_{t+1} = \mu_{t+1} \sum \mu_t$ represent the revision in investors' value estimates induced by the news arriving between time t and t +1, also known as the "innovation" in value. As the innovation at any time captures the effect of news, it cannot be forecast using past information: otherwise, news would not really be news! This means that $\mathrm{E}(\varepsilon_t\varepsilon_s) = 0$, for $s \neq t$ (otherwise past innovations could be used to forecast future ones), and that $\mathrm{E}[\varepsilon_{t+1}|\Omega_t] = 0$, which implies that the expectation of μ_{t+1} as of time t is simply μ_t:

(3.2)

$$\mathrm{E}\left[\mu_{t+1}|\Omega_t\right] = \mu_t.$$

As $p_t = \mu_t$ at each date, it follows immediately that

(3.3)

$$\mathrm{p}_t = \mathrm{E}\left(\mathrm{p}_{t+1}|\Omega_t\right).$$

Equation (3.3) carries an important implication for the dynamics of stock price: namely, that the best predictor of future prices, given currently available information, is the current price. In other words, under the EMH transaction prices follow a *martingale*: price changes over a given interval are serially uncorrelated. This can be seen by computing the change of p_t from **(p.83)** equation (3.1):

(3.4)

$$\Delta p_{t+1} = p_{t+1} - p_t = \mu_{t+1} - \mu_t = \varepsilon_{t+1},$$

so that cov(Δp_t, Δps) = $cov(\varepsilon_t, \varepsilon_s) = 0$, since $E(\varepsilon_t \varepsilon_s) = 0$, for $s \neq t$.

Under the EMH, what drives intraday price changes is new information, and assets' prices adjust immediately to the fundamental values given the available information. Hence, any change in price must be due to completely unanticipated information, so that changes cannot be predicted from past information, and particularly from previous price changes.

Let us return to the football club example. On the trading day after the match with Juventus, Ajax opens at €7.20. According to the EMH all subsequent trades should be at this price as long as no new information about Ajax arrives. But suppose that later Ajax announces that its goalkeeper was injured during the match and will be unable to play in future matches. If this worsens Ajax's prospects for the rest of the season, traders mark down their expectations regarding the value of Ajax ($\varepsilon_{t+1} < 0$), and the price immediately drops by $|\varepsilon_{t+1}|$.

The question, however, is how markets become informationally efficient. In other words, what is the *process* whereby equation (3.1) comes to hold? To answer, we need to be more specific about the details of the trading process and the nature of the information that market participants have. Suppose, for instance, that (i) dealers are risk neutral and competitive, (ii) investors do not have more information than dealers, and (iii) trading is cost free. Assumption (ii) implies that the order flow does not contain information; that is, that innovations ε_t and the direction-of-trade indicator, d_t, are independent. If so, dealers have no reason to update their value estimate when they receive buy or sell orders.

Under these circumstances, there is a zero bid-ask spread, and prices do not change in response to orders. To see this, consider how a dealer i sets his ask price at time t, a_t^i. If he sells the stock at price a_t^i, he obtains an expected profit equal to

$$\mathrm{E}\left[\left(a_t^i - v\right)|\Omega_t\right] = a_t^i - \mu_t.$$

Thus, the lowest price at which the dealer is willing to sell is his estimate of the stock's value, μ_t. Since this is the case for all market makers, competition drives their ask price to μ_t. A similar argument shows that the bid price is also μ_t. Thus, dealers offer to execute all orders at price μ_t, the bid-ask spread is nil, and any price movement is entirely attributable to the arrival of public information, not to the order flow.

This is a useful benchmark model, but it fails to capture some important aspects of the intraday trading process. First, empirical studies have shown that **(p.84)** intraday price volatility is too great to be explained solely by news (French and Roll, 1986, and Roll, 1988). This suggests that the trading process itself is a source of volatility. Second, the model fails to capture the simple fact that positive bid-ask spreads are the norm ($a_t - b_t >$ 0). Lastly, in practice, intraday changes in prices are often negatively correlated. For instance, for the data reported in table 3.1, the correlation between successive price changes is negative (−0.45), in line with the empirical evidence summarized by Stoll (2000).[5]

In the rest of this chapter, we show that these different features of the intraday trading process can be captured by relaxing assumptions (i), (ii), and (iii), which is to say, by introducing*frictions*. As we shall see, then, it is no longer the case that the transaction price p_t is equal to the dealers' estimate of the asset value, μ_t. What does determine the price changes between one transaction and the next, Δp_t, and their volatility, var(Δp_t)? What are the determinants of the bid and ask quotes, a_t and b_t? What is the relationship between price changes and order flow, q_t? Are changes in prices still serially uncorrelated? We now turn to these questions.

3.3. Price Dynamics with Informative Order Flow

We have seen that under the EMH the price of securities reflects investors' beliefs about these securities' fundamental values, which are continuously updated on the basis of new information, as captured by the term ε_t in equation (3.4). Where does this information come from? It could be generated by public announcements about company performance and macroeconomic news (say, interest rate changes). For instance, prices react to companies' announcements of dividends, insofar as the announcement is really a surprise. But not all information is broadcast simultaneously to all market participants: some investors may be privy to price-relevant information before other market participants, possibly thanks to tips from the company's management or because they have superior analytical skills.[6]

When these investors trade on their private knowledge, their orders convey information to the rest of the market, over and above what is already publicly available. In this environment, market participants will revise their estimate of **(p.85)** securities' values in light of the order flow: unusual buying pressure will induce price increases; unusual selling pressure, price declines.

This feature can explain the existence of a bid-ask spread. To understand this point intuitively, consider the following argument, first presented by Jack Treynor in 1971 under his pseudonym "Bagehot." In principle, dealers are not always as well informed as each and every customer. Traders with superior information will exploit any mispricing by dealers, buying when the ask price is lower than the fundamental value and selling when the bid price is higher. Dealers lose money when they trade with such investors. This is known as "adverse selection": due to the informational asymmetry, market makers tend to attract customers who expect to make a profit at the dealer's expense.[7] To recoup their losses on informed orders, dealers must gain on their business with other traders. They achieve this by means of the bid-ask spread vis-à-vis all customers.

We develop these ideas in the context of a simplified version of the model proposed by Glosten and Milgrom (1985), a cornerstone of empirical and theoretical developments in this field.[8] In this model the ask price exceeds the bid because the former is set in anticipation of receiving a buy order, the latter a sell order:

(3.5)

$$\begin{aligned} a_t &= \mathrm{E}\left(v|\Omega_{t-1}, d_t = +1\right), \\ b_t &= \mathrm{E}\left(v|\Omega_{t-1}, d_t = -1\right), \end{aligned}$$

where Ω_{t-1} is the information known right up to the last trade d_t (recall that in this chapter $q_t = d_t$, since order sizes are normalized to 1). A positive bid-ask spread arises because of the informational content of the order flow d_t, which leads dealers to see orders as a source of information additional to that previously available, Ω_{t-1}.

3.3.1 The Glosten-Milgrom Model

Suppose that some traders have better information than dealers: assume that the order at time t is placed by a risk neutral *informed* trader with probability π. An informed trader is defined as one with advance knowledge of v. With the complementary probability, $1 - \pi$, the order comes from a *liquidity* trader, who **(p.86)** places a market buy or sell order with probability 1/2 each. A liquidity trader is an investor who trades for reasons unrelated to information about the value of the security: he may be an individual who needs cash for an unanticipated contingency, or a fund manager, who has to invest a recent cash inflow or rebalance the portfolio.

For simplicity, the terminal value v has binary distribution, that is, it can take two values: v^H or v^L, with $v^H \rangle v^L$. Let θ_t and $1 - \theta_t$ be the probabilities that dealers assign to the occurrence of a high value, v^H, and a low value, v_L. These probabilities reflect the market makers' views *before* observing the $(t+1)^{th}$ order. The dealers' estimate of the value *after* the t^{th} order is therefore

(3.6)

$$\mu_t = \theta_t v^{\mathrm{H}} + (1 - \theta_t) v^{\mathrm{L}}.$$

As is explained below, dealers' beliefs about the value of the security evolve because the order flow brings new information. In reality, dealers also get information from other channels (economics news, corporate announcements, etc.). But in order to focus on the informational role of the order flow, here we assume that orders are the sole source of new information for dealers. Formally, this means that $\Omega_t = \{d_1, d_2, ..., d_t\}$.

Notice that we include the order submitted at date t in the information set used by dealers to determine their quotes at this date.[9] How can that be, if dealers set their quotes *before* observing the order flow at date t? Here, it is important to recall that bid and ask quotes are *contingent* offers: a bid price is the price the dealer offers contingent on receiving a sell order ($d_t < 0$), and the ask price is an offer contingent on receiving a buy order ($d_t > 0$). Thus, the answer to the question, "What is the price at which I should execute an order?" must be identical before and after the order is actually received (in this sense, quotes are "with no regrets").

The setting is very similar to the framework developed in section 3.2, with the difference that now we are more specific about the source of dealers' information. Thus, using the findings of section 3.2, the ask and bid quotes set by competitive dealers will be

(3.7)

$$a_t = \mu_t^+ = \mathrm{E}(v|\Omega_{t-1}, d_t = +1),$$
$$b_t = \mu_t^- = \mathrm{E}(v|\Omega_{t-1}, d_t = -1),$$

where μ_t^+ is the dealers' estimate of the security's final value if they receive a buy order at time t, and μ_t^- if they receive a sell order. If the order flow contains **(p.87)** information, then it affects dealers' beliefs and we necessarily have $\mu_t^+ \neq \mu_t^-$; hence; $a_t \neq b_t$.

How does the order flow affect dealers' value estimate? For instance, on receiving a buy order should they revise it upward or downward? The answer depends on the *correlation* between the order flow and the value. which is ultimately determined by the way informed traders behave. In this model, they are assumed to be risk neutral and to have only a single trading opportunity. They therefore simply maximize their expected gain from trading, $(v - p_t\, d_t$, where d_t is their order and p_t is the transaction price (at for a buy and b_t for a sell order). As long as bid and ask quotes lie between v^L and v^H (which will be seen to hold in equilibrium), informed investors will always buy ($d_t = +1$) if they observe $v = v^H$ and sell ($d_t = -1$) if they observe $v = v^L$.

This observation has an immediate implication: the likelihood of a buy order is greater when $v = v^H$ than when $v = v^L$. Conversely, the likelihood of a sell order is lower when $v = v^H$ than when $v = v^L$. This is easily checked. A buy order arises in either of two ways: with probability $1 - \pi$ a liquidity trader arrives and buys with probability 1/2; with probability π, an informed trader arrives and buys (with probability 1) only if $v = v^H$. Hence, if $v = v^H$, the probability of a buy order is $(1 - \pi)/2 + \pi = (1 + \pi)/2$; but if $v = v^L$, the probability is just $(1 - \pi)/2$ because the informed traders do not buy. Thus, whenever $\pi > 0$, the market maker is more likely to receive a buy order when the security has high rather than low value. For sell orders, the converse is true.

This observation captures the intuitive idea that a sequence of buy (sell) orders is more likely when informed traders are privy to good (bad) news about the actual value of the security. Thus, informed trading induces a positive correlation between order flow and value. Buy orders are in fact good news: a *signal* that the true value of the security is high. Conversely, sell orders are bad news, signalling low true value. Accordingly, dealers must mark their value estimate up when they receive a buy order, and down when they get a sell order:

$$\mathrm{E}(v|\Omega_{t-1}, d_t = +1) > \mathrm{E}(v|\Omega_{t-1}, d_t = -1),$$

which implies, $a_t > b_t$. The bid-ask spread is therefore a natural consequence of the fact that buy and sell orders convey different messages: a dealer is willing to execute buy orders at a premium and sell orders at a discount relative to his own estimate μ_{t-1} of the security's value before the t^{th} transaction.

3.3.2 The Determinants of the Bid-Ask Spread

We now compute the ask and bid prices in this model. First consider the determination of the ask price at time t. In a perfectly competitive market with risk neutral dealers, it will be driven down to the point where dealers make zero **(p.88)** expected profits, that is, the point at which, on average, the losses they make on trading with informed investors are offset by their profits on business with liquidity traders. The market maker must consider that he may be in one of two possible situations:

(i) He may sell to an informed investor and so lose $a_t - v^H$, as informed investors buy only when $v = v^H$. At time $t-1$, when he sets the ask price a_t, the market maker assigns a probability $\pi\theta_{t-1}$ to an informed buy (i.e., the probability π of an informed investor being active on the market multiplied by the probability θ_{t-1} that the asset has value v^H).
(ii) He may sell to a liquidity trader and so book a profit of $a_t - \mu_{t-1}$: *conditional on trading with a liquidity trader*, the dealer's estimate of the final value of the security remains unchanged at μ_{t-1} since no new information has emerged since time $t-1$. The probability of receiving a buy order from a liquidity trader is $(1-\pi)/2$ (i.e., the probability $1-\pi$ of a liquidity trader being active on the market multiplied by the probability $\frac{1}{2}$ that he buys).

The possible trading events that can occur on the buy and sell sides at time t are summarized in table 3.2.

Using these probabilities, we can write the dealer's expected net profit from transactions at the ask price a_t:

(3.8)

$$\underbrace{\theta_{t-1}\pi\cdot\left(a_t - v^{\mathrm{H}}\right) + 0\cdot\left(a_t - v^{L}\right)}_{\text{expected profit from trading with in formed customer}} + \underbrace{\frac{1}{2}(1-\pi)\cdot\left(a_t-\mu_{t-1}\right)}_{\text{expected profit from trading with uninformed customer}} + \underbrace{\left\{(1-\theta_{t-1})\pi + \frac{1}{2}(1-\pi)\right\}}_{\text{probability of no ask-side customer}}.0.$$

To give intuition some help, we focus first on the special case of a market maker who is choosing, at time 0, the first ask and bid quotes of the trading day,

Table 3.2. Transaction Probabilities and Underlying Values by Direction of Trade and Trader Identity

Transaction	Identity of trader	Joint probability	Conditional value
	Informed: $v = v^H$	$\theta_{t-1}\,\pi$	v^H
Buyer ($d_t = +1$) at a_t	Informed: $v = v^L$	0	v^L
	Uninformed	$\frac{1}{2}(1-\pi)$	μ_{t-1}
	Informed: $v = v^H$	0	v^H
Seller ($d_t = -1$) at b_t	Informed: $v = v^L$	$(1-\theta_{t-1})\pi$	v^L
	Uninformed	$\frac{1}{2}(1-\pi)$	μ_{t-1}

(p.89) a_1 and b_1. Assume that this market maker estimates the security's value at its unconditional mean $\mu_0 = (v^H + v^L)/2$ (i.e., assigns probability $\theta_0 = 1/2$ to the value being high). Then the expression for the expected net profit (3.8) becomes:
(3.9)

$$\underbrace{\frac{1}{2}\pi\cdot\left(a_1 - v^{\mathrm{H}}\right)}_{\text{expected profit from trading with informed customer}} + \underbrace{\frac{1}{2}(1-\pi)\cdot\left(a_1-\mu_0\right)}_{\text{expected profit from trading with uninformed customer}}.$$

If dealers are competitive, a_1 will be such that this expected profit is zero, so that

(3.10)

$$b_1 = \mu_0 + \pi\left(v^L - \mu_0\right) = \mu_0 - \underbrace{\frac{\pi}{2}\left(v^H - v^L\right)}_{s_1^b},$$

Hence the ask price at time 1 includes a markup $s_1^a = \frac{\pi}{2}\left(v^H - v^L\right)$ over the initial best estimate of the security's value. Naturally, this markup occurs only if there are informed traders ($\pi > 0$) and if they have price-relevant information (that is, $v^H > v^L$). Symmetrically, the bid price b_1 will be

(3.11)

$$b_1 = \mu_0 + \pi\left(v^L - \mu_0\right) = \mu_0 - \underbrace{\frac{\pi}{2}\left(v^{\mathrm{H}} - v^L\right)}_{s_1^b},$$

so that the bid price is at a discount $s_1^b = \frac{\pi}{2}\left(v^H - v^L\right)$ on the initial estimate. Hence, the bid-ask spread for the first transaction of the day will be:
(3.12)

$$\mathrm{S}_1 \equiv a_1 - b_1 = \pi\left(v^H - v^L\right).$$

This makes it clear that the bid-ask spread is a compensation required by dealers to cover the loss they incur when trading with better informed investors. This loss is often referred to as the cost of adverse selection, and accordingly known as the adverse-selection cost component of the spread. And while it refers only to the first spread of the day, equation (3.12) reveals that the adverse selection cost is an increasing function of two variables:

(i) *The proportion of informed traders, π*: The more prevalent are the orders placed by informed traders, the greater the risk of loss for dealers, who accordingly require more compensation for supplying liquidity. This shows how asymmetric information generates illiquidity, motivating policies, such as bans on insider trading and disclosure requirements, that are intended to reduce informational asymmetries.
(p.90) (ii) *The volatility of the security's value, as proxied by its range of variation, $v^H - v^L$*: The broader the range of possible values, $v^{\mathrm{H}} - v^L$, the larger the losses incurred by dealers in trading with informed investors. Accordingly, dealers require more compensation (i.e., a wider bid-ask spread) as volatility increases. And in practice, other things being equal, spreads tend to be wider for more volatile securities (see Stoll 2000, table I).

The same reasoning allows us to derive the quotes the dealer will set later during the day, after revising his value estimate on the basis of the orders received: the probability θ_{t-1} that he places on the security being of high value, at some later time $t-1$, will presumably no longer be 1/2. Let us turn back to expression (3.8) for the profit the market maker expects from time t transactions at the ask. Setting this equal to zero, we get the competitive ask price at time t:

(3.13)

$$a_t = \mu_{t-1} + \frac{\pi\theta_{t-1}}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}\left(v^H - \mu_{t-1}\right) = \mu_{t-1} + \underbrace{\frac{\pi\theta_{t-1}\left(1-\theta_{t-1}\right)}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}\left(v^H - v^L\right)}_{s_t^a}.$$

Thus, at any time during the trading day, dealers set their ask price a_t above the fundamental value μ_t-1. From the first expression, one sees that the difference $a_t - \mu_{t-1}$ is proportional to the probability of receiving an order from an informed trader, conditional on it being a buy.[10] The second expression is obtained by replacing the term μ_{t-1} in brackets with the lagged value of expression (3.6), that is, rewriting it in terms of the probabilities of the high and low outcomes. The same reasoning yields the bid price b_t (left to the reader as an exercise):

(3.14)

$$b_t = \mu_{t-1} - \underbrace{\frac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi(1-\theta_{t-1}) + (1-\pi)\frac{1}{2}}\left(v^H - v^L\right)}_{s_t^b},$$

implying that dealers bid to buy the security at a discount s_t^b relative to μ_{t-1}

(p.91) Using equations (3.13) and (3.14), we obtain the bid-ask spread S_t at time t:

(3.15)

$$S_t \equiv a_t - b_t = s_t^a + s_t^b$$
$$= \pi\theta_{t-1}(1-\theta_{t-1})\left(\frac{1}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}} + \frac{1}{\pi(1-\theta_{t-1}) + (1-\pi)\frac{1}{2}}\right)\left(v^H - v^L\right).$$

This expression for the spread is more general than (3.12), which is the special case of $\theta_{t-1} = \frac{1}{2}$. It remains true that the spread increases with the proportion of informed traders (π) and the volatility of the security's value ($v^H - v^L$). In addition, expression (3.15) shows that the adverse selection trading cost S_t has a third determinant, namely, dealers' beliefs about the value of the security, θ_{t-1}. Unlike the first two, this determinant varies over time, as the market maker changes his value estimate. The spread is greatest when θ_{t-1} = 0.5, as posited in expression (3.12), and falls to zero as θ_{t-1} goes to 1 or 0. To understand this result, suppose that θ_{t-1} is close to 1. This means that dealers are quite confident that the fair value of the security is v^H, so a new buy order is not a surprise-no news-and does not move their value estimate: a_t is very close to μ_{t-1}. But when θ_{t-1} = 0.5, dealers are perfectly uncertain about the direction of the market, so their beliefs are highly sensitive to the new orders they receive. Accordingly, the revision of their value estimates is substantial. Therefore the spread is greater when dealers are more uncertain about the value of the security. This is why bid-ask spreads often increase in advance of the release of important information, such as macroeconomic data or earnings announcements. This may also explain why the spreads tend to be larger at market openings.

Let us highlight two features of this model. First, it is a highly stylized model of market making, with adverse selection costs the sole component of the bid-ask spread. As we shall see in sections 3.4 and 3.5, there may be additional determinants as well. Second, since in this model dealers' expected profit is zero, the adverse selection cost eventually bears on the liquidity traders, who-by "paying" the bid-ask spread-lose exactly what informed traders gain. In our setting, liquidity traders' demand is inelastic: it is independent of the price. Intuitively, in reality, one would expect them to be less likely to place orders when the bid-ask spread is greater. As exercise 6 shows, this situation can lead to a market breakdown, where the spread is so wide that no trade occurs at all. This suggests that asymmetric information may also lead to a complete market freeze, as in the interbank market during the crisis of 2007–08.[11]

(p.92) 3.3.3 How Do Dealers Revise their Quotes?

Up to now, in deriving the zero-profit bid and ask prices that competitive dealers will set, we have taken their beliefs about the underlying value of the security as *given*, that is, μ_{t-1} and θ_{t-1} are given. As bid-ask quotes depend on these beliefs (see equation (3.15)), in order to understand how quotes evolve over time, we must explain how dealers form their beliefs in the light of new orders. For instance, on receiving a flurry of buy orders, a dealer should attach more and more weight to the possibility that the security has a high value.

To see how this is done, consider how dealers receiving a buy or sell order at time t, revise the probability that they attach to the value being high, starting from an initial probability θ_{t-1}. Define θ_t^+ as the probability that dealers assign to high value, $v = v^H$, after they receive a buy order at date t, and let θ_t^- be the corresponding probability in the wake of a sell order. Formally,

$$\theta_t^+ \equiv \Pr\left(v = v^H \middle| \Omega_{t-1}, d_t = +1\right),$$
$$\theta_t^- \equiv \Pr\left(v = v^H \middle| \Omega_{t-1}, d_t = -1\right).$$

To compute θ_t^+ using Bayes' Rule,[12] let A be the event $v = v^H$ and B the arrival of a buy order at time t. Then $\Pr(A) = \theta_{t-1}$; $\Pr(B) = \pi\theta_{t-1} + (1-\pi)\frac{1}{2}$ (from table 3.2); and the probability of a buy order when $\pi \cdot 1 + (1-\pi)\frac{1}{2} = (1+\pi)\frac{1}{2}$, since in this case any informed trader will be a buyer. So the probability that $v = v^H$, conditional on a buy order at time t, is given by:

$$\theta_t^+ = \Pr(A|B) = \frac{\Pr(B|A)\cdot\Pr(A)}{\Pr(B)} = \frac{(1+\pi)\frac{1}{2}\cdot\theta_{t-1}}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}.$$

The derivation of θ_t^- is analogous:

(3.16)

$$\theta_t^+ = \frac{(1+\pi)\frac{1}{2}}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}\theta_{t-1},$$

(3.17)

$$\theta_t^- = \frac{(1-\pi)\frac{1}{2}}{\pi(1-\theta_{t-1}) + (1-\pi)\frac{1}{2}}\theta_{t-1}.$$

It is easy to see that $\theta_t^+ \rangle \theta_{t-1}$. Dealers become more bullish about the security when they get a buy order, insofar as buy orders signal that informed traders **(p.93)** may have good news. In a symmetrical way, $\theta_t^- \langle \theta_{t-1}$: dealers become more bearish when they get a sell order, as this indicates that informed traders may have bad news.

Upon receiving a buy order at time t, the dealers' updated expectation of the security's value is the weighted average of v^H and v^L, where θ_t^+ and $1-\theta_t^+$ are the updated probability weights:

(3.18)

$$\mu_t^+ = \theta_t^+ v^H + \left(1-\theta_t^+\right) v^L.$$

Recalling that the dealers' estimate before receiving the buy order was ε_{t-1}, their revision of the value in the wake of a buy order is:

(3.19)

$$\begin{aligned}\mu_t^+ - \mu_{t-1} &= \theta_t^+ v^H + \left(1-\theta_t^+\right) v^L - \left[\theta_{t-1} v^H + (1-\theta_{t-1}) v^L\right] \\ &= \frac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}\left(v^H - v^L\right) = s_t^a.\end{aligned}$$

Analogously, in the wake of a sell order

(3.20)

$$\begin{aligned}\mu_t^- - \mu_{t-1} &= \theta_t^- v^H + (1-\theta_t^-) v^L - \left[\theta_{t-1} v^H + (1-\theta_{t-1}) v^L\right] \\ &= -\frac{\pi\theta_{t-1}(1-\theta_{t-1})}{\pi(1-\theta_{t-1}) + (1-\pi)\frac{1}{2}}\left(v^H - v^L\right) = s_t^b.\end{aligned}$$

So s_t^a and s_t^b can be interpreted as the changes to dealers' estimates of the asset value: they correspond to

ε_{t+1} in the model described in section 3.2. The model here simply recognizes that the order flow itself is a source of information when orders can come from informed traders. Now that we know the dynamics of dealers' beliefs, we also know how they will set their quotes. As equation (3.7) shows, they will set their ask price in anticipation of a buy and therefore will set $a_t = \mu_t^+$. Conversely, they will bid $b_t = \mu_t^-$.[13]

Note that we have used two different but equivalent ways to interpret and derive bid and ask prices. In section 3.3.2, we obtained them by positing that dealers set price markups s_t^a and s_t^b so as to offset the potential losses on trades with better informed agents with the profits on business with uninformed traders. Here, instead, as illustrated by equations (3.19) and (3.20), we derived the markups s_t^a and s_t^b as the revisions to the dealers' estimate of the value of the **(p.94)** security following buy and sell orders. That this change is equal to the markup required by dealers to execute a buy order reflects the fact that the order flow itself contains information.

To sum up, the wedge between the dealers' ask price and their prior estimate of fundamentals is both a compensation for the risk of trading with better informed investors *and* an adjustment to the information contained in order flow. Of course, the two approaches produce the same result: equations (3.19) and (3.20) yield the same ask and bid prices as equations (3.13) and (3.14). The advantage of looking at the revision in beliefs is that it enables us to characterize how dealers change their quotes over time in response to the order flow. Therefore, if we specify a time series for the order flow, we can now map out the corresponding time path of quotes, as we do in the next section.

3.3.4 Price Discovery

Buy orders execute at the ask price, sell orders at the bid, so the transaction price at date *t* is

(3.21)

$$p_t = \begin{cases} a_t = \mu_t^+ & \text{if} d_t = +1, \\ b_t = \mu_t^- & \text{if} d_t = -1. \end{cases}$$

Hence

(3.22)

$$p_t = \mu_t = \theta_t v^H + (1 - \theta_t) v^L,$$

where $\theta_t = \theta_t^+$ if $d_t = +1$, and $\theta_t = \theta_t^-$ if $d_t = -1$. Thus, transaction prices reflect all the information available to market makers at time *t*, which is to say the information Ω_{t-1} they had at time $t - 1$ plus the direction of the order flow that they receive at time *t*.

Can we therefore conclude that transaction prices always reflect all available information, as claimed by the EMH? The notion of "all available information" is problematic in an environment in which traders have asymmetric information. Does it mean both public and private information, or only public information? In other words, in this context does the EMH apply in strong, semi-strong, or weak form?

The semi-strong form states that transactions take place at fair values given all *public* information. In our model, public information at date *t* is the order flow observed up to this date, $\Omega_t = \{d_1, d_2, \ldots, d_t\}$. As $p_t = \mu_t = \mathrm{E}(v \mid \Omega_t)$, the semi-strong EMH holds true in this model. This might seem surprising, insofar as the bid-ask spread is usually construed as a friction that causes temporary deviations of transaction prices from fundamental asset values. But in this model the difference between the execution prices for buy orders and sell orders is due entirely to the fact that they convey different information. Thus, far from being a symptom of inefficiency, the bid-ask spread is part of the mechanism by which **(p.95)** dealers incorporate the information contained in the order flow into the price process.

The strong version of the EMH states that transactions always take place at fair values given all public *and* private information. For instance, suppose that informed traders learn that $v = v^H$. If the EMH holds in strong form, then all trades take place at prices equal to v^H (insofar as market participants get no further information). At first glance, it seems impossible for markets to be informationally efficient in this sense. How could prices ever reflect information that is not yet available to all market participants? Yet in this model this is a possibility. To see this, suppose that $\pi = 1$. In this case, the quotes posted by dealers at time 1 (setting $\pi = 1$ in equations (3.13) and (3.14)) will be:

$$a_1 = v^H,$$
$$b_1 = v^L.$$

If $v = v^H$, then informed traders buy and the first transaction of the day is crossed at $p_1 = a_1 = v^H$.[14] Upon observing this transaction, dealers infer that the value of the security is high since all traders are informed and they are buying. Thus, $\theta_1 = 1$, and all subsequent transactions are at v^H (only insofar as no further information arrives, of course). Similar reasoning applies when $v = v^L$. Thus, the strong form version of the EMH holds, despite the fact that dealers initially have less information than some market participants. This is because dealers draw rational inferences from the order flow; that is, they *learn* about private information from the order flow and adjust their quotes accordingly.

Of course, this example is extreme (positing that $\pi = 1$), but it does suggest that prices can adjust gradually to the fair value of the security as dealers learn it from the order flow. The process by which private information is incorporated into prices and the market becomes informationally efficient is called *price discovery*. Thus, one advantage of the model presented here is that it allows us to study whether and how fast price discovery occurs.

Ultimately these questions bear on the dynamics of dealers' beliefs about the asset's fair value. For instance, suppose as before that $v = v^H$. Equation (3.22) shows that transaction prices will converge to v^H if θ_t goes to 1, in other words, if dealers are increasingly confident that the value of the security is high. Thus, the speed of price discovery is the speed at which θ_t goes to 1. In this model, it can be shown formally that price discovery always obtains in the long run if there are some informed traders in the market ($\pi > 0$) and that its speed increases with the fraction π of informed investors.

(p.96) A formal derivation of these claims is tedious, but the intuition is simple. In the absence of informed trading, the order flow is balanced, with 50 percent buy and 50 percent sell orders. However, with informed trading, the flow is unbalanced and its tendency provides information on value. For instance, if $v = v^H$, the proportion of buy orders will be more than 50 percent; if $v = v^L$, sell orders will exceed 50 percent. The speed of the learning process is then determined by how fast dealers become confident that buy orders make up more or less than 50 percent, which in turn is determined by the proportion of informed traders: buy orders will dominate very quickly if π is close to 1 and $v = v^H$; sell orders will predominate if $v = v^L$.

A good way to illustrate these intuitions is to simulate the model. Assume that $v^H = 102$, $v_L = 98$, and $\theta_0 = 0.5$. Thus, at the start of the trading day, dealers estimate the asset value at $\mu_0 = 100$. Initially, suppose that $\pi = 0.3$ and that the true value of the security is $v = v^H$. On this basis, we draw a sequence of one hundred orders (each of which is a buy with probability $\pi + (1-\pi)/2 = 0.65$), and record the evolution of dealers' beliefs θ_t associated with this specific sequence of order arrivals, as well as the evolution of transaction prices p_t. For instance, suppose that the two first orders are buys and the third is a sell. Equation (3.16) yields the corresponding dealers' beliefs: $\theta_1 = 0.65$, $\theta_2 = 0.77$, and $\theta_3 = 0.65$.[15] Equation (3.22) gives the corresponding sequence of transaction prices: $p_1 = 100.6$, $p_2 = 101.10$, $p_3 = 100.6$.

We repeat this experiment ten times, drawing ten different random sequences of one hundred orders with the same value of π. Figure 3.1 shows the pattern of dealers' beliefs in each experiment. Clearly, the probability assigned to the high value by the dealers converges to 1 (they eventually discover the correct value), and relatively quickly.

Next, we measure price discovery by computing the squared difference between the transaction price p_t at date t and v^H:

(3.23)

$$PD_t = \left(p_t - v^H\right)^2.$$

(p.97)

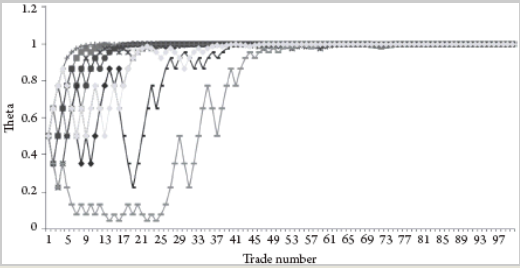


Figure 3.1. Evolution of dealers' beliefs

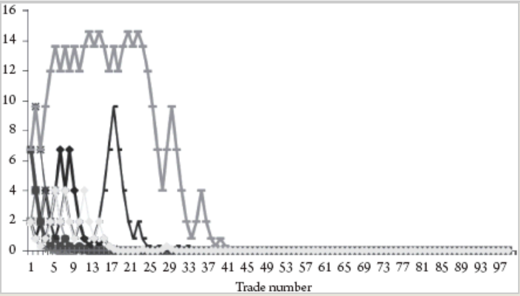


Figure 3.2. Evolution of pricing errors

As dealers learn about the value of the security, PD_t goes to zero; that is, it is an inverse measure of price discovery. Figure 3.2 shows the corresponding evolution for the pricing error in each experiment. Again, it declines quickly as dealers discover the true value.

To see how the proportion of informed traders π affects price discovery, we run the experiment again with three different values of π: 0.1, 0.5, and 0.9. Figure 3.3 shows the evolution of the squared pricing errors PD_t for these three values of π, averaged across the 10 simulations: $\sum_{i=1}^{i=10} PD_{it}/10$. The middle line summarizes the results for π = 0.5: on average, full price discovery is achieved after fifteen trades. The bottom line shows that when informed trading is predominant, to the point that nine in ten orders are placed by **(p.98)**

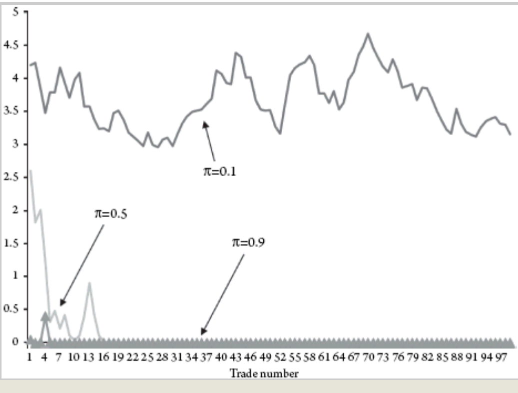


Figure 3.3. Speed of price discovery and the level of informed trading

informed traders, price discovery speeds up considerably: on average, full price discovery occurs after just five trades. In contrast, when informed trading is rare (only one in ten orders placed by informed traders), convergence is much slower–almost imperceptible even after one hundred transactions–because of the greater proportion of noise.

These observations on price discovery pose a conundrum for market organizers: more frequent informed trading widens bid-ask spreads and tends to make the market illiquid, as shown in equation (3.15), but it also speeds price discovery. Intuitively, the presence of informed traders is necessary for information to be incorporated in prices. At least in the short run, therefore, there is a tradeoff between liquidity and informational efficiency.

3.3.5 The Implications for Price Movements and Volatility

Now let us go back to the trade and quote data for the AGF stock in the introduction (table 3.1). A graphical representation of the price dynamics for AGF is given in figure 3.4 (considering the first fifty trades on March 26, 2001).

The upper and lower curves chart the best ask and bid quotes, respectively. The dots indicate the prices at which trades take place. It is apparent from the figure that prices adjust in the direction of the order flow. For instance, most of the first ten trades are initiated by sellers placing market orders, and the **(p.99)**

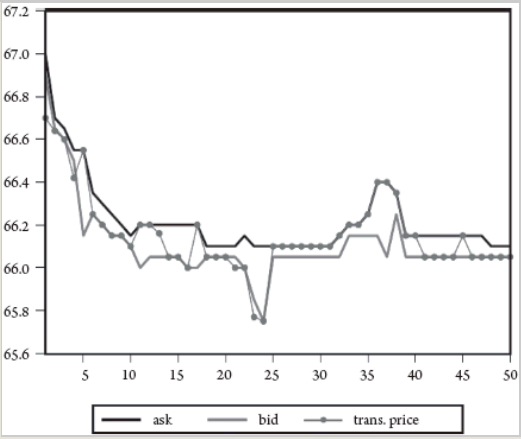


Figure 3.4. Quotes and transaction prices for AGF (50 first trades of the day)

midprice declines. Conversely, trades twenty-five to forty are mostly triggered by buy market orders, and the midprice rises.

The model developed in this section provides a framework for explaining this effect of order flow on price movements. Recall that (see equations (3.19) and (3.20)):

$$\mu_t^+ = \mu_{t-1} + s_t^a,$$
$$\mu_t^- = \mu_{t-1} - s_t^b,$$

where the expressions for $s^a{}_t$ and $s^b{}_t$ are given by equations (3.13) and (3.14). These equations can be written more compactly as:
(3.24)

$$\mu_t = \mu_{t-1} + s\left(d_t\right) d_t,$$

where

$$s\left(d_t\right) \equiv \begin{cases} s_t^a & \text{if} d_t = +1, \\ s_t^b & \text{if} d_t = -1. \end{cases}$$

The dealers' estimate of the value of the security after the t^{th} transaction, μ_t, depends on the direction of the order flow; market orders are informative.

Now recall that dealers set their quotes at time t so as to bracket their prior estimate of the security's value, μ_{t-1}, since

(3.25)

$$a_t = \mu_t^+ = \mu_{t-1} + s_t^a,$$

(3.26)

$$b_t = \mu_t^- = \mu_{t-1} - s_t^b.$$

(p.100) Thus they will revise their quotes upward after buy orders and downward after sell orders, exactly as observed in Figure 3.4 for AGF. As an illustration, consider the numerical example analyzed in the previous section ($v^H = 102$, $v^L = 98$, $\theta_0 = \frac{1}{2}$, and $\pi = \frac{1}{2}$). On these assumptions we readily find, from equations (3.13) and (3.14), that $a_1 = 101$ and $b_1 = 99$. Now, assume that the first order isabuy($d_1 = +1$). This leads dealers to revise the probability of $v = v^H$ upward to $\theta_1 = \frac{3}{4}$ and correspondingly mark up their value estimate to $\mu_1 = 101$. Again using equations (3.13) and (3.14), dealers will raise their bid and ask quotes to $a_2 = 101.6$ and $b_2 = 100$, respectively. If the first order is a sell order, then $\mu_1 = 99$ instead, and $\theta_1 = \frac{1}{4}$. After this trade, dealers lower their bid and ask quotes to $a_2 = 100$ and $b_2 = 98.4$.

As a result, trade-to-trade changes in transaction prices are correlated with the order flow. All transactions take place either at the ask price or the bid price, so that$p_t = \mu_t$. Therefore, equation (3.24) also implies:

(3.27)

$$p_t - p_{t-1} = \mu_t - \mu_{t-1} = s\,(d_t)\,d_t.$$

Hence, in this framework the difference between the price of the $(t-1)^{\text{th}}$ transaction and the t^{th} transaction is entirely determined by the direction of the t^{th} order.[16] The last equation is similar to equation (3.4), except that it explicitly relates the change in price ε_t to the order flow d_t: here the price innovation ε_t is equal to $s(d_t)d_t$. This makes sense because, under asymmetric information, the order flow is informative.

Finally, from equation (3.27) we can compute the variance of price changes:

(3.28)

$$\text{var}(\Delta p_t) = \text{var}(s\,(d_t)\,d_t).$$

Return volatility at time t is thus determined both by the size of the bid-ask spread $s(d_t)$ and by the uncertainty of the direction of the order flow d_t. This shows that trading is also necessarily a source of volatility, being a source of information. Moreover, equation (3.28) suggests that return volatility might not be constant during the trading day, and that it should be correlated with the bid-ask spread. As dealers' value estimate becomes more accurate, their quotes should become less sensitive to order flow, the spread should narrow, and return volatility should decrease. Interestingly, this pattern is observed in equity markets: from the opening to the mid-session, both volatility and bid-ask spreads decline on average, before picking up again as the market close **(p.101)** approaches. Of course, our interpretation presupposes that private information is obtained mostly at the start of the trading day or before: if during the day dealers come to suspect that some new price-relevant information–say, a rumor of a takeover bid–is guiding the order flow, they will widen their spreads again, to protect themselves against traders who may have a better sense of the likely outcome.

Expression (3.28) for return volatility is based on the idea that new information affects stock prices only via the orders placed by informed traders. In reality, much new information takes the form of public news, which leads all market participants to revise their quotes and therefore leads to price changes without triggering any trades. Hence, the volatility of stock returns also reflects the variance of price changes induced by such public news. Models with informed trading can easily be adapted to include public news, as will be seen in section 5.2.1 of Chapter 5.

3.4. Price Dynamics with Order-Processing Costs

The risk of losing money in trading with better informed investors is the only cost borne by liquidity suppliers that we have considered so far. In reality, however, processing a transaction costs time and money: trading fees, clearing and settlement fees, paperwork and back office work, telephone time, and so on.[17] Naturally, liquidity suppliers also require a compensation for these so-called "order-processing costs." To explore how they affect the bid-ask spread and the resulting dynamics of transaction prices, we extend the framework developed in the previous section by assuming an order-processing cost equal to γ per share. In reality, some of these costs are per transaction, others per dollar traded or per share traded. Most of our results here would also hold if order-processing costs were γ per dollar traded or per transaction (see exercise 1).

3.4.1 Bid-Ask Spread with Order-Processing Costs

Consider again the determination of the bid-ask spread for, say, the t^{th} transaction. Suppose that a buy order is received. To break even, a dealer's ask price must now cover not only the expected loss from trading with potentially **(p.102)** informed buyers s_t^a but also the order-processing cost γ. So equation (3.25) must be modified as follows:

(3.29)

$$a_t = \mu_{t-1} + \gamma + s_t^a.$$

Dealers simply pass the processing cost on to liquidity demanders. By a similar reasoning, the bid price posted by dealers at time t is

(3.30)

$$b_t = \mu_{t-1} - \gamma - s_t^b.$$

Hence, the bid-ask spread

(3.31)

$$S_t \equiv a_t - b_t = 2\gamma + s_t^a + s_t^b,$$

now has two different components: order-processing cost (2γ) and adverse-selection cost $\left(s_t^a + s_t^b\right)$. For practical purposes, it is important to gauge their relative importance in determining the spread. For instance, the policy measures needed to alleviate illiquidity are not the same in the two cases: technological upgrades and rules encouraging competition among trading platforms can reduce processing costs, while action against insider trading may mitigate adverse selection.

How the two components of the bid-ask spread can be measured separately is not obvious, since at first glance they have the same effect. In the next section, however, we will see that the two components carry very different implications for the dynamics of transaction prices, which gives us a way to estimate the components of the spread using data on order flow and transaction prices.

3.4.2 Price Dynamics with Order-Processing and Adverse-Selection Costs

As trades take place at bid and ask prices, the transaction price at time t can be written as:

(3.32)

$$p_t = \mu_{t-1} + \left(s\left(d_t\right) + \gamma\right) d_t,$$

where $s\left(d_t\right) = s_t^a$, and $s\left(d_t\right) = s_t^b$. As $\mu_t = \mu_{t-1} + s(d_t)d_t$, we obtain:

(3.33)

$$p_t = \mu_t + \gamma d_t.$$

Thus, in the presence of order-processing costs, transaction prices deviate from fair value given all public information, including the direction of trade initiation. **(p.103)** The deviation is equal to the processing cost (as $|p_t - \mu_t| = \gamma$). Intuitively, dealers cover this cost by executing buy (sell) orders at a markup (discount) relative to their value estimate based on all public information.

Thus, order-processing costs induce *transient* deviations of transaction prices from fundamental values, so called because these deviations do not correspond to a revision in dealers' value expectations. Accordingly, they are subsequently corrected, at least in part by opposite price movements ("reversals").

To see this, consider the arrival of a buy order at time t and define its short-term (ST) price impact as the deviation of the transaction price from the immediately preceding fundamental value, μ_{t-1}:

$$\text{ST impact} \equiv p_t - \mu_{t-1} = a_t - \mu_{t-1}.$$

Using equation (3.29), one can see that the immediate effect of a buy market order is to push the price up by

(3.34)

$$\text{ST impact} = s_t^a + \gamma \rangle 0.$$

Now consider the long-term (LT) price impact, that is, its effect on the average price at a distant time $t + T$, p_{t+T}. At that time, by equation (3.33), the price will be given by $p_t + T = \mu_{t+T} + \gamma d_{t+T}$. Its expected value as of time t is then

(3.35)

$$\begin{aligned} \mathrm{E}\left(p_{t+T}|\Omega_{t-1}, d_t = 1\right) &= E\left(\mu_{t+T}|\Omega_{t-1}, d_t = 1\right) + \gamma E\left(d_{t+T}|\Omega_{t-1}, d_t = 1\right) \\ &= \mu_t + \gamma E\left(d_{t+T}|\Omega_{t-1}, d_t = 1\right) \\ &= \mu_{t-1} + s_t^a + \gamma E\left(d_{t+T}|\Omega_{t-1}, d_t = 1\right), \end{aligned}$$

where the first step follows by the law of iterated expectations and the second by (3.32).

Suppose that $t + T$ is so far ahead that by then the currently pending uncertainty about true value of the asset (v^H or v^L) will have already been resolved-the fundamental publicly revealed (say, $t + T$ is beyond the end of the trading day). Then, the direction of trade at time t (d_t) has no predictive power for its direction at $t + T$ (d_{t+T}). Hence $\mathrm{E}(d_{t+T}|\Omega_{t-1}, d_{t=1}) = 0$, so that

$$\mathrm{E}\left(p_{t+T}|\Omega_{t-1}, d_t = 1\right) = \mu_{t-1} + s_t^a.$$

Thus the long run impact of the buy order at time t includes only the informational component of the bid-ask spread, and not the order-processing **(p.104)** costγ:[18]

(3.36)

$$\text{LT impact} = s_t^a.$$

Hence, in the long term, only the informational effect persists:

(3.37)

$$\text{ST impact} - \text{LT impact} = \gamma.$$

In the absence of order-processing costs ($\gamma = 0$), the short term impact of the buy order is equal to the long term impact, because the impact is due entirely to the informational content of the buy order. In contrast, in presence of order-processing costs, the short run impact exceeds the long run impact by an amount equal to the processing cost, creating a further temporary price blip at time t. Thus, after the initial price increase, the price level tends to revert. A similar decomposition can be obtained if the order at time t is a sell order: its immediate effect is to reduce the price, but in the long run the price bounces back and partly reverts to its initial level. Thus, with order-processing costs, trade-to-trade changes in prices will show a negative serial correlation. As Chapter 2 explains, Roll's model exploits this feature to derive an estimator of the bid-ask spread and corresponds to a special version of the current model where the bid-ask spread is due only to order-processing costs. These effects can also be observed in figure 3.4. For instance, the sequence of buy orders from the twenty-fifth to the fortieh transaction trigger a price increase, but part of it is transient so that after the fortieh transaction the price partly reverts.

To summarize, buy orders and sell orders create upward and downward price pressures, which are permanent only to the extent that illiquidity is due to asymmetric information. The transient component is instead a compensation for the costs borne by liquidity suppliers to accommodate temporary order imbalances.

(p.105) As order-processing costs and asymmetric information typically coexist, market orders have both permanent and temporary effects. This observation can be used to measure order-processing and adverse selection costs, as Chapter 5 explains.

Box 3.1 Dealers'Rents as an Additional Component of the Bid-Ask Spread

In this section, we interpret γ as a cost per share borne by dealers to execute market orders. But γ may equally well be viewed as a measure of dealers' rents from market power. To see this, suppose that order-processing costs are equal to γ^c and that dealers require an expected profit per share *at least* equal to γ^r on each transaction. It follows immediately that to achieve an expected profit of γ^r for executing a buy market order, dealers must charge an ask price equal to

$$a_t = \mu_{t-1} + \gamma^c + \gamma^r + s_t^a,$$

so that the term $2\gamma = 2(\gamma^c + \gamma^r)$ in expression (3.31) for the bid-ask spread S_t will reflect not only the dealers' operating costs γ^c but also their non-competitive rents γ^r.

By the same token, we can repeat the reasoning of section 3.4.2 above to conclude that after the execution of a buy market order the price level will revert by an amount averaging $\gamma^c + \gamma^r$. Similar remarks apply to sell orders. Thus, considering only the dynamics of prices after a trade, one cannot measure separately the order-processing component and the rent component of the bid-ask spread. This implies that what is termed the "order-processing cost" component must be interpreted with care, since it may in fact capture both processing costs proper and the rents captured by dealers in supplying liquidity.

In reality, dealers often earn rents. Indeed, some markets (e.g., Nasdaq or NYSE) charge a fee on market participants to become dealers. The level of this fee is an indicator of the economic value of being a dealer. Competition among dealers maybe imperfect also due to collusion. In the 1990s, Nasdaq dealers were accused of implicitly colluding on excessive spreads (see Christie and Schultz, 1994, and Christie, Harris and Schultz, 1994). Chapter 4 offers a more systematic analysis of the way trading rules can affect liquidity suppliers' ability to earn rents at the expense of liquidity demanders.

(p.106) 3.5. Price Dynamics with Inventory Risk

In continuous markets, buy and sell orders from traders seeking liquidity do not arrive at the same time. This creates temporary order imbalances, and one important function of liquidity suppliers is to serve as counterparty when the order flow is unbalanced. For instance, a dealer may execute a sell order for five thousand shares of a stock, which increases his position in it. Next, ten minutes later, he receives a buy order for one thousand shares, which allows him to pare his extra inventory down. Finally, he receives another buy order for four thousand shares, and so reverts to his initial position. Over time, the net position taken by the dealer is zero. His role was simply to balance demand and supply over time.

However, this role exposes the dealer to inventory risk, that is, the possibility of a change in the value of his holdings because of, say, news about the underlying fundamentals. Assuming that dealers are risk averse, they will require a compensating risk premium, known as inventory holding cost-even if they are not at risk of being picked off by traders with superior information. Thus, inventory risk is another important determinant of the bid-ask spread, as was first shown formally by Stoll (1978). Here we analyze the effect of inventory risk on dealers' pricing policy and discuss its implications for the spread and for price dynamics.

To simplify the analysis, we assume that dealers are competitive and *short sighted:* in considering whether to fill an incoming order at time t, a dealer acts *as if* his holdings will be marked to market at time $t + 1$ and liquidated at the market price at that time.[19] In pricing an order arriving at time t, the dealer must take account of the fundamental risk from t to $t + 1$, namely, the risk that public information ε_t about the underlying fundamental value of the security μ_t will emerge and alter the price during the holding period. The per-period standard deviation of this risk is denoted by σ_ε.

In order to isolate the impact of inventory holding costs, we make two simplifying assumptions throughout this section. First, the order flow is not correlated with news about fundamentals; that is, it is not driven by traders with **(p.107)** an informational advantage over market makers ($\pi = 0$). Second, dealers have no order-processing costs ($\gamma = 0$).

Before trading at any time t, the representative dealer has cash c_t and a starting inventory of the risky security z_t, where $z_t > 0$ indicates a long and $z_t < 0$ a short position.[20] Marked to market, his initial wealth w_t is the value of his cash and security position before trading at time t, evaluated at time t market price p_t:

(3.38)

$$w_t = p_t \cdot z_t + c_t.$$

The dealer's concern is how his trade at time t will affect his next-period wealth $w_{t+1} = p_{t+1} \cdot z_{t+1} + c_{t+1}$, where z_{t+1} and c_{t+1} are his stock inventory and cash position after trading at time t.

The market is organized as a call auction where the representative dealer is assumed to behave competitively, that is, to take the price p_t as given in choosing the number of shares y_t that he is willing to supply:[21] $y_t > 0$ indicates the dealer's offer to sell y_t shares, and $y_t < 0$ means that he is willing to bid to buy $|y_t|$ shares. Thus the dealer's inventory after selling y_t shares is

(3.39)

$$z_{t+1} = z_t - y_t,$$

and his corresponding cash position after receiving payment for those shares of $p_t y_t$ is
(3.40)

$$c_{t+1} = c_t + p_t y_t.$$

Thus, the dealer's end-of-period wealth, as a function of the amount y_t he sells at time t, is

(3.41)

$$w_{t+1} = p_{t+1}\left(z_t - y_t\right) + c_t + p_t y_t,$$

using (3.39) and (3.40) in the definition of w_{t+1}.

For ease of exposition, in section 3.5.1 we start with a simple two-period setting where dealers trading at time t expect to liquidate their holdings at a price equal to the fundamental value $v = \mu_t + \varepsilon$. Then, in section 3.5.2, we turn to a simple multi-period setting where at time t each dealer anticipates that at time $t + 1$ the value of his inventory will be determined by a price p_{t+1} that is endogenously determined. In this dynamic setting, the order flow is assumed to respond to prices; in equilibrium, the pattern of orders, prices, and dealers' inventories over time is jointly determined.

(p.108) 3.5.1 A Two-Period Model

In a two-period setting, dealers can be assumed to value their final inventories z_{t+1} at their fundamental value. Hence, in expression (3.41) for the dealer's final wealth, the end-of-period price p_{t+1} can be replaced by the fundamental value v:

(3.42)

$$w_{t+1} = v\left(z_t - y_t\right) + c_t + p_t y_t.$$

Since dealers are risk averse, their objective function is increasing in the expected value of their end-of-period wealth $E_t(w_{t+1})$ and decreasing in its riskiness, which we measure by its variance $\text{var}_t(w_{t+1})$. We consider two possible formulations of this objective function.

(i) Mean-variance preferences.

The most commonly used formulation in finance theory (and one that will be used again in Chapter 4) is the linear mean-variance function

(3.43)

$$U = \mathrm{E}_t\left(w_{t+1}\right) - \frac{\rho}{2}\mathrm{var}_t\left(w_{t+1}\right),$$

where ρ is a measure of risk aversion. Computing the objective (3.43) for the dealer's end-of-period wealth in expression (3.42), we get

$$\begin{aligned} U &= \mathrm{E}_t\left(v\right)\left(z_t - y_t\right) + c_t + p_t y_t - \frac{\rho}{2}\left(z_t - y_t\right)^2\sigma_\varepsilon^2 \\ &= \mu_t\left(z_t - y_t\right) + c_t + p_t y_t - \frac{\rho}{2}\left(z_t - y_t\right)^2\sigma_\varepsilon^2, \end{aligned}$$

recalling the definition $\mathrm{E}_t\left(v\right) \equiv \mu_t$. The dealer chooses his supply of shares y_t so as to maximize this objective function. The dealer's first-order condition with respect to y_t is

$$\frac{\partial U}{\partial y_t} = -\mu_t + p_t + \rho\left(z_t - y_t\right)\sigma_\varepsilon^2 = 0,$$

which yields his inverse supply function (i.e., the price at which he is willing to supply a given amount of shares): (3.44)

$$p_t = \mu_t + \rho\sigma_\varepsilon^2\left(y_t - z_t\right).$$

Thus, as shown in figure 3.5, if the price equals the fundamental value $\mathrm{E}_t(\mu_{t+1})$, so that the market offers no compensation for inventory holding costs, the dealer supplies his entire initial inventory z_t, so as to reduce his end-of-period inventory to zero. But if the market offers a higher price and therefore a risk premium, the dealer is willing to supply more shares, thus taking a risky short position. The size of the short position depends on the dealer's degree of risk aversion ρ and on the stock's fundamental volatility σ_ε^2: the more risk averse the dealer and the riskier the stock, the fewer shares the dealer will **(p.109)**

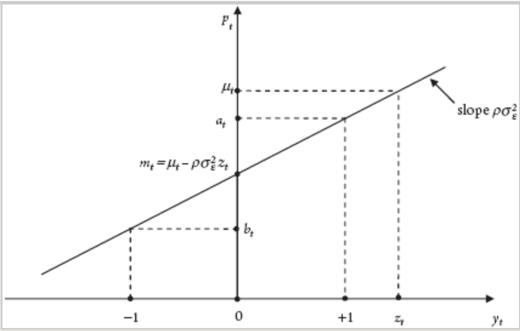


Figure 3.5. Dealers' supply with mean-variance preferences

supply. Conversely, if the price is below the fundamental value, the dealer will supply less than his initial inventory: if shares trade at a discount relative to their fundamental value, he is willing to hold a long position until time $t + 1$.

In equilibrium, the representative dealer must supply exactly the amount of shares needed to satisfy the incoming order flow, that is, $y_t = d_t$. Replacing this condition in the dealer's supply function (3.44) yields the equilibrium price:

$$p_t = \underbrace{\mu_t - \rho\sigma_\varepsilon^2 z_t}_{\text{midquote } m_t} + \rho\sigma_\varepsilon^2 d_t = m_t + \rho\sigma_\varepsilon^2 d_t.$$

Hence the equilibrium midquote m_t reflects not only the stock's expected fundamental value, but also an inventory risk adjustment: it is the price at which the dealer is willing to hold precisely his initial inventory. As shown in figure 3.5, the equilibrium price depends on whether the dealer receives a buy or a sell order. The ask price for one share ($d_t = +1$) exceeds the midquote, reflecting the need to reward the dealer for supplying one unit out of his inventory, while the bid price ($d_t = -1$) incorporates a discount to reward him for adding one extra unit to it:

(3.45)

$$p_t = \begin{cases} a_t = m_t + \rho\sigma_\varepsilon^2 & \text{if} d_t = +1, \\ b_t = m_t - \rho\sigma_\varepsilon^2, & \text{if} d_t = -1. \end{cases}$$

Hence the bid-ask spread is determined by the dealers' risk aversion ρ and by the stock's fundamental volatility σ_ε^2, in other words, by inventory holding costs:

(3.46)

$$S_t = 2\rho\sigma_\varepsilon^2.$$

(ii) Mean-standard deviation preferences.

In the multi-period analysis further on, it will be more convenient to adopt a less commonly used formulation **(p.110)** of preferences, which is linear in the mean and the standard deviation of wealth:

$$U = \mathrm{E}_t\left(w_{t+1}\right) - \rho \mathrm{sd}_t\left(w_{t+1}\right),$$

where "sd" denotes the standard deviation of a random variable. Computing this objective for the dealer's end-of-period wealth in expression (3.42), we have

(3.47)

$$\begin{aligned} U &= E_t(v)\left(z_t - y_t\right) + c_t + p_t y_t - \rho sd_t\left(v z_{t+1}\right) \\ &= \mu_t\left(z_t - y_t\right) + c_t + p_t y_t - \rho\left|z_t - y_t\right|\sigma_\varepsilon \end{aligned}.$$

Differentiating with respect to y_t, we get

$$\frac{\partial U}{\partial y_t} = \begin{cases} p_t - \mu_t - \rho\sigma_\varepsilon & \text{if } y_t \rangle z_t, \text{ that is}, z_{t+1} \langle 0, \\ p_t - \mu_t + \rho\sigma_\varepsilon & \text{if } y_t \langle z_t, \text{ that is}, z_{t+1} \rangle 0. \end{cases}$$

Intuitively, the change in the dealer's utility from the sale of a share is given by the expected gain $p_t - \mu_t$ minus the implied increase in risk $\rho\sigma_\varepsilon$ if the dealer ends up with a short position, or plus the implied decrease in risk if even after the sale the dealer is still long in the stock. If, on balance, selling an extra share is beneficial ($\partial U/\partial y_t \rangle 0$), dealers would want to sell infinitely many shares at the price p_t, thus generating an excess supply. Conversely, if $\partial U/\partial y_t \langle 0$, dealers would want to buy infinitely many shares, generating an excess demand. Accordingly, in equilibrium the price must be such that the dealers' marginal utility is equal to zero; that is, the price must lie in the range

(3.48)

$$p_t \in \left[\mu_t - \rho\sigma_\varepsilon, \mu_t + \rho\sigma_\varepsilon\right].$$

In this case the dealers' supply takes the stepwise form shown in figure 3.6. At any price within the interval (3.48), the representative dealer is willing to dispose of his initial inventory, as illustrated by the fact that at his initial inventory z_t, his supply curve is vertical. If the price is at the upper bound $\mu_t + \rho\sigma_\varepsilon$, the dealer is also willing to supply any further amount required by investors, because the premium per share $\rho\sigma_\varepsilon$ compensates him for the risk arising from a short position. If, instead, the price is at the lower bound $\mu_t - \rho\sigma_\varepsilon$, the dealer is not only willing to hold his initial inventory z_t but also to buy any additional shares that investors may want to sell, because the discount $\rho\sigma_\varepsilon$ is sufficient to compensate him for the risk of taking a longer position.

Imposing the equilibrium condition $y_t = d_t$, as before, one obtains the ask and bid prices charged by the dealer for a unit-sized order, as figure 3.6 indicates. If the buy order ($d_t = +1$) exceeds the dealer's initial inventory, forcing him to take a short position ($z_{t+1} \langle 0$), the price must be high enough to reward him for the implied risk; conversely, a sell order ($d_t = -1$), which adds to the initial **(p.111)**

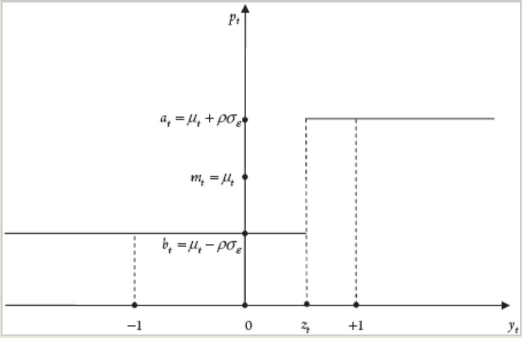


Figure 3.6. Dealers' supply with mean-standard deviation preferences

inventory, will be absorbed at a price low enough to reward the risk of a long position ($z_{t+1} \rangle 0$):[22]
(3.49)

$$p_t = \begin{cases} a_t = \mu_t + \rho\sigma_\varepsilon & \text{if} d_t \rangle z_t \text{, that is, } z_{t+1} \langle 0, \\ b_t = \mu_t - \rho\sigma_\varepsilon & \text{if} d_t \langle z_t \text{, that is, } z_{t+1} \rangle 0. \end{cases}$$

Hence the midquote in this case is simply the expected value of the fundamental, conditional on information known at time t, that is, $m_t = \mu_t$. And the bid-ask spread is again determined by dealers' inventory holding cost, namely, the compensation they require for the cost of holding a risky inventory:

(3.50)

$$S_t = 2\rho\sigma_\varepsilon,$$

the only difference from expression (3.46) being that risk is now measured by the standard deviation of the stock's fundamental value rather than the variance.[23] Chapter 4 shows that in presence of inventory risk the bid-ask spread is also determined by the size of customers' orders (which determines the incremental risk that must be borne by a dealer) and the number of dealers in the market (which affects their per-capita risk).

(p.112) 3.5.2 A Multi-Period Model

We are now ready to consider a multi-period setting in which at each date t the future value of dealers' inventories is determined by their future price. Hence, the representative dealer's utility is:

(3.51)

$$U = \mathrm{E}_t\left(p_{t+1}\right)\left(z_t - y_t\right) + c_t + p_t y_t - \rho s d_t\left(p_{t+1}\right) \cdot \left|z_t - y_t\right|,$$

which differs from its two-period analogue (3.47), because the future price p_{t+1} replaces the fundamental value v. In this multiperiod setting, one must specify what determines the order flow, since future orders determine the prospective path of dealers' inventories, which in turn affects the path of future prices. We assume that the order flow is price sensitive, in the sense that customer orders respond to the possibilities of profit and loss inherent in dealers' prices and so lead to inventory rebalancing. For simplicity, we suppose that the order flow response to prices is perfectly predictable: if at time t the midquote is below the stock's fundamental value ($m_t \langle \mu_t$), a customer will place a buy order with the dealer ($d_t = +1$); if above ($m_t \rangle \mu_t$), a sell order ($d_t = -1$).[24] If $m_t = \mu_t$, instead, no order will arrive. In short, to simplify, we do not allow the order flow to contain some noise (i.e., random and price insensitive orders).[25]

Maximizing the representative dealer's objective function (3.51) and imposing equilibrium as in the two-period case, one obtains the following expression for the equilibrium price:

(3.52)

$$p_t = \begin{cases} \mathrm{E}_t\left(p_{t+1}\right) + \rho \mathrm{sd}_t\left(p_{t+1}\right) & \text{if} d_t \rangle z_t \Rightarrow z_{t+1} \langle 0, \\ \mathrm{E}_t\left(p_{t+1}\right) - \rho \mathrm{sd}_t\left(p_{t+1}\right) & \text{if} d_t \langle z_t \Rightarrow z_{t+1} \rangle 0. \end{cases}$$

Unlike its two-period analogue (3.49), the equilibrium price at time t now depends on its own expected value and its standard deviation at time $t + 1$. These in turn depend on the distribution of the clearing price at time $t + 2$, and so on. We seek a stationary solution for the price, that is, an expression for p_t such that the expected value and standard deviation of its future value p_{t+1} is consistent with the equilibrium relationship (3.52). This is an instance of a "rational expectations equilibrium," a relationship between economic variables such that there is market clearing, and if correctly anticipated, it induces people to behave consistently with it.

(p.113) To identify such an equilibrium relationship, one generally starts from a conjecture about it and then goes on to verify for which functional form and parameters the relationship is consistent with (i) equilibrium and (ii) rationality of expectations-an approach known as the "method of undetermined coefficients." In the case at hand, we conjecture that in equilibrium the relationship between the price and dealers' inventories is time invariant and linear, with p_t centered around the true underlying value of the stock μ_t with a markdown proportional to the dealers' net inventory *after* the transaction, zt_{+1}:

(3.53)

$$p_t = \mu_t - \beta z_{t+1} = \mu_t - \beta(z_t - y_t),$$

for some $\beta \rangle 0$. Since in equilibrium $y_t = d_t$, this conjecture implies that bid and ask prices should be $a_t = \mu_t - \beta z_t + \beta$ and $b_t = \mu_t - \beta z_t - \beta$, and thus the midquote should be
(3.54)

$$m_t = \mu_t - \beta z_t.$$

We will show that conjecture (3.53) is indeed verified in equilibrium, given our assumptions about the order flow process (for a particular value of the parameter β that we identify). Intuitively, taken together with our assumption about the order flow's response to prices, the conjectured equilibrium relationship implies that upon increasing their inventories from z_t to z_{t+1} after filling a sell order ($d_t = -1$), dealers will mark down the current price p_t. But to determine the magnitude of this price drop (i.e., the value of β), we must consider that the drop in p_t (and therefore in the new midquote m_{t+1}) will trigger a feedback effect:[26] it will attract buy orders from customers at $t + 1$, and thus push the price p_{t+1} back up, inducing a reversion of the price towards its initial level. This feedback effect on the future price p_{t+1} must be taken into account in solving the model, since equation (3.52) tells us that *pt* depends on the expected value and variability of p_{t+1}.

Consider now a scenario where, at the start of period t, the representative dealer has a positive inventory that is too large to be worked off in a single period; that is, let z_t be an integer z strictly greater than 1. Then, according to conjecture (3.54), we should have $m_t \langle \mu_t$, triggering a buy order at time t and reducing the inventory at time $t + 1$ to $z_{t+1} = z - 1$. The same will occur in the subsequent period: since also at time $t + 1$ the dealer's inventory is positive, at $t + 1$ the price will still be depressed ($m_{t+1} \langle \mu_{t+1}$), triggering a new buy order at time $t + 1$ and reducing inventories further to $z_{t+2} = z - 2$. If $z \rangle 2$, then at time $t + 2$ inventories will not be depleted yet, and the process will continue: for z periods the price stays below the fundamental value and the inventory is reduced by one unit per period. It will stop only at time $t + z$, when the **(p.114)** inventory has been depleted. Thus, based on conjecture (3.53), the expected value and variance of the price at time $t + 1$ are

(3.55)

$$\mathrm{E}_t\left(p_{t+1}\right) = \mu_t - \beta(z_{t+1} - 1), \quad \mathrm{var}_t\left(p_{t+1}\right) = \sigma_\varepsilon^2.$$

Combining expressions (3.53) and (3.55), we see that the conjecture implies an expected price increase of β between period t and $t + 1$:

$$E_t\left(p_{t+1}\right) - p_t = \beta.$$

But the equilibrium relationship (3.52), taken jointly with the expression for price volatility from (3.55), requires

$$\mathrm{E}_t\left(p_{t+1}\right) - p_t = \rho\sigma_\varepsilon,$$

whenever the inventory after trade at time t (z_t +1) is still positive. Hence, for the initial conjecture to be consistent with equilibrium, we must have $\beta = \rho\sigma_\varepsilon$. Replacing the parameter β with $\rho\sigma_\varepsilon$ in the initial conjectures (3.53) and (3.54) yields the following expression for the equilibrium price at time t:
(3.56)

$$p_t = \mu_t - \rho\sigma_\varepsilon z_{t+1}$$

and the equilibrium midquote:
(3.57)

$$m_t = \mu_t - \rho\sigma_\varepsilon z_t.$$

Since, in our example, dealer's inventories decrease between time t and $t + z$, equation (3.56) implies that the equilibrium price increases at any time $t + \tau$ between t and $t + z - 1$, and, once the initial inventory z is depleted, stays at its fundamental value:

$$p_{t+\tau} = \begin{cases} \mu_{t+\tau} - \rho\sigma_\varepsilon (z - \tau - 1) & \text{for} \tau \in \{0, 1, \ldots, z-1\}, \\ \mu_{t+\tau} & \text{for} \tau \geq z. \end{cases}$$

Thus, as illustrated in figure 3.7, the price path is the mirror image of dealers' inventories (the light grey line): as the initial inventory z decreases by one share per period, the price increases linearly at rate $\rho\sigma_\varepsilon$ per period. The fundamental value μ_t, shown as a solid black line, is assumed to start at 10 and evolve as a random walk, with innovations ε_t that are normally distributed with a mean of zero and standard deviation of 0.5. The dealers' risk aversion ρ is fixed at 0.2, and their initial aggregate inventory at time 0 is z = 20. As dealers have a large initial long position, the initial midprice is below the fundamental value. Over time, the midprice converges to the fundamental μ_t.

As in section 3.2, the expected fundamental value is $\mu_t = \mu_{t-1} + \varepsilon_t$, since dealers know the current public news ε_t when they set their time-t quotes.[27] **(p.115)**

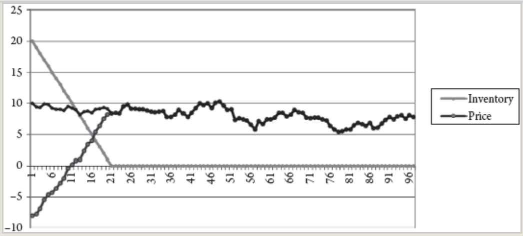


Figure 3.7. Inventory and price reversals

The midprice m_t in (3.57) can be interpreted as the representative dealer's marginal valuation for the stock adjusted for the risk arising from his inventory-that is, the price at which he is willing to change his inventory by a very small amount. In contrast to the models analyzed so far, here this marginal valuation is not determined solely by dealers' estimate of the expected payoff: it is also inversely related to the average current size of the inventory. The value $\rho\sigma_\varepsilon$, which measures how much dealers reduce m_t in response to inventories z_t, is increasing with their risk aversion ρ and with the security's fundamental risk σ_ε: both of these parameter changes induce dealers to require a larger discount in order to add an extra share.

The equilibrium price relationship (3.56) also yields the ask and bid prices that dealers will quote at date t: these are the prices that correspond to the case in which the customer places a market buy order (d_t = +1), so that $z_t + 1 = z_{t-1}$, and the case in which he places a sell order ($d_t = -1$), so that $z_t + 1 = z_{t+1}$. Hence, a buy order executes at the ask price:

(3.58)

$$a_t = \mu_t - \rho\sigma_\varepsilon z_t + \rho\sigma_\varepsilon,$$

and a sell order at the bid price:

(3.59)

$$b_t = \mu_t - \rho\sigma_\varepsilon z_t - \rho\sigma_\varepsilon,$$

so that the equilibrium bid-ask spread is $S_t = 2\rho\sigma_\varepsilon$, as in expression (3.50) derived in the two-period model.

3.5.3 The dynamics of prices and inventories

The main insight from this multi-period model is that when a dealer has a long position before trading at date t ($z_t \rangle 0$), buying additional shares is not **(p.116)** attractive, as it increases his risk exposure. By the same token, selling shares is attractive, as it reduces the exposure. Thus, a dealer with a long position is willing to trade shares at a low price relative to the fair value, μ_t. Conversely, a dealer with a short position is willing to trade at a relatively high price. Hence, the mid-price at date t will deviate from the fair value of the security, and the size of this deviation ($m_t - \mu_t$) is inversely related to dealers' aggregate inventory. This deviation can be called the "price pressure" arising from inventory holding costs.

For stocks listed on the NYSE, Hendershott and Menkveld (2010) find evidence consistent with this prediction. They provide an estimate of $\rho\sigma_\varepsilon$, which measures the price pressure per unit of inventory, and find that it is much greater for small-capitalization than large-capitalization stocks. Specifically, a $1,000 inventory results in price pressure of 1.01 basis points for small stocks and 0.02 basis points for large stocks. This makes sense, in that fewer people are interested in trading smaller stocks, so it takes longer to unwind the inventories and dealers remain exposed to risk for a longer time. In turn, this price response will enable dealers to rebalance their inventories by eliciting inventory-reducing order flow from market participants. In line with this prediction, researchers— such as Reiss and Werner (1998) and Hansch, Naik, and Viswanathan (1998)—have found empirically that dealers with long positions are more likely to execute buy market orders; those with short positions, sell market orders.

The dynamics of quotes and transaction prices is driven both by changes in price fundamentals μ_t *and* by changes in dealers' inventories. Since dealers' inventories change in response to the order flow, the inventory risk model links the dynamics of prices to those of the order flow, just like asymmetric information and order-processing costs. To see this point, note that equation (3.57) implies:

(3.60)

$$m_t - m_{t-1} = \mu_t - \mu_{t-1} - \rho\sigma_\varepsilon\left(z_t - z_{t-1}\right) = \rho\sigma_\varepsilon d_{t-1} + \varepsilon_t.$$

Thus after a buy order at date $t - 1$, dealers will increase their quotes by $\rho\sigma_\varepsilon$, on average: the order generates a rise in the midquote, since it leads the dealer to decumulate (maybe even to take a short position), and thus to become less willing to sell shares. Conversely, after a sell order, dealers will reduce their quotes by $\rho\sigma_\varepsilon$, on average: execution of a sell order leads the representative dealer to build up his inventory (or decrease his short position), which makes him less willing to buy additional shares.

The immediate impact of a buy order on the midprice is:

(3.61)

$$\text{ST impact} \equiv p_t - m_t = \rho\sigma_\varepsilon \rangle 0.$$

Thus, $\rho\sigma_\varepsilon$ determines both the bid-ask spread at a given date and the immediate price pressure exerted by incoming market orders. Just like asymmetric information, inventory risk implies a positive short-term correlation between **(p.117)** the change in the midprice (the "position of dealers' quotes") and the order flow. However, in contrast to the case of asymmetric information, the short-term impact of trades due to inventory risk does not persist in the long term: it vanishes, though in general more gradually than the impact of order-processing costs. To understand why, we must solve for the joint dynamics of prices and inventories, which we turn to now.

As shown by equation (3.60), the dynamics of the midprice are driven by changes in dealers' inventories, which ultimately depend on order flow (recall that in equilibrium $z_t + 1 = z_t - d_t$). But the order flow in turn responds to changes in the midprice: customers are more likely to place buy orders when quotes are below the security's fundamental value, and more likely to place sell orders when quotes are above it. In particular, under our assumptions the order flow will shrink the dealers' inventory. For this reason, aggregate inventory is *mean-reverting.* That is, conditional on their aggregate position z_t after trading at time t, dealers have a smaller aggregate net position at time $t + 1$:

(3.62)

$$\left|z_{t+1}\right| = \begin{cases} |z_{t-1}| & \text{for} |z_t| \neq 0, \\ 0 & \text{for} z_t = 0, \end{cases}$$

where z_t is assumed to be an integer. Thus, the dealers' aggregate position tends to revert to its long run value of zero: when dealers start with a long position, they seek to lower their inventory; hence they shade their price to attract buy orders, which bring their inventory back down toward zero. The converse is true if they start with a short position.

The fact that trades have only transient impact on dealers' inventories implies also that their impact on the midprice due to inventory holding costs will eventually be dissipated, and at a speed that is directly related to that at which dealers' inventories revert to their initial level. (In our simple model this has been assumed to be one share per period, but in practice may obviously vary depending on stock and market characteristics.) Hence, the deviation between the midprice m_t and the fundamental value μ_t also follows a mean-reverting process: when m_t deviates from μ_t, it tends to revert back to it as inventory gets rebalanced, as is illustrated in figure 3.7.

This analysis suggests that the price impact of trades due to inventory holding costs eventually dissipates. To see this precisely, note that the mid-price T periods ahead is $m_t + T = \mu_{t+\mathrm{T}} - \rho\sigma_\varepsilon z_{t+T}$, from equation (3.57). Thus, the expected value of this price conditional on information Ω_{t-1} and on an order d_t at time t is:

$$\mathrm{E}_t\left(m_{t+T}\right) = \mathrm{E}_t\left(\mu_{t+T}\right) - \rho\sigma_\varepsilon \mathrm{E}_t\left(z_{t+T}\right) = \mu_t, \quad \text{for} T \text{large},$$

since μ_t follows a random walk, so that $\mathrm{E}_t(\mu_{t+T}) = \mu_t$, and in the long run the inventory converges to zero. This is intuitive since, as we have seen, dealers' **(p.118)** aggregate inventory after a trade revert towards its average long run value of zero. Thus, like order-processing costs, inventory holding costs generate price reversals. The key difference is speed: reversion is immediate with order-processing costs, gradual with inventory holding costs.

In our simple framework, the speed of price adjustment arising from inventory holding costs is fixed. In a more complex framework, the response of the order flow would depend on market characteristics, such as the extent of investors' monitoring of the stock, and this response would depend on the size, not just the sign, of price discrepancies. Hence, the speed of reversion would also depend on dealers' risk aversion and the stock's riskiness. Moreover, in practice, dealers actively seek to control their inventory not just by pricing but also by advertising opportunities and soliciting customer orders, which also affect the speed of adjustment. Finally, the speed of adjustment will reflect regulatory limits on the size of dealers' positions, for instance, short-sales constraints or margin constraints on leverage. Intuitively, the more stringent these constraints, the more apparent the footprints of inventory holding costs on price dynamics.

Hendershott and Menkveld (2010) estimate the size and duration of price pressures for NYSE stocks. As explained, they find that price pressure is stronger for small-capitalization stocks. It also lasts longer for these stocks (presumably because trading is less frequent and liquidity demand less elastic). They also measure the contribution of price pressure induced by inventory holding costs to daily volatility, finding that it ranges from 0.17 percent for large stocks to 1.20 percent for small stocks. **(p.119)**

Box 3.2 Dealers' Inventories, Liquidity, and Volatility Before and after the Crisis

This inventory holding cost model shows that dealers' willingness to accumulate and decumulate inventories in order to serve investors' orders is a key to the market's ability to provide liquidity, and that this willingness depends on the volatility of the fundamentals. So it is no surprise that, with the increase in risk during the financial crisis of 2008–09, dealers' willingness to hold inventories diminished. Their ability to hold large inventories was further impaired by regulatory restrictions on their proprietary trading. Nicole Bullock's article "Wall Street: Inventory reductions 'the death of trading'" (*Financial Times*, March 12, 2012) argues that this has reduced liquidity and increased the volatility of prices in the corporate bond market, as the model analyzed above predicts:

> *Before the global financial crisis, large Wall Street dealers used to hold big inventories of corporate bonds to facilitate trading for their clients.*
>
> *When a bond manager such as Jesse Fogarty, of Cutwater Asset Management, wanted to sell a large number of bonds, the dealer would sell some of them to other clients for him and buy some itself to sell down over time, hopefully for a profit.*
>
> *Since the financial crisis, however, large Wall Street dealers have sharply cut the number of corporate bonds they hold to facilitate this kind of trading for clients or for proprietary positions.*
>
> *Mr. Fogarty calls this move "the death of trading."*
>
> *Known in the financial markets as "dealer inventories," these bond holdings reached highs of more than $200bn in 2007, but dropped to less than half that by the end of 2008, as dealers reduced their risk because of the banking crisis.*
>
> *In the second half of last year, dealers cut their bond holdings again, as they dropped from about $90bn at the start of June to $40bn by the end of February 2012, the lowest level in a decade.*
>
> *These lower inventories, analysts and investors say, reflect an environment of lower risk-taking by Wall Street banks in the aftermath of the financial crisis and ahead of tighter regulations.*
>
> *The result is likely to be higher volatility in bond prices, as the buffer is far smaller now.*
>
> *In addition, the so-called Volcker Rule, a part of the 2010 Dodd Frank Act, could make it impossible for dealers to take these kinds of positions, because it prohibits proprietary trading by banks. At the same time, higher capital costs under Basel III also loom as an impediment to holding bonds on a bank's balance sheet. Many in the market believe the rules, which still need to be finalised, are a big driver of the reduction in inventory that was seen last year.*
>
> *[…] market participants generally expect lower inventories will increase volatility, as they say happened during last year's sell-off.*
>
> *As Europe's debt crisis escalated, risk premiums on investment grade and high-yield bonds widened sharply.*
>
> *"The takeaway is that market liquidity has been reduced," says Chris Taggert, analyst at CreditSights, a research group. "You end up with higher volatility for the simple fact that, without the dealers taking positions, trading volumes will move prices faster."*

(p.120) 3.6. The Full Picture

In this chapter, we have seen that an order's short run and long run effects on transaction prices differ depending on what determines the bid-ask spread, that is, whether the source of market illiquidity is adverse selection costs, order-processing costs or rents, or inventory holding costs. In this section we bring these factors together, illustrating qualitatively the type of price response to the order flow that we should observe depending on which of the three underlying forces determines it. Figure 3.8 plots the expected path of the transaction price over time, conditional on a buy order coming to the market at time t. The reason why we plot the expected rather than the actual price path is that, at any time, transaction prices are affected by news on fundamentals as well as by new orders, so that the actual price path is subject to continuous shocks that may conceal the effect of the buy order at time t. By taking expectations of the price at time t, we filter out this noise and highlight the "impulse response" to the buy order submitted at that time.

The top panel of the figure shows that in a setting with adverse selection the price impact of an order is permanent: the buy order induces traders to revise their beliefs about fundamentals upwards, and embed this change in beliefs fully and permanently in their quotes. By the same token, in this setting there

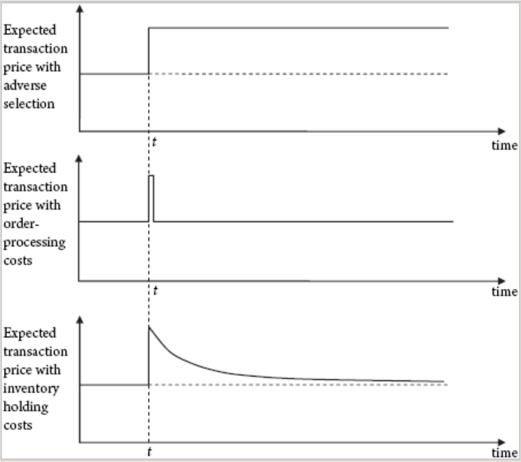


Figure 3.8. Price reaction to a market buy order in different settings

(p.121) is no difference between the short run and the long run effect of the order. The middle panel shows that with order-processing costs (but no adverse selection) the buy order induces a temporary upward blip in the transaction price: the effect is present only in the very short run and disappears in the longer term, generating short-term negative autocorrelation in transaction price changes. Merging the time impacts illustrated in the top and middle panels, we have the short-term and long-term price reactions described by equations (3.34) and (3.36), which merge the adverse selection and order-processing cost models.

The bottom panel displays the price path when a buy order is submitted to risk averse dealers: the price reaction is positive in the short term but vanishes in the long term. So there is a price reversal as in the middle panel. Here, however, the price impact does not vanish instantly but dissipates slowly over time, as the inventories are gradually brought back to target. So again in this case returns should exhibit negative autocorrelation, but at lower frequencies and for longer lags than in the case of order-processing costs.

In the real world, of course, all three effects may be present at once. This is illustrated in figure 3.9, which shows how all three may contribute to some extent to the positive short-term impact of a buy order, with some reversal occurring quickly as a result of order-processing costs, some further reversal occurring more gradually due to inventory holding costs, and in the long run only the informational effect associated with adverse selection persisting.

Market microstructure researchers have repeatedly sought to decompose the bid-ask spread into the three components of adverse selection, order-processing costs (plus rents), and inventory holding costs. To identify these components of market illiquidity, researchers exploit the fact that each one has a different effect on the time-series properties of prices and orders, as is shown in the previous figure. Chapter 5 will explain in detail the econometric techniques that

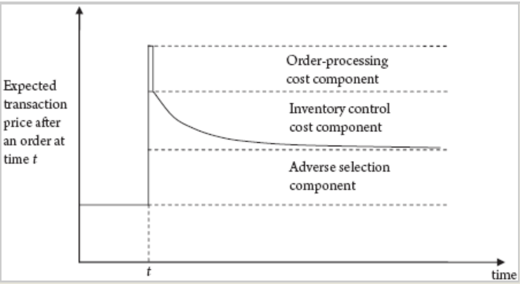


Figure 3.9. Short-term and long-term price responses to a market buy order

(p.122) are applied to identify these effects and measure the contribution of each source of market illiquidity to the bid-ask spread.

3.7. Further Reading

The model of trading with asymmetric information developed in section 3.3 is based on Glosten and Milgrom (1985). This model has been extended in many directions. For instance, Easley and O'Hara (1987) consider the possibility of multiple trade sizes for the informed investor; Easley and O'Hara (1992) introduce the possibility of information event uncertainty (that is, the possibility of there being no change in the asset value). In this case, dealers learn not only from the order flow but also from trading volume (the total number of trades over a given period).

One important feature of this model is that liquidity suppliers have less information than some liquidity demanders. In a model related to Glosten and Milgrom (1985), Calcagno and Lovo (2006) provide a theoretical analysis of price competition among dealers when some dealers are better informed than others. In this case, dealers' quotes are informative; the order flow not. Bloom-field, O'Hara, and Saar (2005) show experimentally that informed investors optimally use both market and limit orders when they have the requisite flexibility to do so. Overall, Calcagno and Lovo (2006) and the experimental evidence in Bloomfield, O'Hara, and Saar (2005) suggest that in securities markets quotes also contain information, in addition to order flow.

The model by Glosten and Milgrom (1985) has also been used to address various policy issues, as we do in Chapter 8. For instance, Diamond and Verrecchia (1987) use it to study the effects of short-sales constraints on price discovery. They show that transaction prices remain efficient in the semi-strong form with short-sales constraints, as in Glosten and Milgrom (1985), but that short-sales constraints slow the process of price discovery. Intuitively, such constraints prevent informed investors from selling a stock when they have bad news, which eventually reduces the speed at which prices reflect informed investors' private information.

Stoll (1978) was the first to relate the bid-ask spread to inventory holding costs. Ho and Stoll (1981) analyze the optimal dynamic pricing policy for a monopolist dealer in presence of inventory risk (see also Madhavan and Smidt, 1993, and Hendershott and Menkveld, 2010). Amihud and Mendelson (1980) also consider a dynamic model in which dealers are risk neutral but face position limits (i.e., a constraint on the maximum size of their inventory). They show that position limits also generate mean reversion in inventories. The empirical literature exploits the idea that inventory effects dissipate in the long run to **(p.123)** separate adverse selection from inventory holding costs (see Chapter 5). Our presentation in section 3.5.3 is related to Hasbrouck (1988). Given inventory risk, information on the aggregate inventories is informative on the short-term path of prices. Such information may also be a source of adverse selection; its effect is analyzed by Vayanos (1999). Ho and Stoll (1983), Biais (1993), and Yin (2005) study competition between dealers when they have different inventories in various market structures.

The speed of dealers' inventories' reversion to the mean is sometimes used by researchers as a gauge of the importance of inventory risk in securities markets (see, for instance, Madhavan and Smidt, 1993, Hasbrouck and Sofianos, 1993, Lyons, 1995, and Hendershott and Menkveld, 2010). Interestingly, the rate of reversion to the mean varies across assets. For instance, Madhavan and Smidt (1993) and Hasbrouck and Sofianos (1993) empirically find a slow rate of mean reversion for the inventories of the specialists in NYSE stocks, whereas Lyons (1995) finds a very high rate of reversion for a dealer in the Deutsche Mark/Dollar market. These differences may reflect differences between assets in price volatility, risk aversion of dealers or responsiveness of order flow. Recent years have seen the emergence of high frequency market-makers. These market-makers automate the posting of their quotes and use their speed of access to various trading platforms and information to manage inventory risk very efficiently. As a result, they hold their inventories for a very short period of time (see, for instance, evidence in Menkveld, 2011). This suggests that the speed of reversion to mean in dealers' inventories and prices may have increased considerably in recent years.

3.8. Exercises

1. Bid-ask spread with order-processing costs.

In section 3.4.1 order-processing costs are assumed to be γ per share traded. Consider the following alternative assumptions:

a. Assume that order-processing costs are k per transaction. Compute the bid-ask spread in this case and show that it is decreasing with the size of the transaction. Which features of the technology of trading would lead you to think that this is a realistic model of order-processing costs?
b. Assume that order-processing costs are k per euro traded. Show that the absolute bid-ask spread is increasing in the security's underlying value and the relative bid-ask spread is constant, in contrast with the expressions found in the text where order-processing costs are a constant γ per euro traded, irrespective of the share value.
(p.124) c. Stoll (2000) reports that Roll's measure ranges from 6.45 cents for small-company NASDAQ stocks to 13.17 cents for large-company NASDAQ stocks (p. 1494). Note that, on average, small-company stocks have a lower price. Which of the following hypotheses is more consistent with this empirical evidence?
i) Order-processing costs are constant per share traded.
ii) Order-processing costs are constant per dollar of value traded.

2. Bid-ask spread and insider trading.

A small risky company's stock is worth either \$10 (v^L) or \$20 (v^H) with probability $\frac{1}{2}$ each ($\theta = 1-\theta = 0.5$).

a. Compute the bid and ask prices set by risk neutral competitive market makers in the absence of informed trading.
b. Compute the bid and ask prices set by risk neutral competitive market makers when they expect one in ten of trade initiators to be informed (to know the stock's true value) and to trade as profit maximizers, while the other nine out of ten are uninformed and buy or sell with equal probability. Assume that all transactions are of the same size.
c. Compute the average trading cost to an uninformed trader and the average gain to an informed one, assuming a unit trade size in both cases.
d. Do you agree with the following statement? "Insider trading does not harm most market participants: it harms only those who are unlucky enough to trade with an insider." Why? Refer to the example in this exercise to illustrate your argument.

3. Imperfectly informed investors in the Glosten-Milgrom model.

Consider the one-period Glosten-Milgrom model, where the security's true value v can be high (v^H) or low (v^L) with probability $\frac{1}{2}$ each. Market makers are competitive and risk neutral, and do not know v. In each period, a single trader comes to the market: with probability $1 - \pi$, he is a noise trader, who buys or sells one unit with probability $\frac{1}{2}$ each; with probability π he is an informed trader, who observes a signal about security's true value. With probability $\rho \in (\frac{1}{2}, 1]$ the signal is accurate, that is, it coincides with the true value of the security. With probability $1 - \rho$, instead, the signal is mistaken, so that the insider assigns the wrong value to the security. Hence, ρ measures the accuracy of the signal observed by the informed trader: for ρ close to $\frac{1}{2}$, the informed would be similar to a noise trader; for $\rho = 1$, the insider trader would be perfectly informed.

a. Write down dealers' expected profits when they receive both a buy and a sell order. [Hint: assume that the informed trader buys the security **(p.125)** when his signal equals v^H and sells when it is v^L, so that in each instance with probability ρ he makes profits and with probability $1 - \rho$ he makes losses.]
b. Compute the bid and ask prices set by risk neutral competitive market markers.
c. Derive the bid-ask spread as a function of signal's informativeness. How does this result compare with the case of perfectly informed insider trading? Is the market more or less illiquid? Intuitively, why?
d. Verify whether, given the bid and ask prices derived at point b, the insider is actually willing to buy when his signal equals v_H and sells when it is v_L, that is, whether this strategy yields positive expected profits in equilibrium.

4. Endogenous information acquisition by insiders in the Glosten-Milgrom model.

Consider the one-period Glosten-Milgrom model, where the security's true value v can be high (v^H) or low (v^L) with probability $\frac{1}{2}$ each. Market makers are competitive and risk neutral, and do not know v. In each period, a single trader comes to the market: with probability $1 - \pi$, he is a "noise trader," who buys or sells one unit with probability $\frac{1}{2}$ each; with probability π he is a "potential insider," who learns the security's true value v if he pays a cost c, in which case he will trade on his information to make a profit. (If he does not elect to acquire information, he does not trade.)

a. Compute the bid and ask prices and the bid-ask spread that market makers set, assuming that they believe the insider will acquire information with some given probability $\phi \in [0,1]$?
b. Given these prices, determine the trading profit of an insider who has decided to acquire information.
c. Determine the condition on the cost parameter c in terms of π and $v^H - v^L$ under which the potential insider will never choose to acquire information, even in the most auspicious situation in which market makers do not expect to face any insider trading (i.e., $\phi = 0$).
d. Determine the condition on the cost parameter π in terms of π and $v^H - v^L$ under which the potential insider will always choose to acquire information, even in the least auspicious situation in which market makers expect to face insider trading with probability π (i.e., $\phi = 1$).
e. Now consider intermediate values of c for which neither of the conditions determined under (c) and (d) is satisfied. Determine the value of ϕ that will make the potential insider indifferent between acquiring and not acquiring information. This endogenous value of ϕ describes a "mixed-strategy equilibrium" in which the insider randomizes between **(p.126)** the two options. How does this value of ϕ depend on the parameters of the model? Explain your results intuitively. **f.** Characterize the equilibrium in the three ranges of the values of c considered in (c), (d), and (e). Plot the bid-ask spread and the potential insider's net expected profit as a function of the cost c of acquiring information. Explain your graph intuitively.

5. Equilibrium with price-sensitive uninformed trading.

Consider the one-period version of the model developed in section 3.3, assuming $v^H = 1$ and $v^L = 0$ with equal probabilities, and the fractions of both informed and uninformed investors are π and $1 - \pi$, respectively. Uninformed investors refuse to trade at a price that is more than δ away from the public estimate μ of the security's value.

a. Compute the zero-expected-profit bid and ask quotes posted by the dealers.
b. Plot how bid and ask prices and the probability of trading vary with π.

6. Market breakdown in the face of excessive informed trading.

Risk-neutral competitive market makers set bid and ask prices for a security whose final liquidation value v is distributed on $(0, \infty]$ with density:

$$f(v) = \frac{2}{(v+1)^3}.$$

The liquidation value of the security becomes publicly known after trading; but during trading, it is known only to informed traders who come into the market with probability π and trade a unit to maximize their profit. Noise traders come with probability $1 - \pi$; they buy a unit of the security if their personal valuation is higher than the ask price set by market makers and sell a unit if their personal valuation is lower than the bid price. Noise traders' personal valuations for the security are also distributed with the same density $f(\cdot)$, though independently of the true value v.

Compute equilibrium bid and ask prices, and show that there is a critical value of π beyond which the market breaks down, in the sense that the market makers are unable to set a profitable ask price.

In solving this exercise, you can use the fact that the cumulative distribution function corresponding to density function $f(\cdot)$ is:

$$F(v) = 1 - \frac{1}{(v+1)^2},$$

and also that for this distribution, for any given ask price a and bid price b,

$$\mathrm{E}(v) = 1,$$

$$\mathrm{E}(v) = 1,$$
$$\mathrm{E}(v|v \geq a) = 2a + 1,$$
$$\mathrm{E}(v|v \leq b) = \frac{b}{b+2}.$$

(p.127) 7. Differences in price pressure across stocks.

Can you explain the empirical finding by Hendershott and Menkveld (2010) that the price pressure per unit of inventory is much greater for small-capitalization than for large-capitalization stocks, based on the inventory holding cost model presented in this chapter? How?

8. Short-sale constraints, liquidity, and market efficiency (inspired by Diamond and Verrechia, 1987).

Consider the market for a risky security with uncertain final value v, which can be 1 or 0 with equal probability. As in this chapter, quotes for unit trade size are posted by risk neutral market-makers, and market orders are submitted either by informed traders, who arrive with probability π and know the true value of the stock, or by liquidity traders, who arrive with probability $1 - \pi$ and wish to buy or sell with equal probability.

Traders may be subject to constraints on short selling: an informed trader is short-sale constrained with probability κ_i, and a liquidity trader is short-sale constrained with probability κ_l. If a trader is constrained, he cannot sell when he would like to do so and in this case does not trade. Let d be the direction of the market order: $d = 0$ (*no trade* if the trader wants to sell and is short sale constrained), $d = +1$ (buy market order) and $d = -1$ (sell market order).

a. Draw the tree for the order arrival process in this model.
b. Calculate the bid and ask quotes, and show that the bid-ask spread does not depend on κ if short-sale constraints are identical for liquidity traders and informed traders, that is, if $\kappa_i = \kappa_l = \kappa$. What happens to the bid-ask spread if $\kappa_i > \kappa_l$ or if $\kappa_i < \kappa_l$? What is the economic intuition for these results?
c. Compute dealers' expectations of the value of the security *if there is no trade*. Show that it is strictly lower than the unconditional expected value of the security if and only if $\kappa_i > 0$. What is the intuition for this finding?
d. Assume that $\kappa_i = \kappa_l = \kappa > 0$. Let $p(d) = \mathrm{E}(v \mid d)$ be the "price" of the security at time 1 for $d \in \{1, 0, -1\}$. (Note that $p(0)$ is not observable in the form of a quote, because it is the market maker's valuation of the security on receiving no order.) Show that

$$\mathrm{E}(p(d)) = \mathrm{E}(v)$$

and that

$$\mathrm{E}(p(d)\,|d \neq 0)\,|\rangle \mathrm{E}(v)\,\mathrm{if}\kappa\rangle 0.$$

(p.128) e. Suppose that you have data for the transaction prices for stocks traded in a market before the removal of short-sale constraints (time $t - 1$) and after its removal (time t). According to the model presented above, would you expect, on average, to find a negative, zero, or positive return around the date in which the constraint is removed? Can we use this result to evaluate the merits of short-sale constraints?

9. Bid-ask spread and order size in the inventory holding cost model.

Consider the two-period model with inventory risk in section 3.5.1 where the representative dealer's utility is $U = E_t(w_{t+1}) - \rho\, sd_t(w_{t+1})$. The dealer can receive a buy order $y_t > 0$ or a sell order $y_t < 0$ and is endowed with an initial inventory z_t.

a. Compute the bid-ask spread as a function of the order size y_t when the dealer has a long initial position ($z_t > 0$).
b. Compute the bid-ask spread as a function of the order size y_t when the dealer has a short initial position ($z_t < 0$).
c. Represent these two cases graphically (plot price on the vertical axis and order size y_t on the

horizontal axis).

10. Inventory holding cost model with order flow risk.

This exercise extends the model of inventory holding costs presented in section 3.5.2 to the case of random order flow, so that impending orders cannot be fully anticipated by dealers. As in that section, suppose that the representative dealer's objective function is linear in the mean and the standard deviation of his wealth at time $t + 1$:

$$U(w_{t+1}) = E(w_{t+1}) - \rho sd(w_{t+1}),$$

and that dealers are competitive and short-sighted. The dealer's wealth is defined as in section 3.5.1. In deciding how to price an order arriving at time t, the dealer must take into account that over his holding period $[t, t + 1]$ he faces not only *fundamental risk* σ_ε (arising from new public information ε_t about the fundamental value μ_t) but also *order flow risk*, that is, the risk that new orders will further unbalance his inventory, leading to an adverse movement in the market clearing price at the end of the period. Assume that the two sources of risk are independent; that is, the order flow is not driven by traders with an informational advantage over market makers ($\pi = 0$), and that dealers have no order-processing costs ($\gamma = 0$).

Suppose that in every period just one customer arrives. With probability $1-\delta$, this customer simply places a buy or sell order with probability $\frac{1}{2}$ each, irrespective of the prices charged by dealers. With probability δ, instead, the customer is price-sensitive and compares dealers' prices to the fundamental value, based on publicly available information. That is, he only places an order if **(p.129)**

Table 3.3. Order Flow Response to Observed Market Prices

Price level	$\Pr(d_{t+1} = +1)$	$\Pr(d_{t+1} = -1$	$E_t(d_{t+1})$	$\text{var}_t(d_{t+1})$
$p_t \langle \mu_t$	$\frac{1}{2}(1+\delta)$	$\frac{1}{2}(1-\delta)$	δ	$1-\delta^2$
$p_t = \mu_t$	$\frac{1}{2}(1-\delta)$	$\frac{1}{2}(1-\delta)$	0	$1-\delta$
$p_t \rangle \mu_t$	$\frac{1}{2}(1-\delta)$	$\frac{1}{2}(1+\delta)$	$-\delta$	$1-\delta^2$

he expects to gain. As a result, if the market price is below the fundamental value ($p_t \langle \mu_t$), the probability of observing a buy order is $\frac{1}{2}(1-\delta)+\delta = \frac{1}{2}(1+\delta)$, while that of observing a sell order is only $\frac{1}{2}(1-\delta)$: buys are more likely than sells. The opposite is the case if the market price is higher than the fundamental value ($p_t \rangle \mu_t$). Hence, the order flow is assumed to respond to observed market prices as shown in Table 3.3.

The parameter δ captures the responsiveness of the order flow to the difference between fundamental value and price: if $p_t \langle \mu_t$, the dealer expects to receive a net order flow δ in the next period, whereas if $p_t \rangle \mu_t$ he expects a net order flow $-\delta$. As in section 3.5.2, the representative dealer's inventory after trading is $z_{t+1} = z_t - y_t$ and market clearing requires $y_t = d_t$, from which $z_{t+1} = z_t - d_t$.

To solve for equilibrium, start from the conjecture that p_t has a time-invariant and linear relationship with dealers' inventories z_t, centered around the true underlying value μ_t with a markdown proportional to the dealers' net inventory *after* the transaction, namely:

(3.63)

$$pt = \mu_t - \beta(z_t - y_t) = \mu_t - \beta z_{t+1}.$$

a. Compute the expected price at time $t + 1$ and its variance by using the distribution of the order flow at time $t + 1$ shown in table 3.3 together with the market-clearing condition and conjecture (3.63). (Note that: (i) there are three different expressions for the expected price depending on whether z_{t+1} is positive, negative or zero, while the variance takes different values depending on whether z_{t+1} is equal to zero or not; (ii) from (3.63), $z_{t+1} \gtreqless 0$ implies $p_t \gtreqless \mu_t$.)
b. Solve for the equilibrium pricing policy. (Hint: combine the result obtained under (a) and the first-order condition of the dealer's problem.)

c. Show that in equilibrium:

$$\beta = \frac{\rho\sigma_\varepsilon}{\sqrt{\delta^2 - \rho^2\left(1-\delta^2\right)}}.$$

(p.130) Compare this value of β with that obtained in the model without order flow risk in section 3.5.2, and give an economic interpretation of the relationship between β and order flow responsiveness δ.
d. Compute the equilibrium bid-ask spread. How is it related with order flow responsiveness δ and why?

11. Predicting future order flow.

In section 3.4.2, we discuss how the direction of the order flow at future dates t and beyond can be predicted given the current belief θ_{t-1} about the probability of a high value of the stock:

a. Use table 3.2 to determine $\mathrm{E}(d_{t+T})$ for any $T \geq 0$, given the current belief θ_{t-1}.
b. Use equation (3.16) to update beliefs in the wake of a buy order and thus compute:

$$\mathrm{E}\left(d_{t+T}|\Omega_{t-1}, d_t = 1\right) - \mathrm{E}\left(d_{t+T}|\Omega_{t-1}\right).$$

12. Empirical analysis of the determinants of spreads and the Amihud illiquidity measure.

The data for this exercise are contained in the Excel file, Ch3_ex12_data.xls or in the Stata data file Ch3_ex12_data.dta available on the companion website for this book. These files provide a record of the average values of the following variables, for a sample of 1128 stocks traded in U.S. stock markets (based on daily data for April 2009):

- • ticker code that identifies each stock (*ticker*);
- • bid price (*pb*), ask price (*pa*), and closing price (*p*);
- • number of shares (in thousands) traded per day (*vo*);
- • trading volume (in millions of dollars) per day (*vp*);
- • number of shares (in thousands) outstanding (*ibnosh*);
- • sector according to GICS classification (*gics*);
- • relative bid-ask spread, in percent (*bas100*);
- • Amihud illiquidity measure (*ami100*);
- • market capitalization, in millions of dollars (*mktcap*);
- • daily return (*ret*);
- • realized volatility (twenty-trading-day moving standard deviation of returns, *vola*).

a. Compute the correlation matrix of the closing price, trading volume, market capitalization, bid-ask spread, average return, and return volatility. Which variables are most closely correlated? What does this suggest for the specification of regressions whose dependent variable is the bid-ask spread or the Amihud illiquidity ratio?
b. Estimate regressions where the dependent variable is the bid-ask spread, in six specifications that all include a constant, volatility, and **(p.131)** *alternatively* the following other variables: 1) market capitalization, 2) trading volume, 3) log of market capitalization, 4) log of trading volume, 5) turnover rate (defined as *vo/ibnosh*); 6) turnover rate and log of closing price. Compare the explanatory power of these alternative specifications and, based on the models studied in this chapter, interpret the estimates so obtained. In particular, indicate the change in the bid-ask spread associated with a 1 percent increase in trading volume, according to your estimates.
c. Repeat the empirical analysis under (b) using the Amihud illiquidity measure as dependent variable.

How do the results compare with those under (b)?
d. Would the theories studied in this chapter lead one to suppose that financial stocks should have been more illiquid than non-financial, other things being equal, in the estimation period of April 2009? Investigate by generating a financial sector dummy variable (*fin*) that takes the value 1 when the *gics* variable is equal to 40 (the code of the financial sector according to the Global Industry Classification Standard), and then by re-estimating specifications 4 and 6 described under point (b) of this exercise with the addition of this dummy among the regressors. Comment on your results.
e. The specification estimated in answering the previous question allows for the possibility that financial stocks may have been more illiquid than other stocks for given values of all the other explanatory variables. In fact, in the financial sector, the relationships of illiquidity with capitalization, trading volume, and price may not have been the same as in other sectors, in April 2009. Test this hypothesis within specification 6 under point (b) of this exercise (testing the equality of the coefficients of the explanatory variables first separately, and then jointly), and offer an interpretation of your results.

Notes:

(1.) The companion website for this book reports the same data for the entire day (519 trades).

(2.) The transaction price is not necessarily equal to the bid or the ask price just before the transaction. In a limit order market, the depth at the best quotes may be too little to fully execute a marketable order. In this case, the order walks up or down the book (depending on its direction). So the average transaction price can be higher than the current ask or lower than the current bid price. The price can also be strictly inside the quotes if it is a matched trade pre-negotiated outside the main market and entered into the system by a special procedure. For a description of the trading operations in a limit order market, see Chapter 1; for a formal analysis of this trading mechnaism, see Chapter 6.

(3.) The role of trade size is considered in Chapter 4.

(4.) Palomino et al. (2009) actually find some evidence that contradicts the EMH: the stock price does not seem to adjust immediately to bad news, or to incorporate all relevant public information.

(5.) There is also considerable debate among scholars on whether the EMH is a good description of price formation at lower frequencies, for instance, monthly or even yearly: see the overview by Shleifer (2000).

(6.) Insiders (i.e., employees of a firm, board members, or people with close connections to them) are very well placed to obtain private information. Insider trading is strictly regulated (e.g., insiders cannot trade in the run-up to major company announcements) precisely in order to mitigate the risk of informed trading in the marketplace. Nevertheless, cases of illegal insider trading are frequent and constitute a good example of informed trading.

(7.) For instance, in the extreme case where *all* investors are better informed than the market maker, he will be singled out for a trade *only* when his quotes generate a loss for him. Clearly no one would want to make a market under these circumstances.

(8.) Like Glosten and Milgrom (1985), we suppose that all market participants are risk neutral, market makers are perfectly competitive, and traders place orders of a fixed size. But we simplify their setting by assuming that the value of the security has a binary distribution and that the orders placed by liquidity traders are price inelastic.

(9.) We also assume that all dealers have identical information, which means in this case that they all observe the order flow realized at each date. This is a reasonable assumption in a market where trade characteristics (price and size) are reported immediately to all market participants, as in many equity markets. But it is unrealistic in more opaque markets, such as the FX market or the bond market. We discuss these points in greater detail in Chapter 8.

(10.) Denoting by I the arrival of a buy order from an informed trader, U the arrival of a buy order from an uninformed trader, and B the arrival of a buy order, the dealer's probability of trading with an informed investor on the buy side is

$$\Pr(I|B) = \frac{\Pr(I \cap B)}{\Pr(B)} = \frac{\Pr(I \cap B)}{\Pr(I \cap B) + \Pr(U \cap B)} = \frac{\pi\theta_{t-1}}{\pi\theta_{t-1} + (1-\pi)\frac{1}{2}}.$$

(11.) The insight that adverse selection can shut a market down dates back to Akerlof's celebrated piece in 1970 on the market for "lemons."

(12.) Bayes's rule states that the conditional probability of an event A given an event B is:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(B|A) \cdot \Pr(A)}{\Pr(B)}.$$

(13.) Note that the ask price at time t can be expressed in terms of the belief θ_{t-1} only (without reference to μ_{t-1}) by using (3.16) in (3.18), and similarly for the bid price:

$$a_t = \mu_t^+ = \frac{(1+\pi)\theta_{t-1}}{2\pi\theta_{t-1} + 1 - \pi} v^H + \frac{(1-\pi)(1-\theta_{t-1})}{2\pi\theta_{t-1} + 1 - \pi} v^L,$$
$$b_t = \mu_t^- = \frac{(1-\pi)\theta_{t-1}}{2\pi(1-\theta_{t-1}) + 1 - \pi} v^H + \frac{(1+\pi)(1-\theta_{t-1})}{2\pi(1-\theta_{t-1}) + 1 - \pi} v^L.$$

(14.) In this case, informed investors are just indifferent between buying or not trading. We assume that they buy.

(15.) A convenient way to express the updating of dealers' beliefs θ_t is to recast equations (3.16) and (3.17) in terms of the odds ratio, which yields a linear first-order difference equation:

$$\frac{\theta_t^+}{1-\theta_t^+} = \frac{1+\pi}{1-\pi} \cdot \frac{\theta_{t-1}}{1-\theta_{t-1}} \quad \text{and} \quad \frac{\theta_t^-}{1-\theta_t^-} = \frac{1-\pi}{1+\pi} \cdot \frac{\theta_{t-1}}{1-\theta_{t-1}}.$$

Therefore the odds ratio at time t is simply a function of the order imbalance xt, defined as the cumulative difference between buy and sell orders up to time t:

$$\frac{\theta_t}{1-\theta_t} = \frac{\theta_0}{1-\theta_0}\left(\frac{1+\pi}{1-\pi}\right)^{x_t}, \ \text{where } x_t \equiv \sum_{\tau=1}^{t} d_\tau.$$

(16.) Equation (3.27) implies that the price change in any given time period is proportional to the order imbalance during the period. Thus, it lays the foundation for the price impact regression discussed in Section 2.3.2 of Chapter 2. Here, the order size is fixed, so only the direction of orders is informative. Chapter 4 considers a more complex environment in which both order direction and size are informative.

(17.) Automation reduced some of these costs considerably, but they are still not negligible. For instance, on Euronext, from 2000 to 2004 liquidity providers paid an execution fee between e1.00 and e1.60 per trade.

(18.) Working out the medium-term effect of the buy order on price is somewhat trickier. Suppose that time $t + T$ is not so distant, and that by then the true value of the security is not yet publicly revealed. In this case dt has some predictive value for the future order flow $dt+T$. To see this, consider that a buy order at t leads to an upward revision from $\theta t\text{-}1$ to θ_t^+ in the probability that the stock has a high value, as described in equation (3.16), and so it increases the probability of further buy orders from informed investors at $t + T$, that is,

$$\mathrm{E}(d_{t+T}|\Omega_{t-1}, d_t = 1)\rangle \mathrm{E}(d_{t+T}|\Omega_{t-1}).$$

This increased probability is associated with an additional medium-term price impact beyond s_t^a. To see this, compare the expected price at $t + T$ before and after the buy order at time t:

$$\mathrm{E}\left(p_{t+T}|\Omega_{t-1}, d_t = 1\right) - \mathrm{E}\left(p_{t+T}|\Omega_{t-1}\right)$$
$$= s_t^a + \gamma\left[\mathrm{E}(d_{t+T}|\Omega_{t-1}, d_t = 1) - \mathrm{E}(d_{t+T}|\Omega_{t-1})\rangle s_t^a, \right]$$

where we have used equation (3.35). In exercise 11 the reader is asked the precise value of the revision in the expectation of $dt+T$ in this expression.

(19.) In a multi-period model such as that analyzed in Section 3.5.2 below, a far-sighted dealer would instead recognize that he could change his holdings again at time $t + 1$, depending on the new market situation. To deal with this more complex problem, one would have to resort to dynamic optimization methods, which are technically more challenging than those used in the simple static setting considered in the text. But the main insights into the effect of inventory risk on prices are already conveyed by the simpler case of short-sighted dealers. See Ho and Stoll (1981), Madhavan and Smidt (1993), or Hendershott and Menkveld (2010) for the analysis of dealers' dynamic optimization problem under various assumptions.

(20.) Since we are considering a representative dealer, zt must be interpreted as the dealers' aggregate inventory.

(21.) In Chapter 4, section 4.3.2, we analyze the imperfectly competitive case in which dealers take account of the impact of their orders on the clearing price (i.e., exert market power). But as we explain there, the results for the effects of inventory risk on price dynamics, our topic in this section, do not depend on whether dealers are competitive or not.

(22.) In the knife-edge case where the order exactly wipes out the dealer's initial inventory ($d_t \rangle z_t$), the price is indeterminate, i.e. it can take any value within the interval (3.48).

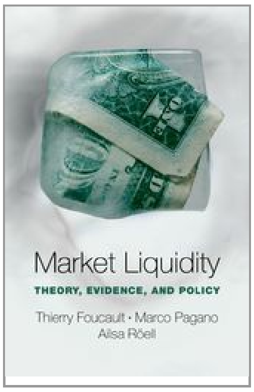
(23.) Strictly speaking, the bid-ask spread is given by equation (3.50) for orders of size at least $|z_t|$; otherwise, the bid and ask are equal. Given that in this chapter we consider only unit order size, a nonzero spread will only appear if the intial inventory is of size $|z_t| \leq 1$. Exercise 9 analyzes the more general case where the order yt can be of any size relative to the dealer's initial position z_t.

(24.) Also for simplicity, the customer's decision to buy or sell is assumed to depend only on whether the market price ishigheror lower than the fundamental value, and noton the magnitude of the discrepancy.

(25.) The model can be extended to consider noise. With such orders, prices also respond to the random component in the order flow, which creates additional price volatility on top of fundamental risk. As a result, dealers' holding costs will be affected not only by fundamental risk but also by order flow risk. This extension is developed in exercise 10 of this chapter.

(26.) Note that from (3.53) and (3.54), $p_\text{t} - \mu_t = m_{t+1} - \mu_{t+1} = -\beta z_{t+1}$. Hence a larger z_{t+1} is associated with a lower transaction price at t and a lower midquote at t+1.

(27.) In this section, the order flow direction d_t is uninformative, so μ_t is unaffected by the time t order direction d_t.



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Trade Size and Market Depth

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter provides a framework for analyzing the determinants of market depth (i.e., how and why transaction prices react to order size). Market depth depends on three of the factors that affect the bid-spread: asymmetric information, risk aversion, and rents due to imperfect competition among market makers. Section 4.2 presents these ideas, building upon one of the most popular models of price formation under asymmetric information: the static version of the Kyle (1985) model of informed trading and market liquidity. The model is extended to imperfectly competitive market-making, in the context of a call market, where all traders submit supply or demand schedules and a Walrasian auctioneer crosses them at a common market-clearing price. Section 4.3 analyzes a call market where market makers have inventory concerns, being risk averse. It shows that in this setting the price impact of orders is increasing in order size, and depends on the

underlying volatility of the stock and the risk aversion and market power of the market makers. The final sections provide suggestions for further reading and exercises.

Keywords: market depth, price formation, asymmetric information, informed trading, imperfectly competitive market-making, call market

Learning Objectives:

- • How trade size affects prices
- • Market depth
- • How traders choose their trade size
- • Why the number of dealers is a determinant of market liquidity

4.1. Introduction

On Friday January 18, 2008, the top management of Société Générale (the world's leading equity derivative trading house at the time) discovered that one of their traders, Jérôme Kerviel, had accumulated massive unauthorized positions in European equity derivatives (estimated at somewhere between forty and fifty billion euros), which had already accrued losses of about €1.5 billion. On Monday January 21 the bank began liquidating these positions in a series of small trades and waited six days to go public with the news, giving itself time to unwind the positions and avoid greater losses. Even so, by the end of the week, its losses amounted to €4.9 billion. Yet Société Générale Chairman Daniel Bouton commented: "Had we not acted swiftly, the loss could have been ten times worse." As Société Générale was unwinding its positions, stock markets fell so sharply that on Tuesday January 22 the Federal Reserve was prompted to step in with a dramatic cut in interest rates. Société Générale's fire **(p.133)** sales probably contributed to the stock market drop, even though the selling was limited to about 10 per cent of daily trading volume.

This episode illustrates three facts. First, the strategy of gradual sales indicates that when an investor wants to sell a large amount of securities he should try to avoid very large orders that are likely to unsettle market prices. Second, the fact that Société Générale kept its exposure secret for a whole week while unwinding it suggests that the news would have had a serious adverse impact on market prices. Finally, the fact that the market turned down illustrates that even this gradual, covert strategy was not entirely successful at insulating the company from loss. In short, even very liquid markets—such as those for equity index futures–have limited depth, so large orders can unsettle prices.

In this chapter we present a framework for analyzing the determinants of market depth (i.e., how and why transaction prices react to order size). Recall that a deep market is one in which large orders do not have a much greater impact on prices than small orders. We shall see that depth depends on three of the factors that affect the bid-spread (see Chapter 3), namely (i) asymmetric information, (ii) risk aversion and (iii) rents due to imperfect competition among market makers:[1]

(i) Insofar as larger orders reflect more private information than small orders, they induce larger price movements. The sensitivity of prices to trade size is then determined by the degree of asymmetric information between liquidity providers and informed traders: the greater the asymmetry, the shallower the market–that is, the more sensitive prices are to an order of a given size.
(ii) Another factor affecting market depth is the risk aversion of liquidity providers. Filling a large order exposes providers to greater inventory risk; consequently the more risk-averse they are, the larger the price concession that they require to execute trades. To the extent that the order is absorbed by multiple liquidity providers rather than a single one, market depth will depend on their aggregate risk-bearing capacity, hence also on their number.
(iii) Finally, strategic market makers have market power. As liquidity providers' market power is inversely related to their number, this is another reason why market depth increases with the number of liquidity providers.

(p.134) We present these ideas in section 4.2, building upon one of the most popular models of price formation under asymmetric information: the static version of the Kyle (1985) model of informed trading and market liquidity. The price impact of orders is increasing in order size, to an extent that depends on the informational advantage of the informed investor and the volume of liquidity or "noise" trading (trades that are not information driven). Section 4.2.4 extends the model to imperfectly competitive market-making, in the context of a call market (also known as a batch auction), where all traders submit supply or demand schedules and a Walrasian auctioneer crosses them at a common market-clearing price (see Chapter 1). Then, in section 4.3, we analyze a call market where market makers have inventory concerns, being risk averse. We show that also in this setting the price impact of orders is increasing in order size, and depends on the underlying volatility of the stock and the risk aversion and market power of the market makers.

4.2. Market Depth under Asymmetric Information

To understand the determinants of market depth, we start with a model in which some traders have superior information. The logic is similar to that of the Glosten-Milgrom model described in Chapter 3. Here too, the price is set by risk-neutral market makers, but with two differences. First, orders can be of any size. Second, market makers may vary their prices depending on the quantities traded. This extension enables the model to portray the role of trade size in conveying information and driving price movements.

The vehicle for our analysis is the model developed by Kyle (1985), now a standard for studying trading in the presence of asymmetric information. As in Glosten-Milgrom, the demand for liquidity comes from two types of traders, informed and uninformed. But whereas in that model, market makers faced either an informed or an uninformed order, here orders from the two types of traders are batched together. Uninformed investors submit a random aggregate order u, which is normally distributed, with zero mean and variance σ_u^2. The informed trader has advance knowledge of the security's value $v = \mu + \varepsilon$, which other market participants see as a variable drawn from a normal distribution with mean μ and variance σ_v^2, following Kyle's notation (note that σ_v^2 is simply the variance denoted by σ_ε^2 in Chapter 3). Variables u and v are independent, and the informed trader does not know the uninformed order u. He can choose the size of his order and make it contingent on v by placing an order of size $x = X(v)$. Only market orders are allowed, not limit orders. The net batched order (the "order flow") $q = x + u$ is submitted to the market. The market makers do not observe v, but infer it-imperfectly-from **(p.135)** the order flow, q, or from the clearing price (depending on the way trading is organized).

In all the models considered below, the relationship between the equilibrium price and the order flow is linear and given by:

(4.1)

$$p = \mu + \lambda q.$$

From the standpoint of a trader who wants to buy or sell, the slope λ in this expression measures the price pressure exerted per unit of order size. More formally, this slope measures the price impact dp/dq. A market is deep when even a large order does not shift the price by much, that is, when λ is small. Formally, the depth of the market can be measured by $1/\lambda$, the inverse of the price pressure parameter, in other words, the amount of order flow that drives the price up by one unit.

The value of λ in equilibrium depends on how informative the order flow is (as it drives dealers' inferences about the payoff): in section 4.2.1, we explain how market makers update their estimate of the security's value on observing the order flow. Section 4.2.2 considers a setting in which competition eliminates all market makers' rents, so that their price is equal to their estimate of the security's value on observing the order flow. In section 4.2.3, we close the model by deriving both the optimal strategy of a monopolistic informed trader and the equilibrium order flow. This gives us a full description of market participants' strategies in equilibrium, as derived by Kyle (1985). Finally, in section 4.2.4 we allow for market power in a call auction framework, where the strategic interaction between market makers affects price determination, and market makers earn oligopoly rents.

4.2.1 Learning from Order Size

Market makers know that the order flow for a stock reflects information about the fundamental value v, because one of the orders comes from the informed investor, who knows v in advance. But there are two obstacles to extracting even part of this information from the order flow: (i) the informed order x is overlaid by the noise of the order u of uninformed investors and (ii) market makers must guess how the informed investor's trade is related to his information. This guess must be right; if not, it would lead to systematic losses for market makers, so eventually we shall have to check whether informed investors really do follow the type of strategy that market makers imagine. In other words, we have to solve for a rational expectations equilibrium, where market participants' conjectures are verified in equilibrium, as in the model of section 3.5.2 of Chapter 3. The **(p.136)** main difference from that model is that here we posit asymmetric information and strategic behavior.[2]

Market makers will obviously expect the informed trader to buy when he knows that the true value v of the security is higher than other market participants' estimate μ, and sell if it is lower. Specifically, suppose that they conjecture the informed investor to trade an amount x that is proportional to the discrepancy $v - \mu$:

(4.2)

$$x = X(v) = \beta(v - \mu)$$

for some parameter $\beta \rangle 0$.[3] What the market makers actually observe, however, is not the informed trader's order alone, but the total net order flow, including the noise traders' orders:

(4.3)

$$q = x + u.$$

That is, the net order flow is a noisy signal of the asset value v, so market makers can use the noisy signal q to form their expectation of v. The resulting conditional expectation $E(v|q)$ will generally differ from their unconditional expectation μ.

As the aggregate order size from noise traders u is normally distributed and independent of v, the expected value of v conditional on q is provided by the ordinary least squares regression rule:[4]

(4.4)

$$\begin{aligned} E(v|q) &= \mu + \frac{\text{cov}(v,q)}{\text{var}(q)} \cdot q \\ &= \mu + \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} \cdot q \\ &\equiv \mu + \alpha q, \end{aligned}$$

(p.137) where $\alpha = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_u^2}$ measures the sensitivity of expectations to the order flow. That is, α measures how informative the order flow is.

The informativeness of the order flow, a, depends on σ_u^2, σ_v^2 and β. For a fixed value of β, a decrease in the variance of noise trading σ_u^2 makes the order flow a more precise signal of the asset's value, and thereby enhances its informativeness. By the same token, informativeness increases with the variance of the security's value σ_v^2, since this increases the fraction of order flow volatility that can be attributed to the informed trader, and makes the total order flow more informative. The impact of β—the aggressiveness of the informed trader—is non-monotonic. Specifically, it is increasing for small values of β and then decreasing, because an increase in β has two opposite effects: greater aggressiveness by informed traders tends to make the order flow more informative, but at the same time inflates trades for any given value of v, so that the dealer must correspondingly scale down the extent to which he adjusts his estimate of v upon observing any given order flow. The first effect dominates for small values of β; the second dominates for sufficiently large β.

4.2.2 Perfectly Competitive Dealers

We now consider the case in which dealers are risk-neutral and trading is organized in three steps. In the first step, market orders from the two types of trader are batched into a single net order $q = x + u$. Second, each market maker responds with the price at which he is willing to execute this order in full. Third, the entire batched order is routed to the market maker who posts the best price.[5] Effectively, this means that dealers engage in Bertrand competition.

The expected profit of the dealer who wins the order at price p is

$$E\left[q\left(p-v\right)|q\right] = q\left[p - E\left(v|q\right)\right].$$

Competition between dealers drives this expected profit down to zero. For instance, suppose that $q > 0$ and that $p > E(v|q)$. If a dealer just matches the best offer, p, he is not sure he will execute the order, which will be allocated among all the bidders at price p. So he is strictly better off by slightly undercutting his competitors to win the order. This continues until the best offer is driven down to the dealers' conditional estimate of the security's value (4.4). At this point, it is no longer profitable for anyone to undercut the best offer and an equilibrium is reached. Hence, in this case, the equilibrium price is

(4.5)

$$p = E\left(v|q\right) = \mu + \alpha q,$$

(p.138) and therefore:
(4.6)

$$\lambda = \alpha = \frac{\beta\sigma_v^2}{\beta^2\,\sigma_v^2\, + \sigma_u^2}.$$

Thus, market depth is entirely determined by the informativeness of the order flow. The less informative the order flow, the deeper the market. The intuition is straightfoward. In this model, dealers use the order flow as a signal of the asset's value and adjust their prices accordingly. If the order flow is not very informative, it does not greatly affect dealers' value estimate, which means that it has little impact on prices since quotes are competitive.

Consider now a trader placing an order of size u: what price can he expect to get? He obviously knows his own order size u; he knows nothing about the size of other orders, but expects that, on average, they net out to zero. Therefore, based on equation (4.5), he expects his own order to be executed at price $E(p) = \mu + \lambda E(q|u) = \mu + \lambda u$. Figure 4.1 plots the relationship between this price and the investor's average order size, $|u|$.

Relative to traders' prior estimate of the asset payoff (μ), buy market orders execute at a premium and sell market orders at a discount equal to $\lambda\,|u|$. Thus, $\lambda\,|u|$ is the effective half bid-ask spread paid by investors on a transaction of size $|u|$. As Chapter 2 explains, the notion of effective bid-ask spread and market depth are closely related: the effective spread increases with the size of the order at a rate of λ.

What does this model tell us about the volatility of stock returns? We can think of the total return as $v - \mu$, the difference between the security's final value v and investors' pre-trading valuation μ (its initial market price). This total return can be split into two components: an initial return up to the time of trading, $p - \mu$, and a subsequent return from the time of trading to the moment when the final value v is revealed, $v - p$.

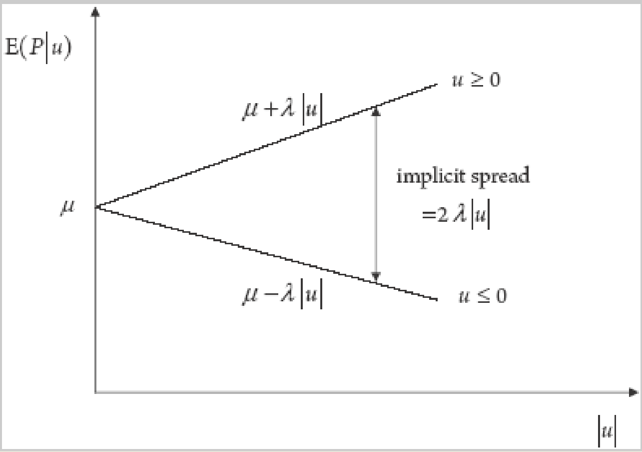


Figure 4.1. Depth and effective bid-ask spread

(p.139) The volatility of the initial return is:

(4.7)

$$\mathrm{var}(p-\mu) = \lambda^2\left(\beta^2\sigma_v^2 + \sigma_u^2\right) = \frac{\beta^2\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}\sigma_v^2,$$

which is increasing in the aggressiveness of informed trading, β. An increase in β means larger trade sizes for the informed trader (the "trade size effect"); thus, for any fixed value of λ, prices deviate more from traders' prior value estimate. This effect is partially offset by the fact that λ decreases with β, for large enough β. But this latter effect is always outweighed by the trade size effect. In short, trading is a source of volatility, because the order flow contains information that moves the price.

The volatility of the return after trading, $v - p$, is:

(4.8)

$$\mathrm{var}(v-p) = \frac{\sigma_u^2}{\beta^2\sigma_v^2 + \sigma_u^2}\sigma_v^2,$$

which is decreasing in the informed trader's aggressiveness, β. Intuitively, this is because if the informed investor trades more aggressively, more of his information is reflected in the market price, so the price is closer to the fundamental v. Price discovery is faster. In this model $\mathrm{var}(v - p)$ equals the "average pricing error," which Chapter 3 defines as $\mathrm{E}[(v - p)^2]$, and presents as an inverse measure of price discovery.[6] This finding parallels our result in Chapter 3 that an increase in the fraction of informed traders accelerates price discovery.

The variance of the total return on the security over both periods (before and after trading) is:

(4.9)

$$\mathrm{var}(v-\mu) = \mathrm{var}(v-p) + \mathrm{var}(p-\mu) = \sigma_v^2,$$

since $\mathrm{cov}(v - p, p - \mu) = 0$.[7] It is important to note that even though informed trading affects the variance of the pre- and post-trade returns ($p - \mu$ and $v - p$), it leaves the volatility of the total return ($v - \mu$) unchanged. Informed trading does not change the fundamental risk of the security: it simply brings forward the resolution of part of the uncertainty from the final date when v is revealed to the date when trading occurs.

(p.140) 4.2.3 The Informed Trader's Order Placement Strategy

So far, we have taken the trading strategy of the informed trader as given. Now we endogenize his behavior. As in Kyle (1985), we posit that dealers are competitive (so that they obtain no rents), and that the informed trader is risk neutral.

Intuitively, the informed trader's strategy depends on the dealers' pricing policy: the informed investor should trade less aggressively on his information when he expects dealers' prices to be very sensitive to the order flow. In turn, as we have seen, dealers' strategies themselves depend on the informed trader's aggressiveness. Thus, the strategies of dealers and of the informed trader are interdependent. In a Nash equilibrium, each agent behaves optimally given the other agents' behavior. We show that the following strategies constitute a Nash equilibrium:

(4.10)

$$\text{dealers' quote: } p(q) = \mu + \lambda q, \quad \text{with } \lambda = \frac{\sigma_v}{2\sigma_u},$$

(4.11)

$$\text{informed investor's trading: } X(v) = \beta(v-\mu) \ \text{ with } \beta = \frac{\sigma_u}{\sigma_v}.$$

Informed investor

Given dealers' pricing strategy (equation (4.10)), the informed investor's expected profit if he trades x shares is:

$$E[(v-p)x] = E[(v - \{\mu + \lambda(x+u)\})x] = (v - \mu - \lambda x)x.$$

Thus, the order size that maximizes his expected profit is:

(4.12)

$$X(v) = \beta(v - \mu),$$

with
(4.13)

$$\beta = \frac{1}{2\lambda}.$$

The trade size of the informed investor ($|X(v)|$) is inversely related to λ. Intuitively, the informed investor will place larger orders in deeper markets because they have less adverse impact on prices.

Dealers

From section 4.2.1, we know that if dealers expect the informed investor to follow a linear strategy $X(v) = \beta\ (v - \mu)$ (equation (4.2)), then their competitive price, for each value of q, is given by:

(4.14)

$$p(q) = \mu + \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} q.$$

Thus, we have

(4.15)

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} \Rightarrow \frac{1}{\lambda} = \beta + \frac{\sigma_u^2}{\beta\sigma_v^2}.$$

(p.141) Combining (4.13) and (4.15) yields the equilibrium values of λ and β:

$$\lambda = \frac{\sigma_v}{2\sigma_u}, \beta = \frac{\sigma_u}{\sigma_v}.$$

The larger the variance of noise trading σ_u^2 or the lower the variance of the fundamental value σ_v^2, the deeper is the market in equilibrium. Intuitively, the greater σ_u^2, the greater the average order flow in the market, and the less likely that an order of any fixed size will be interpreted as information driven. The smaller σ_v^2, the smaller the informed investor's average informational advantage, and the less dealers need to protect themselves against it.

In equilibrium, dealers absorb the aggregate market order. Thus, their net supply is q, and their expected profits can be written as

$$\mathrm{E}\left[(p-v)\, q | q=x+u\right]=\mathrm{E}\left[(p-v)\, x | q=x+u\right]+\mathrm{E}\left[(p-v)\, u | q=x+u\right].$$

The first term in this decomposition is the expected profit on trades with the informed trader; the second is the expected profit on trades with uninformed investors. As the informed investor has an informational advantage, dealers lose on these trades, so that $\mathrm{E}[(p-v)\, x | q=x+u] < 0$. As in equilibrium, dealers just break even, they must recover this loss by the expected profits on trades with the uninformed investors:

$$\mathrm{E}\left[(p-v)\, x | q=x+u\right]=-\mathrm{E}\left[(p-v)\, u | q=x+u\right]$$

Since this equality must hold for all possible realizations of q, it must also hold unconditionally (by the law of iterated expectations):

$$\mathrm{E}\left[(v-p)\, x\right]=\mathrm{E}\left[-(v-p)\, u\right].$$

Hence, the expected profit of the informed investor is just equal to the expected losses of the uninformed, who ultimately bear the adverse selection costs (as explained in Chapter 3).

In equilibrium, the informed investor's expected profit, conditional on a realized value of v, is

$$\begin{aligned}\mathrm{E}\left[(v-p)\cdot X(v)\,|v\right] &= \left[v-\mu-\lambda E\left(q|x\right)\right]\cdot x \\ &= \left[v-\mu-\lambda x\right]\cdot x=\frac{1}{2}\frac{\sigma_u}{\sigma_v}\left(v-\mu\right)^2,\end{aligned}$$

since $x=\frac{\sigma_u}{\sigma_v}\left(v-\mu\right)$. Therefore, his unconditional expected profit is

$$E\left[(v-p)\cdot x\right]=\frac{1}{2}\sigma_u\sigma_v.$$

Thus the informed investor has greater expected profit when there is more uninformed trading (σ_u) or when he has a larger informational advantage (σ_v).

(p.142) A greater volume of uninformed trading, in fact, provides more camouflage, since the aggregate order flow is a noisier signal than the informed investor's actual trade.

Recall that in this model, we can measure price discovery by the average pricing error, $\mathrm{var}(v-p) = \mathrm{E}[(v-p)$ (see section 4.2.2). Given the equilibrium actions of the informed investors and the dealers, price discovery is:[8]

(4.16)

$$\mathrm{E}\left[(v-p)^2\right]=\frac{1}{2}\sigma_v^2.$$

Thus, in equilibrium, half the uncertainty about the security's value is resolved at the time of trading.

An instructive way to represent the model graphically is to draw the two relationships that relate β and λ: equation (4.13), which gives the informed investor's choice of trading intensity λ as an increasing function of the depth of the market $1/\lambda$; and equation (4.15), which shows the market maker's choice of liquidity $1/\lambda$ as a convex function of the informed investor's trading intensity. This second relationship is non-monotonic because the market maker can afford to provide a deep market both when the informed investor's trading activity is very low *and* when the informed investor trades so aggressively as to give away his presence on the market. These two functions are plotted in figure 4.2. The intersection indicates the Nash equilibrium point, which happens to coincide with the point where market depth is minimized ($1/\lambda$ is at its lowest point when β equals σ_u/σ_v).

Depth improves whenever the informed investor's trading intensity differs from the level portrayed in figure 4.2: the line representing the informed investor's response function will then swivel away from the position shown in the figure. This occurs, for instance, if the informed investor is exposed to competition from other informed investors or is risk averse. (We leave the analysis of these two cases to the reader in exercises 3 and 6.) This means that, unlike the simple situation shown in figure 4.2, the equilibrium may actually be to the left or to the right of the minimum-liquidity point. This suggests to an important caveat for anti-insider-trading regulation and enforcement. In practice, one may not know whether the market equilibrium is to the left or to the right of the minimum-depth equilibrium point, so it is hard to say whether cracking down **(p.143)**

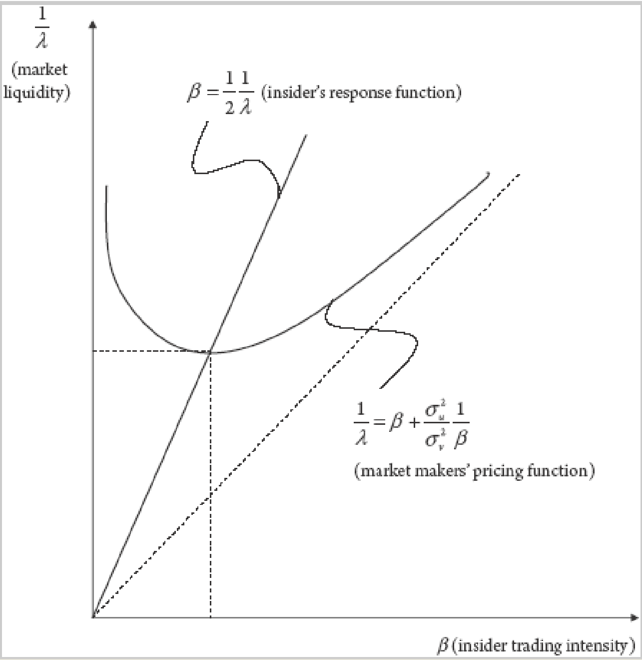


Figure 4.2. Insider's behavior and pricing function in Kyle's model

on insider trading, which steepens the insiders' response function in figure 4.2, would increase or decrease market liquidity.

To conclude this section, let us discuss some possible extensions. First, we have assumed that the informed investor can trade only once, after which his information becomes publicly known, hence obsolete. This means that he does not worry that by trading aggressively today he alerts the market to his presence and so lowers his potential future gains. Kyle (1985) analyzes the trade-offs involved in a multiperiod version of the model. In keeping with intuition, he shows that the informed investor trades less aggressively than in the one-shot case, in order to avoid dissipating his information advantage too quickly. In this case, information is incorporated into market prices only gradually, as the result of the repeated trading by the informed investor. In the limiting case of trading in continuous time, the rate at which information gets embedded in market prices is constant, and so is the volatility of price changes per unit of time.

Second, we have assumed that there is only one informed investor. The static model can easily be extended to consider multiple informed investors (see exercise 3). In this case, the depth of the market increases with the number **(p.144)** of informed investors. Holden and Subrahmanyam (1992) consider the multi-period case and show that competition among informed investors results in a race that dissipates their informational advantage very quickly by comparison with the case of a single informed investor.

4.2.4 Imperfectly Competitive Dealers

The trading mechanism considered in sections 4.2.2 and 4.2.3 is such that competition among dealers results in zero expected profits for the dealers. This is because risk-neutral dealers submit price bids for the *total* quantity demanded, and the auctioneer selects the best bid to fill the entire net order flow q. We now consider a different trading mechanism, namely a call auction (as described in Chapter 1), in which each dealer submits a schedule of offers that specifies the number of shares he is willing to buy or sell at each possible price, without knowing other dealers' offers. An auctioneer then parcels out investors' aggregate demand, q, among the dealers according to their demand or supply at the price that clears the market. Specifically, let $Y^k(p)$ be the total number of shares that dealer k is willing to supply at price p: if $Y^k(p) > 0$, the dealer is willing to sell shares; if $Y^k(p) < 0$, he is willing to buy $|Y^k(p)$ shares. If there are K dealers, their aggregate supply at price p is $\sum_{k=1}^{k=K} Y^k(p)$. Hence, when the aggregate investor demand for the security is $q = x + u$, the clearing price p^* set by the auctioneer is given by the market-clearing condition

(4.17)

$$\sum_k Y^k(p^*) = q.$$

We shall see that with this trading mechanism, competition among dealers does not drive their expected profits to zero unless their number is very large. Moreover, in this setting market depth depends not only on the informativeness of the order flow (as in section 4.2.2), but also on the number of dealers (which determines their market power).

Hence, even fine details of trading arrangements can affect market makers' profits, and market liquidity.

Recall that in section 4.2.2, the starting point was the market makers' conjecture about the trading strategy of the informed trader. In this setting, each market maker has to form a conjecture not only about the informed trader's strategy, but also about the behavior of the other market makers. This is because now, in contrast with the model of section 4.2.2, he cannot take the price as given: he knows he can affect it by changing his own supply of the security, and the response of the equilibrium price depends on how his competitors react to his own action.

(p.145) Thus, in order to determine the optimal behavior of any dealer, we must specify his beliefs about (i) informed investor trading (x) and how that affects his estimate of the security's value based on total customer demand (q), and (ii) the strategic behavior of his competitors, and what that means for the response of the equilibrium price to his own supply of the security. Suppose that because of informed trading, the best estimate of the security's value, given the total order flow q, is $E(v|q) = \mu + \alpha q$, as in section 4.2.2. Suppose further that each dealer k (including himself) will supply shares according to the linear supply function $Y^k(p) = \varphi(p - \mu)$. These two conjectures, together with market-clearing condition (4.17), enable each dealer k to identify a "residual demand function" (i.e., net customer demand q minus the supply from his $K - 1$ competitors for any price p). He can then compute how the equilibrium price $p^*(q)$ will respond to any possible supply of shares y^k he may decide on, and therefore identify his profit-maximizing supply function (his own best response) $Y^k(p)$.

Note that each dealer's best response $Y^k(p)$ depends also on his estimate of the security's fundamental value $E(v|\Omega^k)$, conditional on his information Ω^k. One would think that this information does not contain the aggregate market order q, since dealers do not directly observe q in the call auction and hence they cannot use it to estimate the fundamental value. However, they can infer q from the clearing price, since the clearing price depends on it, by condition (4.17). As dealers submit price-contingent orders, they can take into account the information contained in the clearing price when they bid. Thus, for each possible price$p$, their estimate of the value of the security is $E(v|p^*(q) = p)$, and they optimally choose the quantity $Y^k(p)$ to buy or sell at this price given this estimate and other dealers' offers.[9] Then, noticing that by symmetry all dealers must choose the same supply function and imposing the equilibrium condition (4.17), one finds the value of the supply parameter φ and the relationship $p^*(q)$ consistent with the beliefs assumed. In other words, as in section 4.2.2, here we also seek a rational expectations equilibrium.

In the context of this model, a rational expectations equilibrium is a set of schedules, $\left\{Y^k(p)\right\}_{k=1}^{k=K}$, for the dealers and a price mapping $p^*(q)$ such that: (i) the schedule of offers posted by each dealer maximizes his expected profit, given his belief about the security's value and the price schedules chosen by his competitors; (ii) dealers correctly anticipate that the clearing price is given by $p^*(q)$ and form their beliefs accordingly; and (iii) for each value of q, the market clears for $p^*(q)$. In this equilibrium, dealers' expectations are rational **(p.146)** in the sense that their beliefs are based on the correct relationship between the clearing price and order flow q (the only informative variable about v).

In Appendix A, we show that, if $K \geq 3$, there is a rational expectations equilibrium in which

(4.18)

$$Y^k(p) = \frac{1}{\alpha}\frac{K-2}{K(K-1)}(p-\mu)\forall k \in \{1,\ldots,K\},$$

and the equilibrium price function is
(4.19)

$$p^*(q) = \mu + \lambda q,$$

with $\lambda = \alpha\frac{K-1}{K-2}$ and α defined as in equation (4.4). The equilibrium is linear since both dealers' bidding strategies and the clearing price are linear functions.[10] Note that for equilibrium to exist, there must be at least three market makers. As expected, the market-clearing price reveals information on the payoff v, since it is a linear function of the aggregate order size. In equilibrium, the dealers' estimate of the value of the security when they trade at price p is $E(v|p^*(q) = p) = \mathrm{E}(v|q) = \mu + \alpha q$ (the first equality here reflects the fact that there is a one-to-one mapping between the price p and the aggregate market order size q).

When K goes to infinity, we have:

$$\lambda = \alpha.$$

If the number of dealers is very large, the depth of the market is entirely determined by the informativeness of the order flow, exactly as in section 4.2.2. As dealers trade at the expected value conditional on their information, they obtain zero expected profits. We call this polar case as the competitive case.

In contrast, when the number of dealers is finite, λ is strictly larger than α. Thus, dealers obtain strictly positive expected profits. In fact, we can write equation (4.19) as:

$$p = E(v|q) + \underbrace{\frac{1}{K-2}\alpha q}_{\text{markup}} = \mu + \alpha q + \frac{1}{K-2}\alpha q.$$

Under imperfect competition the dealers' price is higher than their valuation of the security, as illustrated in figure 4.3 below: when the total order size is q, dealers earn an expected profit per share $p - E(v|q) = \frac{1}{K-2}\alpha q \rangle 0$.

In other words, the trading rules in the call auction enable dealers to compete less aggressively than when they must post a single offer for the entire **(p.147)**

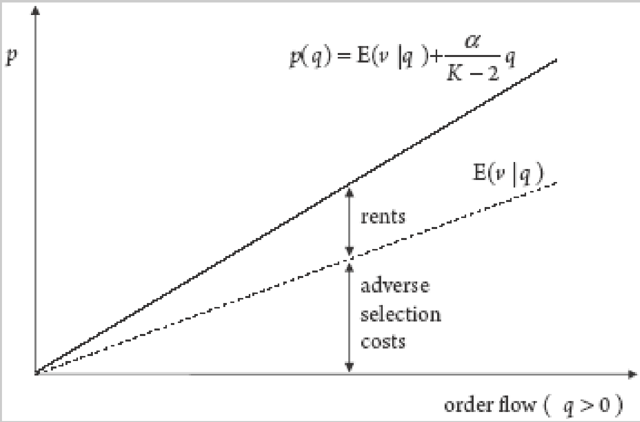


Figure 4.3. Market depth with imperfect competition

order, as was assumed in the previous section. The reason is as follows. The risk of trading against an informed investor leads dealers to limit the number of shares that they offer to buy or sell at any given price. As a result, each dealer has pricing power. To see this, suppose that in the aggregate investors want to be net buyers ($q > 0$). For a given market order size, a dealer can raise the clearing price by reducing his supply, very much as a monopolist would do, since the other dealers supply only a limited number of shares at each price. But this possibility is limited by the extent to which his competitors react by increasing their supply. Intuitively, if a small increase in price generates a large increase in other dealers' aggregate supply,[11] then a dealer must shave his own supply by a large amount in order to have a substantial price effect. In this case, the dealer benefits less from the increase in price by reducing his own sale volume. This happens when K is large and the equilibrium price is therefore close to the dealers' valuation $E(v|q)$. In contrast, when K is small, each dealer has significant market power because the aggregate supply of the others at each price is small. As a result, the wedge between the equilibrium price and dealers' valuation becomes large in markets with few dealers.[12]

For this reason, the depth of the market $1/\lambda$ depends not only on the informational content of the order flow α but also on the number of competing market makers K, as figure 4.3. shows. More dealers tend to mean lower rents and a deeper market. This analysis shows that liquidity providers' market power is a source of market illiquidity.

(p.148) In this section, we have taken the informed investor's trade size as given. It is straightforward to endogenize this trade size as we did in section 4.2.3 (see exercise 8). The behavior of the informed trader is unchanged, except that his trade size now increases with the number of dealers.

4.3. Market Depth with Inventory Risk

As we saw in Chapter 3, the need to compensate market makers for holding risky positions is another determinant, in addition to adverse selection, of the bid-ask spread. Here we extend the analysis of price formation with inventory risk to a situation where investors interact via a call auction mechanism where they can place orders of any size. For simplicity, we suppose that there is no informed trading ($x = 0$ so that $q = u$). First, in section 4.3.1, we posit that risk averse dealers are price-takers (that is, they behave competitively) in the call auction market. In this case, we obtain the same relationship between the equilibrium price and dealers' aggregate inventory as in Chapter 3 (section 3.5.1). Then, in section 4.3.2, we consider risk averse dealers who act strategically in the auction; that is, they realize that the clearing price depends on the way in which they themselves bid. As in the model with asymmetric information, we find that market depth is lower in this case than when dealers behave competitively.

4.3.1 Perfectly Competitive Dealers

Consider again the inventory model of Chapter 3 (section 3.5). The main differences between that model and the model analyzed in this section are that now orders may be of any size and trading is assumed to occur via a call auction, as in section 4.2.4. In the auction, each dealer submits a price schedule $Y^k(p)$ that specifies the number of shares he is willing to buy ($Y^k(p) < 0$) or sell ($Y^k(p) > 0$) if the clearing price is p. For simplicity, we further assume that there is a single trading round (at time $t = 0$) and the asset payoff, v, is realized at its termination (time $t = 1$). In this section, we assume that dealers behave competitively. That is, they neglect the impact of their orders on the equilibrium price (they are "price-takers").

There are K dealers, each with mean-variance preferences: dealer k has risk-aversion coefficient $\rho^k > 0$:

$$U^k = E\left(w_1^k\right) - \frac{\rho^k}{2}\,\mathrm{var}\left(w_1^k\right),$$

where w_1^k is the marked-to-market value of the dealer's portfolio at date 1, that is,

$$w_1^k = v \cdot z_1^k + c_1^k = v\left(z_0^k - y^k\right) + c_0^k + py^k,$$

(p.149) where z_t^k is dealer k's inventory in the risky security and c_t^k is his cash position at time t. If at time $t = 0$ the dealer sells y^k shares, this decreases his risky inventory by y^k, and increases his cash position py^k. Hence, $z_1^k = z_0^k - y^k$ and $c_1^k = c_0^k + py^k$, as shown in the previous equation. As in Chapter 3, the first-order condition for the maximization of dealer k's expected utility is $dU^k/dy^k = 0$, which in the current setting implies:[13]

(4.20)

$$p - \mu + \rho^k \sigma_v^2\left(z_0^k - y^k\right) = 0.$$

Thus, the optimal price schedule for dealer k is

(4.21)

$$Y^k(p) = \frac{p-\mu}{\rho^k \sigma_v^2} + z_0^k.$$

Each market maker k's price schedule is related inversely to his risk aversion ρ^k and directly to his initial inventory z_0^k.

As in section 4.2.4, market-clearing requires that aggregate net sales by dealers equal the net market orders placed by customers:

(4.22)

$$\sum_{k=1}^{K} Y^k(p) = q.$$

In this setting, market orders will typically be filled by several dealers, each taking a portion, with some or all customer orders being directly crossed.

Inserting the supply functions (4.21) into the market clearing condition (4.22) yields:

$$\sum_{k=1}^{K} \left(\frac{p-\mu}{\rho^k \sigma_v^2} + z_0^k \right) = q,$$

so that
(4.23)

$$p = \mu - \rho\sigma_v^2 Z + \rho\sigma_v^2 q,$$

where Z denotes the aggregate initial inventory of the market makers $\sum_{k=1}^{K} z_0^k$ and the constant ρ denotes the "collective" risk aversion of all the market makers, defined as
(4.24)

$$\rho \equiv \left(\frac{1}{\rho^1} + \cdots + \frac{1}{\rho^K} \right)^{-1}.$$

(p.150) For instance, if all market makers have the same absolute risk aversion $\bar{\rho} = \rho^1 = \ldots = \rho^K$, then $\rho = \bar{\rho}/K$. The "collective" risk aversion ρ of the market makers declines as their number K increases, because risk is more widely spread when more dealers are drawn into market-making. So the equilibrium price is:

(4.25)

$$p = \underbrace{\mu_t - \rho\sigma_v^2 Z}_{\text{midquote } m} + \underbrace{\rho\sigma_v^2}_{\lambda} q.$$

As in Chapter 3, the midprice m is inversely related to dealers' aggregate inventory at time $t = 0$. Intuitively, a dealer's marginal valuation for the security is low if his exposure to the risk of the security is already large. Moreover, the previous equation shows that the depth of the market $(1/\lambda)$ is inversely related to dealers' aggregate risk aversion, ρ, and the riskiness of the asset payoff, σ_v^2. So even though dealers are competitive, more dealers means a deeper market when dealers are risk averse, because the higher the number of dealers, the smaller the position that each individual dealer must take to absorb a given market order. Thus, each dealer requires less compensation for bearing inventory risk, and the market is more liquid.

As in section 4.2.4, we can compute the effective bid-ask spread for an order of size $|q|$. Using equation (4.25), we obtain:

(4.26)

$$s = 2\rho\sigma_v^2 \, |q| \, ,$$

which is proportional to the size of the order imbalance as in the Kyle model (figure 4.4).

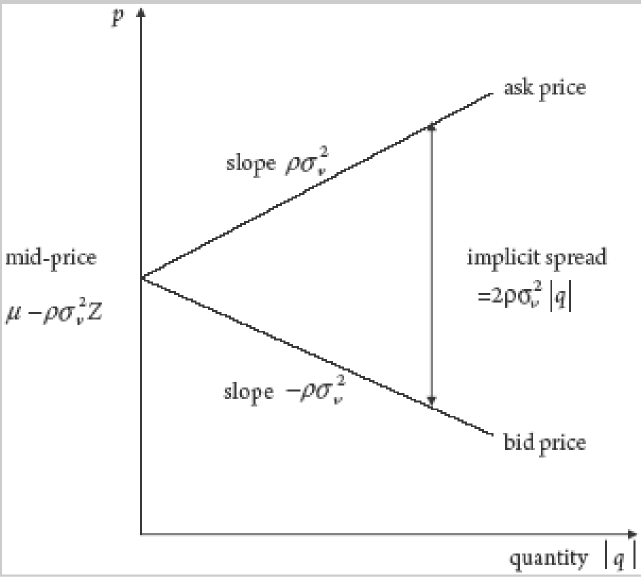


Figure 4.4. Competitive call market price with risk averse market makers

(p.151) Note that the actual price paid in the call market by any particular investor i depends not just on his own order q^i but on the total order flow from other traders, $q - q^i$. From the standpoint of trader i, the aggregate order flow q may be random, and the variance of the price that he will pay is $\rho^2\sigma_v^4$, where $\text{var}(q|q^i)$ is his estimate of the variance of the total order flow from other investors. So in the call market, as the execution price is not certain ex ante, investors face an execution risk whose size increases with trading costs: there is a positive relationship between illiquidity and execution price volatility.

4.3.2 Imperfectly Competitive Dealers

The previous section posits that dealers behave competitively: they ignore the effect of their own orders on the clearing price. When the number of dealers is small, however, this assumption is unrealistic. Suppose for instance that a buy market order arrives. By reducing the number of shares that he offers for sale in the auction, a dealer drives the clearing price up and obtains a better execution price. Of course, the cost of this strategy is that he receives fewer shares at the equilibrium price. Hence, each dealer faces a trade-off between price and quantity, as in models of imperfect competition between firms (and very much as in the model of imperfect competition considered in section 4.2.4). To understand this trade-off and its effect on market depth more clearly, we now relax the assumption that dealers are price takers. For simplicity, we assume that all the K market makers have the same coefficient of risk aversion $\bar{\rho}$, and thus, combined risk aversion $\rho = \bar{\rho}\upsilon/K$.[14]

To analyze the call market equilibrium in this case, we need to account for the interdependence of dealers' bidding strategies. For instance, if a dealer offers a large number of shares at a given price, other dealers will be more inclined to reduce their own orders to obtain a better execution price. We again adopt the concept of Nash equilibrium, which in this case is a set of price schedules $\left\{Y^k\left(p\right)\right\}_{k=1}^{k=K}$ for the dealers such that the schedule of each one maximizes his expected profit given the price schedule chosen by his competitors.

In Appendix B, we show that, if $K \geq 3$, the following price schedules form a Nash equilibrium:

(4.27)

$$Y^k\left(p\right) = \frac{K-2}{K-1}\left(\frac{p-\mu}{\bar{\rho}\sigma_v^2} + z_0^k\right).$$

(p.152) As a consequence, the equilibrium price is a linear function of the market order size q:

(4.28)

$$p = \mu - \rho\sigma_v^2 Z + \left(\frac{K-1}{K-2}\rho\sigma_v^2\right)q.$$

The technique used to derive this equilibrium is very similar to that employed in Appendix A. Indeed, the nature of the competition between dealers in the models with asymmetric information and inventory risk is identical. The only difference is that in the former case dealers need to infer information from the clearing price, an effect that is absent here.

Thus, in equilibrium, at each price dealers offer to sell or buy only a fraction $\frac{K-2}{K-1}$ of the number of shares that they would be willing to trade when they behave competitively (compare equations (4.21) and (4.27)). Intuitively, by restricting the size of his order at each price, the dealer shifts the execution price to his advantage, as explained in the introduction of this section.

As a consequence, the aggregate supply schedule is less elastic, so that the price impact of a market order of a given size is larger, that is, λ is greater than in the benchmark case. Imperfect competition among dealers reduces the depth of the market, regardless of whether the illiquidity arises from adverse selection or from inventory risk. Now, if the number of dealers increases the market becomes deeper (λ decreases) for *two* reasons: (i) the risk-bearing capacity of the market increases, as in the case of perfectly competitive dealers; and (ii) competition intensifies with the number of dealers, whose scope for affecting the equilibrium price by shading their offers is reduced-an effect captured by the fact that the ratio $(K-1)/(K-2)$ is declining in K. This implies that measures of market illiquidity should be inversely related to the number of dealers in a stock, since a larger number of dealers fosters competition and increases overall risk-bearing capacity.

When the number of dealers becomes infinite, the equilibrium converges to that of the benchmark case. Thus, in presence of either asymmetric information or inventory risk, dealers' rents vanish when the number of dealers is very large (strictly speaking, infinite). This observation has important implications. First, it suggests that in many real-world situations liquidity suppliers' rents should be an important determinant of market illiquidity.[15] As section 3.4.2 of Chapter 3 explains, this determinant is often bundled with order-processing costs in empirical analyses. Yet for policy intervention, it is important to distinguish the two causes of illiquidity. Second, trading rules should foster competition among liquidity providers. For instance, rules that restrict the number of market makers in a security are likely to result in excessive rents.

(p.153) Finally, in the model with imperfect competition, the expression for the midprice is the same as in the benchmark case. This means that imperfect competition does not affect the conclusions set out in section 3.5.3 of Chapter 3 regarding the dynamics of quotes and inventories. In fact, those conclusions derive from the relationship between the mid-price and dealers' aggregate inventory, which, as noted, is unaltered by the level of dealers' market power.

4.4. Further Reading

Our analysis in section 4.2 is based on Kyle (1985). The model of imperfect competition among dealers developed in section 4.2.4 is based on Kyle (1989), but that work considers a more general case in which bidders are both risk averse and informed. Pagano (1989a) also considers the case of imperfect competition among risk averse dealers but in a model without asymmetric information, as in section 4.3.2 of this chapter. Madhavan (1992) compares the properties of dealer markets and call markets under assumptions similar to those used in sections 4.2.2 and 4.2.4 above, but in a dynamic setting. At a more general level, the analysis of imperfect competition among dealers is related to papers on competition in price schedules (e.g., Klemperer and Meyer, 1989) and auctions of shares (Wilson, 1979).

Many works have used the framework proposed by Kyle (1985) to study a variety of issues. For instance, Chowdhry and Nanda (1991) analyze multi-market trading (see Chapter 8), and Subrahmanyam (1991a) studies the effects of basket trading. Subrahmanyam (1991b) also analyzes the case of risk averse informed investors and market makers in this framework.

The analyses of sections 4.3 and 4.2.4 emphasize the importance of modeling imperfect competition among liquidity providers, in that dealers' rents are one cause of market illiquidity. Ho and Stoll (1983) study a model of price competition among dealers in the presence of inventory risk, observing that the best ask price should be set by the dealer with the largest position and the best bid price by the dealer with the smallest position, since there is an inverse relationship between a dealer's bid-ask quotes and his inventory (see Chapter 3). They solve for equilibrium quotes assuming that dealers know each other's inventory (see exercise 9). Biais (1993) relaxes this assumption and, applying the revenue equivalence theorem of auction theory, finds that the average bid-ask spread should be the same whether dealers can observe the inventories of their competitors or not. Yin (2005) considers the effect of search costs in the model of Biais (1993). Biais, Foucault, and Salanié (1997) compare the outcome of competition between dealers with inventory risk in three different trading mechanisms: a floor market, a limit order market, **(p.154)** and a dealer market. They find that the limit order market can produce the competitive equilibrium even when the number of dealers is finite, provided that market orders are allocated among dealers tied at the same price on a pro-rata basis (see also Viswanathan and Wang, 2002). Biais, Martimort, and Rochet (2000) present a model of imperfect competition between liquidity suppliers in presence of asymmetric information in a discriminatory auction (a market structure that we analyze in Chapter 6).

4.5. Appendix A

We show that if $K \geq 3$, under the assumptions of the model in section 4.2.4, there is a rational expectations equilibrium in which

(4.29)

$$Y^k(p) = \phi(p-\mu), \quad \forall k \in \{1,\ldots,K\}$$

and the equilibrium price mapping is

(4.30)

$$p^*(q) = \mu + \lambda q,$$

with $\phi = \frac{1}{\alpha}\frac{K-2}{K(K-1)}$, $\lambda = \alpha\frac{K-1}{K-2}$, and $\alpha = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_u^2}$.

We first consider the residual demand that a dealer faces for any given total market demand q, when his competitors use the equilibrium supply strategies of equation (4.29). Let y^k be the number of shares offered by dealer k. Market clearing means that:

(4.31)

$$q = y^k + (K-1)\phi(p-\mu).$$

That is,

(4.32)

$$P(y_k) = \mu + \frac{q - y^k}{(K-1)\,\phi}.$$

is the "residual demand function" faced by dealer k.

We next make an educated guess about what the dealer knows. Even though he does not observe q directly, we can see that he can infer it from the market price because for any given value of his supply y^k, there is a one-to-one relationship between q and the market price given by (4.32). Accordingly, we imagine that he knows q and assume that for each given value of q, with its corresponding residual demand function and the corresponding valuation $E(v|q) = \mu + \alpha q$, he picks a price-quantity pair to maximize his expected profit:

$$\max_{y^k} \; y^k \left[p - \; (\mu + \alpha q)\right],$$

$$\text{where } p = P\left(y^k\right) = \mu + \frac{q - y^k}{(K-1)\,\phi}.$$

(p.155) Taking the first-order condition:

$$\frac{q - 2y^k}{(K-1)\,\phi} - \alpha q = 0,$$

where the dealer is assumed to act strategically, and thus to be aware that the clearing price decreases by $\frac{\partial p}{\partial y^k} = -\frac{1}{(K-1)\phi}$ if he increases his supply y^k by one unit. Thus dealer k's price-quantity pair for any given q is:

$$\begin{aligned} y^k \quad &= \frac{1}{2} q \left(1 - \alpha\,(K-1)\,\phi\right), \\ p \quad &= \mu + \frac{1}{2}\left[\frac{1}{(K-1)\,\phi} + \alpha\right] q. \end{aligned}$$

Eliminating q from these two equations we find that, as the underlying q varies, dealer k's supply function is traced out:

(4.33)

$$Y^k\,(p) = \frac{(K-1)\,\phi\,[1 - \alpha\,(K-1)\,\phi]}{1 + \alpha\,(K-1)\,\phi}\,(p - \mu).$$

If the equilibrium is symmetric and all market makers use supply function (4.29), the slope of the supply functions (4.29) and (4.33) must be the same:

$$\phi = \frac{(K-1)\,\phi\,[1 - \alpha\,(K-1)\,\phi]}{1 + \alpha\,(K-1)\,\phi},$$

which yields

$$\phi = \frac{1}{\alpha} \frac{K-2}{K(K-1)}.$$

Then market clearing implies

$$\sum_k \frac{1}{\alpha K} \frac{K-2}{K-1} (p^* - \mu) = q,$$

so that in equilibrium the market price function is

$$p^* = \mu + \alpha \frac{K-1}{K-2} q.$$

Hence, as claimed,

$$\lambda = \alpha \frac{K-1}{K-2},$$

where $\alpha = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$ as defined in equation (4.4).

(p.156) 4.6. Appendix B

We show that if $K \rangle 3$, under the assumptions of the model in section 4.3.2, there is a Nash equilibrium in which

(4.34)

$$Y^k(p) = \phi(p - \mu) + \varphi z_0^k, \quad \forall k \in \{1, \ldots, K\},$$

with $\phi = \frac{K-2}{K-1} \frac{1}{\bar{\rho}\sigma_v^2}$, $\varphi = \frac{K-2}{K-1}$.

The reasoning is very similar to that in Appendix A. In particular, we can write the residual demand function facing market maker k: to clear the market, for any given value of total demand q, his supply y^k must equal total demand less the supply from his $K-1$ peers, that is,

(4.35)

$$y^k = q - (K-1)\phi(p - \mu) - \varphi \sum_{j \neq k} z_0^j,$$

(4.36)

$$p = \mu + \frac{1}{\phi(K-1)} \left[q - y^k - \varphi \sum_{j \neq k} z_0^j \right],$$

so that the sensitivity of the residual demand function faced by a dealer to his supply is again: $\frac{\partial p}{\partial y^k} = -\frac{1}{(K-1)\phi}$. Dealer k chooses his supply y^k to maximize his mean-variance utility:
(4.37)

$$\max_{y^k} \mu z^k + y^k (p-\mu) - \frac{\bar{\rho}\sigma_v^2}{2}\left(z_0^k - y^k\right)^2,$$

where $p$is determined by (4.36). The first-order condition for this maximization problem is

$$\frac{1}{\phi (K-1)}\left[q - 2y^k - \varphi \sum_{j\neq k} z_0^j\right] + \bar{\rho}\sigma_v^2 \left(z_0^k - y^k\right) = 0.$$

Using the market clearing condition (4.35) to substitute for q, we obtain

$$\frac{\left[y^k + (K-1)\phi(p-\mu) + \varphi \sum_{j\neq k} z_0^j - 2y^k - \varphi \sum_{j\neq k} z_0^j\right] + \bar{\rho}\sigma_v^2 \left(z_0^k - y^k\right)}{\phi (K-1)} = 0,$$

which yields dealer k's supply schedule:
(4.38)

$$Y^k(p) = \frac{1}{\frac{1}{\phi(K-1)} + \bar{\rho}\sigma_v^2}(p-\mu) + \frac{\bar{\rho}\sigma_v^2}{\frac{1}{\phi(K-1)} + \bar{\rho}\sigma_v^2} z_0^k.$$

By equating the coefficients of equation (4.38) with the initially conjectured supply schedule (4.34), it follows that there is a symmetric equilibrium of **(p.157)** the form:

(4.39)

$$Y^k(p) = \frac{K-2}{K-1}\left[\frac{1}{\bar{\rho}\sigma_v^2}(p-\mu) + z_0^k\right].$$

To see how the market price varies with demand q, notice that market clearing requires:

$$\sum_{k=1}^{K} Y^k(p) = \sum_{k=1}^{K} \frac{K-2}{K-1}\left[\frac{1}{\bar{\rho}\sigma_v^2}(p-\mu) + z_0^k\right] = q,$$

which yields

$$p = \mu - \rho\sigma_v^2 Z + \frac{K-1}{K-2}\rho\sigma_v^2 q,$$

as claimed in equation (4.28) in the text.

4.7. Exercises

1. Kyle's model with an imperfectly informed investor.

Suppose that in the model presented in section 4.2 (Kyle, 1985), the informed investor observes a noisy signal $s = v + \eta$ about the final value v of the security, where the noise component $\eta \sim N\left(0, \sigma_\eta^2\right)$ has no correlation either with the security's value v or with the noise traders' order u (cov(η, v) = cov(η, v) = 0).

a. Assume that competitive market makers post the following price schedule:
(4.40)

$$p(q) = \mu + \lambda q,$$

where q is the net order flow. Find the optimal value of λ that they will choose if they conjecture that the informed trader's strategy is the following function of his noisy signal s:
(4.41)

$$X(s) = \beta(s - \mu).$$

How does the market depth $1/\lambda$ chosen by market makers respond to changes in the variance σ_η^2 of the informed investor's error? What is the intuitive explanation for this result? If we plot the depth $1/\lambda$ as a function of the aggressiveness β of informed investors (the former on the vertical axis and the latter on the horizontal axis), how do changes in the variance σ_η^2 affect the position and shape of this curve?

b. In this case, the informed trader too must solve an inference problem in forming his expectation of the security's value. Show that his **(p.158)** expectation of v conditional on the signal s is

$$E(v|s) = \mu + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\eta^2}(s - \mu),$$

and find the value of β as a function of market depth $1/\lambda$. How does the trading aggressiveness β that informed investors choose respond to changes in the variance σ_η^2 of the error term η? What is the intuitive explanation for this result? If we plot β as informed investors' best response to the depth $1/\lambda$ chosen by market makers in the same graph described under (a), how do changes in the variance σ_η^2 affect the position and shape of this line?

c. Compute the equilibrium values of λ and β. How do they respond to changes in the variance σ_η^2 of the error made by the informed investor? Graphically, does the equilibrium still correspond to the point of minimum depth as in the baseline version of Kyle's model?

d. Compute the ex-ante expected profit of the informed investor. What is the effect of an increase in σ_η^2 on this profit? What is the intuitive explanation for this result?

2. Noise trading in Kyle's model.

Consider the model presented in section 4.2 (Kyle, 1985), changing only the assumption about noise trading: assume that there are two groups (1 and 2) of noise traders, whose orders are respectively $u_1 \sim N\left(0, \sigma_{u1}^2\right)$ and $u_2 \sim N\left(0, \sigma_{u2}^2\right)$, both uncorrelated with the asset's future value (i.e. cov(v, u_1) = cov(v, u_2) = 0). All the other assumptions of the model are unchanged: (i) market makers are risk neutral and perfectly competitive, (ii) the asset value is $v \sim N\left(\mu, \sigma_v^2\right)$, (iii) the informed investor's order is $x = \beta(v - \mu)$, and (iv) market makers only observe the total net order $q = x + u_1 + u_2$. [Hint: think whether your answers require computing the results of the Kyle model again.]

a. Derive (i) the price schedule $p = \mu + \lambda q$ chosen by competitive market makers, under the assumption that the informed trader's order is $x = \beta(v - \mu)$; (ii) the optimal informed trader's order, given the price schedule chosen by market makers; and (iii) the equilibrium values of λ and β.
b. Now suppose that market makers can observe the actual realization of the order placed by group 1 (for instance, because these are local investors, while those of group 2 are foreign investors) before trading occurs. Under this further assumption, derive again (i) the price schedule $p = \mu + \lambda q$ chosen by competitive market makers, under the assumption that the informed trader's order is $\lambda = \beta(v - \mu)$; (ii) the **(p.159)** optimal informed trader's order, given the price schedule chosen by market makers; and (iii) the equilibrium values of λ and β.
c. Compare the results obtained under (a) and under (b): does the assumption made under (b) change the equilibrium market depth and informed traders' aggressiveness and, if so, what is the intuitive reason for this difference?

3. Kyle's model with multiple informed traders.

Consider the model presented in section 4.2 (Kyle, 1985), but assume that there are $N \rangle 1$ informed traders, who all perfectly observe the final value of the security v but not the equilibrium price at the time that they determine their quantity demanded q_i.

a. Suppose that market makers post the price schedule described by equation (4.40), where q is the net order flow $\sum_{i=1}^{N} x_i + u$ and $\mu = E(v)$. Assuming that each informed trader uses the following order submission strategy:

$$x_i = X_i(v) = \beta(v - \mu) \quad \text{for } i \in \{1, \ldots, N\},$$

find the value of β for which a Nash equilibrium exists, determine how β is affected by N, and explain intuitively why.
b. Suppose now that investors follow the order submission strategy derived in step a. Show that in this case the market makers' pricing strategy is given by equation (4.40), and find the value of λ that they optimally choose.
c. What is the market depth in equilibrium, and how is it affected by an increase in the number of informed traders, N? What is the economic intuition for this result? Do you think that this result is robust; that is, does it still hold if the assumptions of the model are relaxed? (For instance, discuss informally whether you would

still expect this result to hold if informed traders were risk averse.)

d. Compute the ex-ante expected profit of each informed investor. What is the effect of an increase in N on the aggregate profit of informed investors?

4. Variance of price change and average pricing error in Kyle's model.

Consider the static model by Kyle (1985), where (i) market makers are risk neutral and perfectly competitive, (ii) the asset value is $v \sim N\left(\mu, \sigma_v^2\right)$, (iii) the informed investor's order is $x = \beta(v - \mu)$, and the noise traders' order is $u \sim N\left(0, \sigma_u^2\right)$; and (iv) market makers only observe the total net order $q = x + u$.

(p.160)

a. Based on the price schedule $p = \mu + \lambda q$ chosen by competitive market makers in this model, show that the variance of price changes is:

$$\operatorname{var}(v - p) = \frac{\sigma_u^2}{\beta^2 \sigma_v^2 + \sigma_u^2} \sigma_v^2 ,$$

and explain intuitively why it is decreasing in the informed traders' aggressiveness β.

b. Show that the average pricing error is:

$$\operatorname{var}(p - \mu) = \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \sigma_u^2} \sigma_v^2 .$$

Is this expression increasing or decreasing in informed traders' aggressiveness β? Explain intuitively why.

5. Informed investor with price-contingent orders.

Suppose that in the Kyle (1985)s' model the informed investor is allowed to set a demand schedule conditioned on price rather than a market order.

a. Show that the equilibrium is essentially the same as when the informed investor must place a market order, in the sense that average noise trader transaction costs and informed investor profits are the same. The only difference is that liquidity is provided in part by the informed investor as well as by the competitive market makers. [Hint: imagine that the informed investor submits a demand x after observing the uninformed trader demand u.]

b. What happens under this new assumption if there is more than one informed investor?

6. Risk-averse informed investor.

Suppose that the informed investor is risk averse (with constant coefficient of absolute risk aversion b) and that he liquidates any amount of the security that he buys at a liquidation value $v + \varepsilon$, where ε is a normally distributed random variable with mean zero and variance σ_ε^2. At the time of trading the informed investor knows the realization of v but not that of ε. This noise in his signal implies that in taking long or short positions based on his privileged information, he bears some risk. Show that in this case his trading intensity is reduced relative to the model with risk neutrality, so that the β linear function swivels to the left (in figure 4.2 of the text), increasing the equilibrium liquidity.

7. Extension of the Glosten-Milgrom model to multiple trade sizes

Consider the following simple extension of the high-low-value adverse selection model of bid-ask spreads to two trade sizes, as in Easley and O'Hara (1987). A security's true value is v, which may be high (v^H) or low (v^L) with probability $\frac{1}{2}$ **(p.161)** each. Market makers do not know the true value. In a trading period one trader comes to the market. Market makers cannot tell who the trader is:

- • With probability $1 - \pi$, he is a "noise trader": either a "retail" customer, who wants to trade one unit, or a "wholesale" customer, who always trades two units. Half the noise traders are retail customers, and half are wholesale customers. Both types of noise trader want to buy or sell the security with probability $\frac{1}{2}$ each.
- • With probability π, the trader is an "insider" who knows the security's true value v. He will buy one or two units if the true value is v^H and sell one or two units if the true value is v^L.

Ask and bid prices are set by competitive, risk-neutral market makers.

a. Suppose that the insider always chooses to trade two units. What will be the ask and bid prices quoted by market makers for 1 unit, that is, $a(1)$ and $b(1)$? In this case, will there be a non-zero bid-ask spread for this trade size? What bid and ask prices correspond to the 2-unit trade size, that is $a(2)$ and $b(2)$, under the same assumption?
b. Would the insider always want to trade two units rather than one? Consider this question under two assumptions regarding π: $\pi = \frac{1}{2}$ and $\pi = \frac{1}{4}$. In both cases, compute the insider's profit if he trades two units and compare it with his profit when he trades one unit.
c. Do you have any ideas on what would happen in situations where your answer to part b is "No"? In particular, could it be an equilibrium for the insider to always trade one unit? [Hard question: characterize the equilibrium in which the insider always chooses his trade size to maximize his profit!]

8. Informed traders' optimal trade size with imperfectly competitive dealers.

Consider the model of imperfect competition among dealers in section 4.2.4. Show that informed investors' optimal trade size in this model is: $x(v) = \beta(v - \mu)$, with $\beta = \sqrt{\frac{K-2}{K}} \cdot \frac{\sigma_u}{\sigma_v}$. Why does the informed investor's trade size increase with the number of dealers, K?

9. Competition between dealers with different initial inventories.

This exercise analyzes (along the lines of the model by Ho and Stoll, 1983), how quotes are determined in a competitive dealer market where dealers have different inventories. Consider the two-period model with inventory risk considered in section 3.5.1 of Chapter 3. Instead of a representative dealer, assume that there are two dealers, 1 and 2, with respective inventories z_{t1} and z_{t2}, where $z_{t1} \geq z_{t2}$. These dealers have mean-variance preferences with the same risk aversion, ρ. At time t, an investor contacts the dealers to execute a buy or a sell **(p.162)** order of size q_t. The investor requests a quote simultaneously from both dealers and directs his order to the dealer posting the best offer. If dealers make the same offer, the investor routes his order to either dealer with probability 1/2.

a. Suppose that the investor contacts the dealers to execute a buy order. Compute the ask price that makes dealer j indifferent between selling q_t shares or not. Call this ask price $a_t^r\left(z_{tj}\right)$. Explain why $a_t^r\left(z_{t1}\right) \leq a_t^r\left(z_{t2}\right)$.
b. Suppose that the investor contacts the dealers to execute a sell order. Compute the bid price that makes dealer j indifferent between buying q_t shares or not. Call this ask price $b_t^r\left(z_{tj}\right)$. Explain why $b_t^r\left(z_{t2}\right) \geq b_t^r\left(z_{t1}\right)$.
c. Show that in equilibrium, the investor will buy the security at a price slightly smaller than $a_t^r\left(z_{t2}\right)$ and sell the security at a price slightly higher than $b_t^r\left(z_{t1}\right)$. What is the equilibrium if $z_{t1} = z_{t2}$?
d. How would the results change if there is a third dealer with inventory z_{t3} such that $z_{t1} \rangle z_{t2} \rangle z_{t3}$?

Notes:

(1.) Order-processing costs do not affect market depth, as long as they are proportional to the number of shares traded. However, if they are more than proportional to trade size, they contribute to widen the bid-ask spread as trade size increases; conversely, if they are less than proportional, they tend to reduce the bid-ask spread as trades grow larger.

(2.) The concept of rational expectations equilibrium is an important tool to model situations of trading with asymmetric information; its application to competitive securities markets was developed by Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985), among others.

(3.) In Section 4.2.3, we show that there is a rational expectations equilibrium in which the optimal strategy of the informed investor is linear in $v - \mu$ (this conjecture is verified).

(4.) Consider two joint normally distributed random variables, $X \sim N\left(\mu_x, \sigma_x^2\right)$ and $Y \sim N\left(\mu_y, \sigma_y^2\right)$, and denote their covariance σ_{xy}. A property of the bivariate normal distribution is that the conditional density of Y given $X = x$ is itself normal with conditional mean:

$$E(Y|x) = \mu_y + \frac{\sigma_{xy}}{\sigma_x^2}\left(x - \mu_x\right) = \left(\mu_y - \frac{\sigma_{xy}}{\sigma_x^2}\mu_x\right) + \frac{\sigma_{xy}}{\sigma_x^2}x,$$

which is the predicted value of Y from an ordinary least squares (OLS) regression of the equation $Y = a + bX$, upon setting the explanatory variable $X = x$. The slope coefficient σ_{xy}/σ_x^2 is precisely the OLS estimate of b. In our case, the dependent variable is v and the explanatory variable is q.

(5.) If several post the same price, the order is allocated to one of them.

(6.) Here the average pricing error is equal to var$(v - p)$, since E$(v - p) = 0$.

(7.) This covariance is zero because the price is the best estimate of the fundamental v at the time of trading, so that$v - p$ is the unanticipated component of the fundamental (the portion not yet reflected in the price), and thus uncorrelated with the price p itself.

(8.) We obtain equation (4.16) as follows. We have:

$$v - p = v - \mu - \frac{\sigma_v}{2\sigma_u}\left(x + u\right) = \frac{v - \mu}{2} - \frac{\sigma_v}{2\sigma_u}u,$$

since in equilibrium, $x = (\sigma_u/\sigma_v)\,(v - \mu)$. Computing the variance and observing that v and u are independent, we obtain the result in the text.

(9.) The idea that the clearing price conveys information and that traders should take this information into account in formulating their price-contingent orders is a key insight of rational expectations models such as Grossman (1976).

(10.) There might be other equilibria in which dealers' bidding strategies or the mapping $p^*(q)$ are non-linear.

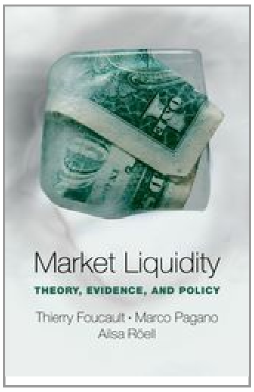
(11.) That is, if the price elasticity of other dealers' aggregate supply is high.

(12.) No linear equilibrium exists when $K = 2$ because in this case, by reducing his supply by a small amount, the dealer obtains an infinitely large increase in price. When $K = 1$, the dealer can charge an infinite price to sell q shares, since the investors' aggregate demand is price insensitive.

(13.) Equation (4.20) corresponds to equation (3.44) in Chapter 3. Beside some slight difference in notation (the time subscripts of variables and the use of σ_v^2 instead of σ_ε^2 to denote the variance of v), the two equations differ because here the initial inventory position and the risk-aversion coefficients can differ between dealers (as indicated by their subscript k), while in Chapter 3 they do not.

(14.) The case where market makers are not all equally risk averse ($\rho^1 \neq \rho^2 \neq \ldots \neq \rho^K$) is analyzed by Röell (1998), who shows that for any given aggregate risk tolerance $(1/\rho)$, more equal distribution of risk tolerance among them improves market liquidity.

(15.) Biais, Bisière, and Spatt (2010) show empirically that competition among liquidity suppliers is imperfect and provide estimates of their rents on one U.S. electronic market.



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Estimating the Determinants of Market Illiquidity

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

The theories set forth in previous chapters have provided a qualitative interpretation of cross-sectional and time variations in market liquidity. This chapter shows how they also serve to quantify the contribution of each source of market illiquidity to these variations, using econometric techniques. The chapter is organized as follows. Sections 5.2 and 5.3 explain how one can use time series of transaction prices and signed trade sizes to estimate the contributions of adverse selection costs, inventory costs, and order processing costs to market illiquidity and price movements. These techniques exploit the fact that market orders have different effects on the dynamics of securities prices depending on the relative prevalence of adverse selection, order processing, and inventory holding costs. Section 5.4 shows how one can use order flow data to build a measure of the frequency of informed trading in a security. Armed with this gauge, one

can test predictions about the effect of informed trading on liquidity and the sources of informed trading (e.g., whether some trading mechanisms are more prone to informed trading). The final sections provide suggestions for further reading and exercises.

Keywords: market liquidity, econometrics, time series, transaction prices, trade size, inventory costs, order processing costs, selection costs, order flow data

Learning Objectives:

- • What a price impact regression is and how it can be used to measure the various sources of market illiquidity
- • How to measure the permanent impact of trades
- • How to estimate the level of informed trading in a security

5.1. Introduction

Liquidity varies dramatically from security to security and over time. As Chapters 3 and 4 explain, these variations should reflect changes in the cost of providing liquidity caused by adverse selection, order processing, and risky inventory holding.

The financial crisis of 2008–09 provides a case in point: during the crisis, most stocks became significantly more illiquid. As an illustration, Figure 5.1 plots the average closing bid-ask spread for NYSE stocks in the financial and consumer staples sectors from April 2006 to April 2009. The spreads start rising in all sectors as the first signs of trouble appear in the summer of 2007, and keep increasing as the crisis deepens, especially after the AIG bailout and the Lehman Brothers bankruptcy in the early fall of 2008.

The crisis also generated a huge increase in the volatility of returns by amplifying uncertainty about public policies and companies' prospects. Moreover, it **(p.164)**

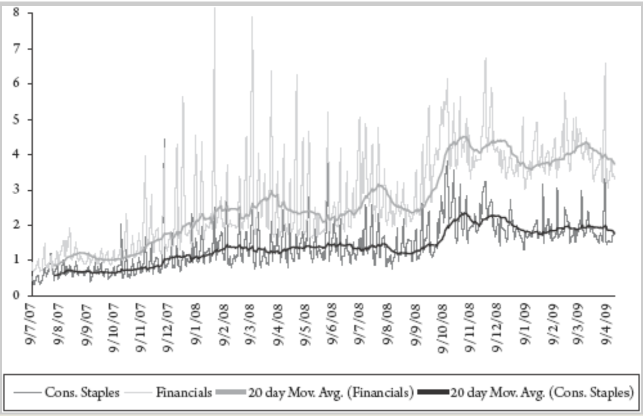


Figure 5.1. Average NYSE closing bid-ask spread, financials and consumer staples, July 2007–September 2009

made it harder for liquidity suppliers to get credit to fund their market-making activity. For these reasons, their inventory holding costs increased, which provides a first explanation for the increase in spreads in the sectors of financials and consumer staples.

In addition, the crisis increased asymmetric information for stocks in the financial sector. By their very position, participants from the financial industry were probably better informed about the soundness of the balance sheets of banks and security firms and the likelihood of government bailouts. Thus, the likelihood of informed trading in financials was now greater than before, which may explain why the crisis widened the bid-ask spread on financials more than on consumer staples stocks.

The role of asymmetric information in explaining variations in market liquidity is also illustrated by figure 5.2, which compares the bid-ask spread for firms that lose analyst coverage (the black line) and firms that do not (the grey line). Financial analysts collect, produce, and disseminate information about the fundamental value of securities. This should level the informational playing field between market participants and thus reduce the opportunities for company insiders. So when analysts' coverage of a company ceases, we expect the bid-ask spread to widen due to adverse selection, and this is precisely what figure 5.2 shows: when a stock loses its last analyst, its bid-ask spread increases dramatically compared to those of comparable stocks that do not.

The theories set forth in previous chapters, then, have given us a *qualitative* interpretation of cross-sectional and time variations in market liquidity. As we **(p.165)**

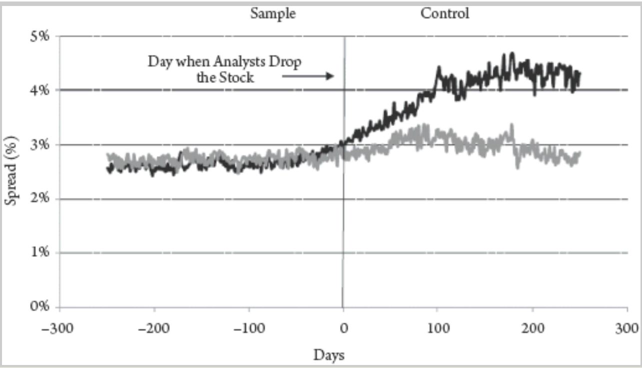


Figure 5.2. Average effective bid-ask spreads for stocks for which analysts coverage is terminated and a control group for which it is continued

(Source: Ellul and Panaydes, 2011)

shall see in this chapter, they also serve to *quantify* the contribution of each source of market illiquidity to these variations, using econometric techniques. In sections 5.2 and 5.3, we explain how one can use time series of transaction prices and signed trade sizes to estimate the contributions of adverse selection costs, inventory costs, and order processing costs to market illiquidity and price movements. The techniques described exploit the fact that market orders have different effects on the dynamics of securities prices depending on the relative prevalence of adverse selection, order processing, and inventory holding costs (see Chapter 3). Finally, in section 5.4, we show how one can use order flow data to build a measure of the frequency of informed trading in a security. Armed with this gauge, one can test predictions about the effect of informed trading on liquidity and the sources of informed trading (e.g., whether some trading mechanisms are more prone to informed trading).

5.2. PRICE IMPACT REGRESSIONS

In this section, we show how to estimate the relative importance of the various costs of providing liquidity by running “price impact regressions,” regressions of changes in prices on contemporaneous and lagged measures of order flow. The exact specification of these regressions depends on assumptions about the sources of market illiquidity and the properties of the order arrival process. In section 5.2.1, we consider the case in which dealers are risk neutral (so as to **(p.166)** ignore inventory holding costs) and market orders are serially uncorrelated. Then, section 5.2.2 explains how to account for inventory holding costs and serial correlation in specifying price impact regressions.

5.2.1 Without Inventory Costs

Consider again the model of trading developed in section 3.4 of Chapter 3: liquidity suppliers are risk neutral and they risk trading with better informed investors. They also bear an order-processing cost for each trade. In Chapter 3’s scenario, the price of the t^{th} transaction is $p_t = \mu_t + \gamma d_t$, where (i) μ_t is liquidity suppliers’ estimate of the asset payoff, (ii) γ is the order-processing cost per share (or a measure of dealers’ rent), and (iii) d_t indicates whether the market order triggering the t^{th} transaction is a buy ($d_t = 1$) or a sell ($d_t = -1$).

Accordingly, consecutive price changes are:

(5.1)

$$\Delta p_t = p_t - p_{t-1} = \mu_t - \mu_{t-1} + \gamma \Delta d_t .$$

As orders contain information, they affect liquidity suppliers’ estimate of the asset’s true value. Specifically:

(5.2)

$$\mu_t = \mu_{t-1} + \lambda d_t + \varepsilon_t ,$$

where λ measures the informativeness of the order flow. We assume that λ is constant, at least over our estimation period. The error term ε_t is white noise (E(ε_t) = 0; E($\varepsilon_t \varepsilon_s$) = 0, $\forall t$, $\forall t \neq s$), which captures the effects of public information other than trades (e.g., macro-economic and corporate announcements) on market participants’ value estimates.

In this model, the bid-ask spread at the time of the t^{th} transaction is equal to

(5.3)

$$a_t - b_t = 2\left(\gamma + \lambda\right).$$

Remember that λ and γ are respectively the adverse selection cost component and the order processing cost component of the bid-ask spread.

Combining equations (5.1) and (5.2) yields

(5.4)

$$\Delta p_t = \lambda d_t + \gamma \Delta d_t + \varepsilon_t.$$

Suppose a buy order arrives at time $t-1$ and then a sell order at time t. The buy order executes at the best ask price, the sell order executes at the best bid. This "bid-ask bounce" generates a price decrease equal to 2γ, which is captured by the second term ($\gamma\ \Delta d_t$) in equation (5.4). Moreover, dealers set the bid price for the t^{th} transaction at a discount from their value estimate after the $(t-1)^{th}$ transaction, reflecting the possibility that the order is from an informed investor. This discount also contributes to the change in price from the $(t-1)^{th}$ to the **(p.167)** t^{th} transaction, and is captured by the first term (λd_t) in equation (5.4). In this case, it amplifies the price decrease due to the bid-ask bounce by an amount equal to λ. Thus, the total change in price from the $(t-1)^{th}$ to the t^{th} transaction is $\Delta p_t = -\lambda - 2\gamma < 0$, as implied by equation (5.4).

Box 5.1 Revisiting Roll's Measure with Adverse Selection

As Chapter 2 demonstrates, we can use the covariance of consecutive changes in prices to estimate the bid-ask spread. For instance, Roll's estimator of the bid-ask spread is $S_{\text{Roll}} = 2\sqrt{-\text{cov}(\Delta p_t, \Delta p_{t-1})}$. As was first noted by Glosten (1987), when the bid-ask spread is in part due to adverse selection costs, this estimator is biased.

To see why, consider again equation (5.4), where only the direction and not the size of trades matters. As d_t and ε_t are independent of each other and over time, equation (5.4) implies that

$$\text{cov}(\Delta p_t, \Delta p_{t-1}) = -\gamma(\lambda+\gamma).$$

Thus, Roll's estimate of the bid-ask spread is

$$\widehat{S}_{\text{Roll}} = 2\sqrt{\gamma(\lambda+\gamma)},$$

whereas the true spread is $S = 2\ (\lambda + \gamma)$. The Roll estimator underestimates it by a factor of $\sqrt{(\lambda+\gamma)/\gamma}$. But if S is observed directly, then Roll's measure can be used to estimate the fraction of it that is due to order-processing costs, since $\widehat{S}_{\text{Roll}}/S = \frac{2\sqrt{\gamma(\lambda+\gamma)}}{2(\lambda+\gamma)} = \sqrt{\frac{\gamma}{\lambda+\gamma}}$. Thus, the order-processing cost component accounts for a fraction $\left(\widehat{S}_{\text{Roll}}/S\right)^2$ of the spread.

More generally, price reversals are a symptom of market illiquidity. For this reason, $-\text{cov}(\Delta p_t, \Delta p_{t-1})$ (a measure of the magnitude of price reversals) is sometimes used as a gauge of market illiquidity. However, if illiquidity is mainly due to asymmetric information, this measure of illiquidity will be small even if the spread is large (since in this case $\text{cov}(\Delta p_t, \Delta p_{t-1})$ goes to zero when γ goes to zero). The price impact of informed trades is permanent, unlike that of trades due to order-processing costs or inventory holding costs.

Equation (5.4) is the cornerstone of what empiricists call a "price impact regression." Indeed, with trade and quote data like those presented in table 3.1 **(p.168)** of Chapter 3, the parameters in this equation can be estimated by running a regression of the trade-to-trade changes in price on the contemporaneous order flow (d_t) and the first difference of this variable (Δd_t). The term "price impact" reflects the fact that the estimates of λ and γ provide a way to evaluate the short-term impact ($\lambda + \gamma$) and the long-term impact (λ) of market orders on transaction prices (see Chapter 3).

In general, researchers have used more complex specifications for price impact regressions than that suggested by equation (5.4). First, the order-processing cost may vary with the size of trades. For instance, if this cost has a fixed component, dealers may be willing to execute large orders at smaller markups or discounts. Alternatively, they may offer better quotes to clients making large trades, as those clients have more bargaining power.[1] These effects work to create a negative relationship between the effective spread and trade size. To account for these "quantity discounts," equation (5.1) can be rewritten as follows:

(5.5)

$$\Delta p_t = \mu_t - \mu_{t-1} + \gamma_0 \Delta d_t + \gamma_1 \Delta q_t,$$

where γ_1 should be negative in presence of quantity discounts.

Moreover, Chapter 4 shows that not only the direction of orders d_t but also their signed size $q_t = d_t\,|q_t|$ is informative, since the size of an informed trader's order will increase in the extent of the deviation between his estimate of the asset's value and the prior estimate made by liquidity suppliers (see section 4.2.3 in Chapter 4). In this case, it is natural to generalize equation (5.2) in this way:

(5.6)

$$\mu_t = \mu_{t-1} + \lambda_0 d_t + \lambda_1 q_t + \varepsilon_t.$$

The coefficient λ_0 should be zero if very small trades have no informational content. Combining equations (5.5) and (5.6), we obtain a more general version of equation (5.4):

(5.7)

$$\Delta p_t = \lambda_0 d_t + \lambda_1 q_t + \gamma_0 \Delta d_t + \gamma_1 \Delta q_t + \varepsilon_t.$$

In estimating such a price impact regression, several econometric problems arise.[2] First, data sets do not always tell whether the trades are triggered by **(p.169)** buy or by sell orders (that is, the trade direction d_t, hence the sign of q_t is not always observed). In this case, one must infer market order direction from recorded trades (by using the Lee and Ready algorithm illustrated in Chapter 3), which typically induces measurement error in d_t.[3] Second, real-world quotes must be positioned on a discrete grid, so the observed transaction price necessarily differs from the theoretical price by a rounding error. This is a source of error in the regression's dependent variable, which tends to induce negative autocorrelation in residuals. Last, the error term in this regression is likely to be heteroskedastic if, for instance, public information does not arrive at a constant rate during the day: this biases the estimated standard errors of the coefficients.

There are several ways of coping with these econometric problems. One way is to estimate equation (5.7) and order direction jointly, using a maximum likelihood approach, as described in Glosten and Harris (1988), even though this approach retains the assumption that the error term is homoskedastic. In more recent studies, data sets are richer, so that order direction is observed directly – as in de Jong, Nijman, and Röell (1996). These authors deal with het-eroskedasticity and autocorrelation of residuals using the Newey-West method to estimate the standard errors of the coefficients. Another method is that adopted by Huang, and Stoll (1997) and Madhavan, Richardson, and Roomans (1997), who estimate an equation similar to (5.7) using the generalized method of moment (GMM), which can accommodate conditional heteroskedasticity of unknown form and serial correlation of residuals.

Glosten and Harris (1988) estimate equation (5.7) using a sample of eight hundred transaction-by-transaction observations for NYSE stocks between December 1, 1981, and January 31, 1983.[4] They first test the parameter restrictions $\lambda_0 = \gamma_1 = 0$, namely, that small orders carry no information and that order-processing costs are unrelated to trade size. For computational reasons, they run this test on a pilot sample of 20 stocks, finding for most of them that the data do not reject these restrictions and accordingly estimating the following restricted specification of equation (5.7) on a larger sample of an other 250 stocks:

(5.8)

$$\Delta p_t = \lambda_1 q_t + \gamma_0 \Delta d_t + \varepsilon_t.$$

They find that at the stock level, λ_1 is significantly positive for 170 stocks and γ_0 is significantly positive for 210 stocks. On average, for the stocks in **(p.170)** their sample, they find that for a one–thousand share lot, $\lambda_1 = 0.0102$ and $\gamma_0 = 0.0465$ (see table 2, panel B on p. 136 of their study). To interpret these figures, consider a stock with these characteristics and suppose that a buy market order for one thousand shares arrives. According to the model, on average, this order executes at a markup (relative to the midquote) equal to: $\gamma_0 + \lambda_1 = 5.67$ cents per share. Thus, the total implicit trading cost on this order is $1{,}000 \times 0.0567 = \$56.70$. However, after such a trade, the price increases permanently by an average amount equal to 1.02 cents (λ_1). So the liquidity suppliers' expected profit on the transaction (the "realized spread") is only 4.65 cents per share. This example shows how one can use price impact regressions to obtain estimates of effective and realized spreads (see Chapter 2 for their definitions).

Using different data, de Jong, Nijman, and Röell (1996) find that on average the adverse selection component of the spread is weakly increasing in order size, as in Glosten and Harris (1988), but unlike the latter they find that the order-processing cost component is strongly decreasing in trade size (i.e., $\gamma_1 < 0$).

5.2.2 With Inventory Costs

If liquidity suppliers require compensation for inventory risk, as explained in Section 3.5 of Chapter 3, the price impact regressions (5.4) and (5.7) are mis-specified. They will accordingly produce biased estimates of order-processing and adverse selection costs. Indeed, as shown graphically in figure 3.9 of Chapter 3, part of the change in liquidity suppliers' quotes after a trade reflects the change in inventory. This will dissipate over time as their aggregate inventory reverts to its long-term average. But in the medium term, this impact persists and moves quotes in the same direction as the trade. So the coefficient of the contemporaneous order flow in equation (5.4) will pick up both the impact of asymmetric information and that of inventory risk, and will therefore overestimate the informativeness of order flow.

To see this point, let us expand the model to include not only asymmetric information but also inventory holding costs. For simplicity, we assume that the informational content of the order flow is entirely contained in the signed trade size q_t (so that $\lambda_0 = 0$ in equation (5.6)):

(5.9)

$$\mu_t = \mu_{t-1} + \lambda q_t + \varepsilon_t.$$

This means that just before observing the order flow q_t, dealers estimate the security's value as $\mu_{t-1} + \varepsilon_t$, assuming that they expect the order flow to be zero on average. Factoring in inventory holding costs, their valuation before **(p.171)** executing the order at time t is:

(5.10)

$$m_t = \mu_{t-1} + \varepsilon_t - \beta z_t,$$

where z_t is dealers' aggregate inventory just before the t^{th} transaction and β reflects inventory holding costs, as in section 3.5.2 of Chapter 3.[5]

Taking the first differences of the midquote in equation (5.10), we obtain

(5.11)

$$\Delta m_t = \Delta\mu_{t-1} + \Delta\varepsilon_t - \beta\Delta z_t = \lambda q_{t-1} + \varepsilon_t - \beta\Delta z_t,$$

where in the second step we have substituted $\Delta\mu_{t-1}$ from equation (5.9). Using the market-clearing condition $\Delta z_t = -q_{t-1}$ (in equilibrium, the change in inventories mirrors

the order flow), we can rewrite equation (5.11) as
(5.12)

$$\Delta m_t = (\lambda + \beta) q_{t-1} + \varepsilon_t.$$

Now the transaction price at time t is the midquote plus or minus half the bid-ask spread, depending on whether the incoming order is a buy or a sell, that is,

(5.13)

$$p_t = m_t + (\lambda + \beta) q_t + \gamma d_t,$$

where we assume that the order-processing cost is γ per share. Taking the first difference of this equation and using equation (5.11), the change in consecutive prices can be written as
(5.14)

$$\Delta p_t = (\lambda + \beta) q_t + \gamma \Delta d_t + \varepsilon_t.$$

Thus even if regression (5.8) is correctly specified, the estimate of the coefficient of q_t will overestimate the informativeness, λ, of the order flow in the presence of inventory effects. The problem is that this specification is not rich enough to distinguish the adverse selection and the inventory cost components, because in the short run they have the same effect on prices.

To overcome this problem, one can enrich the model to account for the fact that, for a variety of reasons, market orders are serially correlated. First, investors often react similarly to events, which generates a flurry of orders on the same side of the market. Moreover, an investor who wants to make a large trade may elect to trickle it into the market as a series of smaller orders to lessen its price impact. This tends to generate positive serial correlation in the order flow. Second, as Chapter 3 explains, dealers react to an unbalanced order flow by changing their quotes to induce more orders from the opposite **(p.172)** side of the market and so rebalance their inventories. This induces negative autocorrelation in the order flow.

Regardless of its sign, the autocorrelation implies that there is a predictable component of the order flow. Let $\eta_t = q_t - \mathrm{E}(q_t|\Omega_{t-1})$ be the unexpected component (i.e., the "innovation") in the t^{th} transaction (Ω_{t-1} being all the relevant information available before the t^{th} transaction relevant for predicting its size). As Hasbrouck (1988) observes, only this unexpected component contains new information and therefore affects dealers' value estimate. Thus equation (5.9) becomes

(5.15)

$$\mu_t = \mu_{t-1} + \lambda [q_t - E(q_t | \Omega_{t-1})] + \varepsilon_t.$$

As a consequence, the transaction price at time t is now:

(5.16)

$$p_t = m_t + \lambda [q_t - E(q_t | \Omega_{t-1})] + \beta q_t + \gamma d_t,$$

where m_t is given by equation (5.10). The adverse selection cost component of the spread (that is, $\lambda\ [q_t - \mathrm{E}(q_t \mid \Omega_{t-1})]$) depends only on the trade innovation for the reasons that we just explained. But the inventory cost component (βq_t) is determined by the actual trade size, since the change in dealers' inventory (and thereby their inventory risk exposure) depends on the entire (not just the unexpected) trade size. This provides a way to measure separately the contribution of asymmetric information (λ) and inventory risk (β) to market illiquidity.

As an example, let us assume, as in Huang and Stoll (1997), that market orders are generated by a first-order autoregressive process:

(5.17)

$$q_t = \phi q_{t-1} + \eta_t,$$

so that $\mathrm{E}(q_t \mid \Omega_{t-1}) = \phi q_{t-1}$. In this case the order flow innovation $q_t - \mathrm{E}(q_t|\Omega_{t-1})$ is $q_t - \phi q_{t-1}$, so that consecutive changes in prices are given by:
(5.18)

$$\Delta p_t = \Delta m_t + \lambda [\Delta q_t - \phi \Delta q_{t-1}] + \beta \Delta q_t + \gamma \Delta d_t.$$

As in equation (5.11), the change in the midquote is $\Delta m_t = \Delta\mu_{t-1} + \Delta\varepsilon_t - \beta\Delta z_t$. Again using the market-clearing condition $\Delta z_t = -q_{t-1}$ and the fact that the change in dealers' value estimate is:

(5.19)

$$\Delta\mu_t = \lambda (q_t - \phi q_{t-1}) + \varepsilon_t,$$

we obtain:
(5.20)

$$\Delta m_t = (\lambda + \beta) q_{t-1} - \lambda\phi q_{t-2} + \varepsilon_t.$$

(p.173) Substituting this expression for Δm_t in equation (5.18), we finally obtain the following specification for the price impact regression:

(5.21)

$$\Delta p_t = (\lambda + \beta) q_t - \lambda\phi q_{t-1} + \gamma\Delta d_t + \varepsilon_t,$$

When estimated jointly with the autoregressive process for the order flow (5.17), this specification of the price impact regression allows us to identify the three determinants of the bid-ask spread: λ, β, γ, and the coefficient of the order flow process, ϕ. Hence, this approach is called a *three-way decomposition* of the bid-ask spread. Identification comes from the additional assumptions that dealers' estimate of the fundamental value is affected only by surprises in order flow and that inventory holding costs are affected by the actual order flow.

Huang and Stoll (1997) apply this methodology on a complete record of trade and quote data for twenty major NYSE stocks in 1992. Specifically, they estimate the following version of equation (5.21):

(5.22)

$$\Delta p_t = \delta_0 d_t + \delta_1 d_{t-1} + \gamma \Delta d_t + \varepsilon_t,$$

where q_t has been replaced by the trade direction indicator, d_t. They estimate the equation separately for various trade size bins. They use the GMM method to estimate equations (5.17) and (5.22) simultaneously.

In their baseline estimation (shown in their table 5, panel A, p. 1018), Huang and Stoll find a surprising result: looking at the average coefficients estimated for all twenty stocks, the order-processing coefficient γ is by far the largest, accounting for 84 percent of the bid-ask spread on average. In contrast, the inventory holding cost and adverse-selection cost coefficients account for 18 percent and −3.14 percent and both are significantly different from zero. Of course, a negative coefficient for the adverse selection component of the bid-ask spread is meaningless theoretically.[6]

Another suspect result is that the autocorrelation parameter ϕ in the order flow is positive and high, at an average of 0.679. This is problematic, since inventory effects should produce a negative autocorrelation in the direction of market orders. Huang and Stoll conjecture that the reason for this finding is that market orders are broken up as they are executed, which is consistent with high serial correlation in trades. Moreover, a given market order may execute against multiple limit orders at a given point in time. Thus, in the data, the trade will generate a distinct record for each executed limit order and so be improperly treated as multiple trades at the same time. Hence, the informational effect of a **(p.174)** single order generating several consecutive small trades will be underestimated by price impact regressions that treat each trade as if it were generated by different orders.[7]

Huang and Stoll discuss various alternative solutions and eventually opt for assuming that an uninterrupted sequence of trades at the same price and on the same side of the market (with no intervening change in quotes) is generated by a single order. They therefore treat trades in such a sequence as a single order and re-estimate equations (5.17) and (5.22) accordingly. They recognize that this method may overcorrect the problem by lumping some independent orders together, and therefore they regard the corresponding estimates as an upper bound on the adverse selection component and a lower bound on the order autocorrelation parameter.

The estimates that they obtain using this modified approach (see their table 6, panel A, p. 1020) are much more sensible. The adverse selection cost component becomes positive on average and accounts for 9.59 percent of the bid-ask spread. The inventory cost and the order-processing cost components now account for 28.65 percent and 61.76 percent respectively. Finally, on average, the autocorrelation parameter ϕ is negative (-0.74). Again, all coefficients are significantly different from zero. There is an economic logic behind this finding: if true inventory holding costs are large, dealers will indeed manage quotes so as to generate a negatively autocorrelated order flow, so that a high estimate of inventory holding costs is naturally in accordance with a negative estimate for ϕ.

Even with this approach, order-processing costs remain the largest component of the spread (about 62 percent): twice as much as inventory holding and six times as large as adverse selection. This suggests that understanding the subcomponents of order-processing costs (imperfect competition, participation fees, etc.) is important, since they contribute significantly to market illiquidity, at least in some cases.

Huang and Stoll (1997) also estimate the price impact regression for various trade size bins (small/medium and large). Interestingly, the estimates of the costs of liquidity provision depend on trade size. In particular, for large trades the inventory holding cost component accounts for about two-thirds of the spread and is more than twice its average value, while the order-processing component is less than half its average value (table 6, panel B, p. 1020). This finding is consistent with theory, since larger orders require dealers to **(p.175)** take a correspondingly larger inventory, while fixed order-processing costs can be spread over a larger trade. The surprising finding is that the adverse selection component of the spread does not increase with trade size, as the Kyle model predicts. This, as Huang and Stoll note, may be because information leaks out before large transactions, so that their actual occurrence has little further information content. Moreover, many large trades are negotiated on the "upstairs market," where brokers can certify that they are not information driven.

Madhavan, Richardson, and Roomans (1997) show how the price impact regression approach can be used to understand the determinants of intraday patterns in bid-ask spreads. They consider a special case of Huang and Stoll (1997), assuming that $q_t = d_t$ (i.e., all trades are posited as being for one share) and $\beta = 0$ (no inventory effect). In this case, equation (5.21) becomes:

(5.23)

$$\Delta p_t = (\lambda + \gamma) d_t - (\lambda\phi + \gamma) d_{t-1} + \varepsilon_t.$$

Using GMM they estimate this equation jointly with (5.17) (to estimate ϕ) for various intervals during the trading day for NYSE stocks.

They show that the adverse selection cost component is relatively high at the opening (about 4 cents in the first half hour of trading, see their table 2) and then declines (to about 2.8 cents in the last half hour). In contrast, the order-processing cost component is lower at the opening (about 3.4 cents) than at the end of the day (about 4.6 cents). This pattern may reflect the fact that dealers' bargaining power increases towards the end of the trading day, since liquidity demanders are more and more impatient to trade. Thus the well-documented U-shaped curve of bid-ask spreads in equity markets (high at the beginning and the end of the trading day, low in the middle) is at least in part generated by the daily evolution of the adverse selection cost and order-processing cost components.

When data on dealers' inventories are available, there is another way to estimate the various components of the bid-ask spread and to relate price changes to order flow. To see this, add and subtract βq_{t-1} from the right hand side of equation (5.21) to obtain:

(5.24)

$$\begin{aligned}\Delta p_t &= (\lambda+\beta)\, q_t - \beta q_{t-1} + \beta q_{t-1} - \lambda\phi q_{t-1} + \gamma\Delta d_t + \varepsilon_t \\ &= \lambda q_t + \beta\,(q_t - q_{t-1}) - (\lambda\phi - \beta)\, q_{t-1} + \gamma\Delta d_t + \varepsilon_t \\ &= \theta_0 q_t + \theta_1\,(q_t - q_{t-1}) + \theta_2\,(z_t - z_{t-1}) + \gamma\Delta d_t + \varepsilon_t,\end{aligned}$$

with $\theta_0 = \lambda$, $\theta_1 = \beta$, and $\theta_2 = \lambda\phi - \beta$, after using the market clearing condition $z_t - z_{t-1} = -q_{t-1}$. Thus, with data on dealers' inventories, we can evaluate the strength of inventory effects by running a regression of price changes on inventories and order flow. In practice, the coefficient of $\Delta q = q_t - q_{t-1}$ may **(p.176)** also reflect the effect of order-processing costs if these have a fixed component, as explained in the previous section. Thus, the coefficient of the price impact regression on the change in inventories, $z_t - z_{t-1}$, is likely to be more informative about the effect of inventory risk on prices. It should be negative if $\beta \rangle \lambda\phi$ (i.e., if inventory effects are strong enough).

Lyons (1995) tests a specification similar to equation (5.24) with data on trades of one dealer in the Deutsche Mark/dollar market in 1992 (over one week, August 3–August 7, 1992). For each transaction in which the dealer acts as a liquidity supplier, Lyons observes its size and the dealer's inventory. He finds very strong inventory effects. One possible reason is that in foreign exchange markets, inventory risk is large, as trades are typically larger there than in equity markets. Moreover, dealers tend to avoid carrying large inventories overnight, so they must adjust their quotes more substantially after trades in order to get faster mean reversion in their positions. Exercise 3 invites the reader to use the data in Lyons (1995) to estimate various specifications of the price impact regressions described in this section.

5.3. Measuring the Permanent Impact of Trades

The previous section models order arrival as an AR(1) process. That is, traders' expectations regarding the next trade depend only on the last trade. Accordingly, the order flow process is assumed to have a very "short memory." This assumption is problematic: with inventory effects, an order's impact is likely to ripple far in the future, suggesting that we should consider autoregressive process with lags greater than one. Consider, say, the arrival of a large sell order at time t. This order leads to a decrease in price, due in part to the increase in liquidity suppliers' risk exposure rather than a true decline in the asset value. Thus, as Chapter 3 explains, the new order at time $t+1$ is more likely to be a buy. But unless dealers' inventories revert very quickly to their long-run level, prices—even after the next trade—will continue to look relatively low to investors. Thus, at time $t + 2$, a buy order is again more likely than a sell. In fact, this will continue to be the case until dealers' aggregate inventory reverts to its long-run level. This suggests that the trade size at time t should affect expectations about trades at time $t + 1$ but also at $t + 2$, $t + 3$, and so on. Moreover, as dealers' inventories revert to their long-run level, a reversal in prices will temper the initial negative impact of the sell order. Hence, in the presence of inventory effects, the permanent impact of a trade is smaller than its immediate impact, and the short-run impact takes time to dissipate (see figure 3.9 in Chapter 3).

To account for these observations, Hasbrouck (1991) proposes to measure the permanent impact of trades (a measure of their informativeness) by jointly **(p.177)** modelling the dynamics of price changes and orders by a vector autoregressive model (VAR). More specifically, changes in midquotes and trades are assumed to behave according to the following processes:

(5.25)

$$\begin{aligned} m_t - m_{t-1} &= \sum_{j=1}^{\infty} a_j \Delta m_{t-j} + \sum_{j=1}^{\infty} b_j q_{t-j} + \varepsilon_t, \\ q_t &= \sum_{j=1}^{\infty} c_j \Delta m_{t-j} + \sum_{j=1}^{\infty} h_j q_{t-j} + \eta_t, \end{aligned}$$

with the assumptions that the innovations ε_t and η_t have zero means and are jointly and serially uncorrelated.[8] If one sets $b_1 = \lambda + \beta$, $b_2 = -\lambda\phi$, $h_1 = \phi$ and all other coefficients to zero, this specification encompasses as a special case the midquote changes given by equation (5.20).

Besides this particular specification, the VAR approach can encompass richer interactions between returns and order flow: per se, this approach does not place any restrictions on these interactions. Typically, these restrictions will follow from an economic model. For instance, in the presence of inventory effects, one would expect the coefficients h_j to be negative, since inventory and quote management induce mean reversion of the order flow. If, instead, investors tend to execute large orders piecemeal, then one would expect a positive autocorrelation in the order flow and positive values for some of the coefficients h_j. Moreover, the order flow could respond to past price movements: recall, for instance, that in section 3.5.3 of Chapter 3, investors were assumed to place buy market orders in response to quotes below the fair value and sell orders in response to high quotes. Such contrarian strategies should result in negative coefficients c_j.

As an illustration, consider the following example inspired by Hasbrouck (1991). Suppose that trades and changes in midquotes for a stock behave according to the following model:

(5.26)

$$m_t - m_{t-1} = 1.998q_{t-1} + 0.096q_{t-2} + 0.053q_{t-3} + 0.025q_{t-4} + \varepsilon_t,$$
$$q_t = -0.486q_{t-1} - 0.269q_{t-2} - 0.124q_{t-3} + \eta_t.$$

Using this specification, we can compute the dynamics of midquote returns and trades after a purchase of size 1 ($q_0 = 1$) when the innovations in the system are set at their average value, namely, zero. Table 5.1 below shows the dynamics of the midquotes and trades from the first transaction to the tenth transaction. For the purpose of this discussion, we normalize the initial estimate of the asset value to $m_0 = 100$.

On receiving the buy order at time $t = 0$, the dealers adjusts his midquote at time $t = 1$ up by 1.998, from 100 to 101.998. Models of trading with adverse **(p.178)**

Table 5.1 Dynamic Responses of Prices and Trades in the Illustrative Model of Hasbrouck (1991)

t	m_t	$m_t - m_{t-1}$	q_t	$\alpha_t(1) = \sum_{\tau=0}^{\tau=t} \Delta m_t$
0	100	–.–	1.000	–.–
1	101.998	1.998	−0.486	1.998
2	101.124	−0.875	−0.033	1.123
3	101.064	−0.060	0.023	1.064
4	101.105	0.041	0.058	1.105
5	101.210	0.105	−0.030	1.210
…	…	…	…	…
10	101.1566	-6×10^{-4}	-3×10^{-4}	1.156

Source: Joel Hasbrouck (1991) “Measuring the information content of stock trades,” Journal of Finance 46, 179–207.

selection and inventory holding costs typically attribute this increase to the informational content of the buy order *and* to the decrease in dealers’ aggregate inventory. In fact, in the aggregate, dealers have sold one unit of the security and now have a short position, so they mark up the price at which they are willing to sell another unit. Hence, the informational content of the trade can be anywhere from 0 to 1.998 depending on the relative magnitude of adverse selection costs and inventory costs.

Hasbrouck (1991) proposes to identify the information content of the trade with its long-run average price effect. The idea is simple: as Chapter 3 explains, a change in the midquote due to inventory holding costs will dissipate over time as dealers reduce their exposure to the desired level by unwinding positions. Hence, the change in price that persists after a trade must correspond to the value estimation of the asset induced by the trade. Accordingly, the average cumulative price change over T periods after a trade of size q_0 (denoted $\alpha_T(q_0)$) can be used as a proxy for the informational content of a trade of this size. In the analysis of VAR models, $\alpha T(q_0)$ is called an *impulse response function*, in this case to an innovation in order flow. Impulse response functions are routinely computed by standard software for statistical analysis.

As an application of this idea, let us look again at table 5.1. The cumulative price change from the first to the tenth transaction, $\alpha_{10}(1)$, is 1.156.[9] This cumulative change in price is less than the initial change ($\alpha_0(1)$), and it can be used as a measure for the informational content of the trade at time $t = 0$. Of **(p.179)** course, one can look at an even longer horizon to get a more accurate estimate of the permanent impact of trade (its true informativeness). The choice of horizon should depend on how fast the price impact of a trade that is not due to its information content dissipates.

Hasbrouck implements this methodology using a continuous sequence of trade and quote data from NYSE, AMEX, and consolidated regional U.S. exchanges, over the sixty-two trading days in the first quarter of 1989. He estimates a specification similar to the previous system of equations, truncated at five lags for each variable.

The impact of trades on quote revisions is generally positive, and convergence of α_T to its limit value is quite rapid (most being completed within five steps) but not instantaneous. Like other researchers (e.g., Huang and Stoll, 1997), Hasbrouck finds that trades are positively autocorrelated (the h_j's are positive) rather than negatively as expected in the presence of inventory effects. But, there is a strong negative correlation with midquote changes (the c_j's are negative), which is more consistent with the inventory control hypothesis.

The measure of private information features interesting cross-sectional relationships with company size. After scaling the persistent price impact α_T by the corresponding average share price, Hasbrouck finds that the overall price impact of a trade innovation is greater for stocks with lower rather than higher market capitalization, implying that information asymmetry is more severe for the former. This may reflect the lesser availability of public information about smaller firms, which generally have little or no analyst coverage. So this finding is consistent with the panel evidence of Ellul and Panayides (2011), quoted at the beginning of this chapter, that stocks that lose analyst coverage have lower liquidity.

5.4. Probability of Informed Trading (PIN)

A completely different approach to evaluating the importance of adverse selection in trading is found in the works of David Easley, Maureen O'Hara, and various coauthors. The method that they propose aims at estimating the probability of informed trading (PIN). Their idea is to infer this probability from the volume and the imbalance of buy and sell orders throughout the trading day: for example, if buy market orders prevail, it seems likely that good news has arrived at the start of trading day, prompting informed buying.

Here we present their approach. The starting point is a specification of the order arrival process over a specific period of time which, in most applications, is put at one trading day. Figure 5.3 gives a graphical summary.

On each trading day, there is a probability α that an information event occurs. An information event is a change in the value of the security, which can be either **(p.180)**

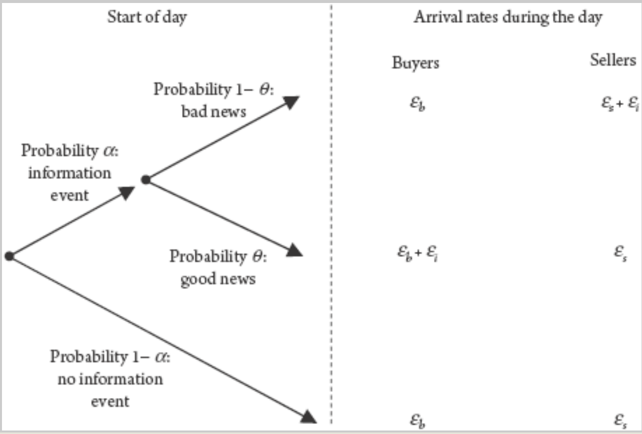


Figure 5.3. Information and order arrival process in the PIN model

positive with probability θ or negative with probability $1 - \theta$. With probability $1 - \alpha$, there is no information event. A day without an information event is defined as a day without a change in the value of the security.

Dealers do not know whether or not an information event has occurred. If it does, some investors are informed. They know whether the information event is positive (the value of the security rises) or negative. As in Glosten and Milgrom (1985), informed investors will buy on days with good news and sell on those with bad. By definition, there is no informed trading on days without an information event.

More specifically, on days with an information event, orders from informed traders arrive according to a Poisson process with intensity ϵi per day. This means that the likelihood of observing k orders from informed traders in a day with an information event is:

$$\frac{\epsilon_i^k e^{-\epsilon_i}}{k!},$$

where $e = 2.71828$ and $k!$ is the factorial of k. Thus, the daily average number of orders from informed traders is ϵ_i. And, whether or not an information event has occurred, buy and sell orders from uninformed investors arrive according to Poisson processes with intensities ϵ_b and ϵ_s, respectively. The order processes for uninformed sellers, uninformed buyers, and informed investors are independent.

The order arrival process differs from the model in Glosten and Milgrom (1985) studied in Chapter 3 in two ways. First, time is continuous and traders arrive at stochastic points in time (in Glosten and Milgrom (1985), time **(p.181)** is discrete). Second, there is uncertainty whether an information event has occurred.[10]

As orders from informed and uninformed traders are generated by independent Poisson processes, the likelihood of any given trade's being is initiated by an informed trader can be shown to be:

(5.27)

$$\mathrm{PIN} = \frac{\alpha\epsilon_i}{\epsilon_b + \epsilon_s + \alpha\epsilon_i},$$

where PIN stands for "probability of informed trading." As one would expect, PIN is the ratio between the rate of arrival of informed traders and the total rate of order arrival. In the limiting case in which uninformed investors do not trade ($\epsilon_b = \epsilon_s = 0$), then PIN = 1. This variable plays the same role as the probability π in Chapter 3.

Using this observation, quotes can be computed as in Chapter 3 (see exercise 6). For instance, consider the bid-ask spread for the opening trade and assume that $\theta = \frac{1}{2}$ and $\epsilon_b = \epsilon_s$, so that uninformed investors are equally likely to buy or sell. In this case, using equation (3.12) in Chapter 3 and replacing π with PIN, we obtain

$$a_1 - b_1 = 2 \times \mathrm{PIN} \cdot (v_H - v_L),$$

where v_H is the realization of the asset's value on days with good news and v_L on days with bad news.

Thus market illiquidity is positively related with PIN, which measures dealers' exposure to informed traders. We now explain how to estimate PIN with order flow data. Suppose there are B_n buy and S_n sell market orders on day n. As the order processes of informed and uninformed investors are independent, the likelihood of these realizations conditional on day n being a bad-news day is

(5.28)

$$\frac{(\epsilon_b)^{B_n} e^{-\epsilon_b}}{Bn!} \cdot \frac{(\epsilon_i + \epsilon_s)^{S_n} e^{-(\epsilon_i + \epsilon_s)}}{Sn!}.$$

Conditional on a good-news day, the likelihood of observing B_n buy orders and S_n sell orders is

(5.29)

$$\frac{(\epsilon_s)^{S_n} e^{-\epsilon_s}}{Sn!} \cdot \frac{(\epsilon_i + \epsilon_b)^{B_n} e^{-(\epsilon_i + \epsilon_b)}}{Bn!}.$$

Finally, on a no-news day, the likelihood of observing B_n buy orders and S_n sell orders is

(5.30)

$$\frac{(\epsilon_s)^{S_n} e^{-\epsilon_s}}{Sn!} \cdot \frac{(\epsilon_b)^{B_n} e^{-\epsilon_b}}{Bn!}.$$

(p.182) Now, recall that the probabilities of the three possible configurations of information are

$$\text{information}=\begin{cases}\text{no news,} & \text{with probability} 1-\alpha,\\ \text{bad news,} & \text{with probability} \alpha(1-\theta),\\ \text{good news,} & \text{with probability} \alpha\theta.\end{cases}$$

Hence, using equations (5.28), (5.29), and (5.30), the unconditional probability of B_n buy orders and S_n sell orders on day n is:

$$\begin{aligned}\Pr(B_n,S_n) &= (1-\alpha)\frac{(\epsilon_s)^{S_n}e^{-\epsilon_s}}{Sn!}\cdot\frac{\epsilon_b e^{-\epsilon_b}}{Bn!}.\\ &+\alpha(1-\theta)\frac{(\epsilon_b)^{B_n}e^{-\epsilon_b}}{Bn!}\cdot\frac{(\epsilon_i+\epsilon_s)^{S_n}e^{-(\epsilon_i+\epsilon_s)}}{Sn!}\\ &+\alpha\theta\frac{(\epsilon_b+\epsilon_i)^{B_n}e^{-\epsilon_b}}{Bn!}\cdot\frac{(\epsilon_s)^{S_n}e^{-(\epsilon_i+\epsilon_s)}}{Sn!}.\end{aligned}$$

Assuming that the occurrence and direction of information events are independent across days, the likelihood of observing a series of daily number of orders B_n and S_n over N days is therefore:

(5.31)

$$\Pr((B_1,S_1),\ldots,(B_N,S_N))=\prod_{n=1}^{N}\Pr(B_n,S_n).$$

This probability is a function of the five structural parameters: α, θ, ϵ_i, ϵ_b, and ϵ_s. Thus we can estimate these parameters, and the PIN, by using the maximum likelihood approach.[11]

Easley, Hvidkjaer, and O'Hara (2002) estimate the PIN of all NYSE common stocks from 1983 to 1998 and obtain a median PIN of about 19 percent. Their estimate does not vary greatly across stocks (figure 5.4) and is negatively correlated with firm size (with a correlation coefficient of about −0.58): larger firms are characterized by relatively less informed trading, as in Hasbrouck (1988) and many other studies. Meanwhile, the PIN is positively correlated in the cross-section with volatility (0.239) and the bid-ask spread (0.353). The correlation with volatility may reflect the fact that information events are more likely for volatile stocks (α is higher), which also present greater profit opportunities for informed traders (so that ϵ_i is higher for more volatile stocks). **(p.183)**

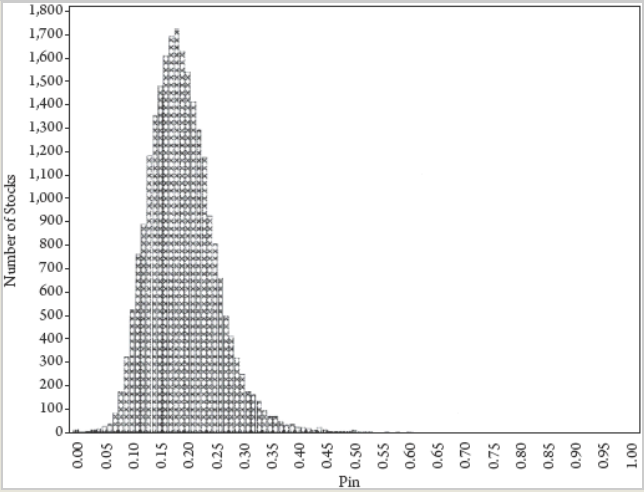


Figure 5.4. Distribution of PIN across NYSE stocks
(Source: panel C of figure 4 in Easley et al., 2002, p. 2207)

Easley, Hvidkjaer, and O’Hara (2002) also find that the PIN is very stable over time: figure 5.5 shows the fifth, twenty-fifth, fiftieth, seventy-firth, and ninety-fifth percentiles of the PIN in each year covered by their study. Finally, the PIN does not appear to be correlated with trading volume, perhaps because informed investors adjust their strategies to the trading intensity of noise traders so as to maintain the PIN invariant to volume.

This methodology has been applied to many different questions. For instance, Grammig, Schiereck, and Theissen (2001) use it to test the hypothesis that there is less informed trading in non-anonymous trading venues. Intuitively, non-anonymous trading can mitigate informational asymmetries by helping dealers to tell apart informed and uninformed traders (see Chapter 8 for further discussion). Accordingly, when an anonymous and a non-anonymous trading venue coexist, informed traders should choose the anonymous one. The German Stock Exchange offers an ideal laboratory for testing this hypothesis, because its stocks are traded in both an electronic market and a floor market. Using data on trades in both markets, Grammig, Schiereck and Theissen (2001) estimate the PIN for thirty stocks. As predicted, they find that the PIN is higher in the electronic market for almost all the stocks. Exercise 4 in this chapter is based on their study. Easley, O’Hara, and Paperman (1998) gives **(p.184)**

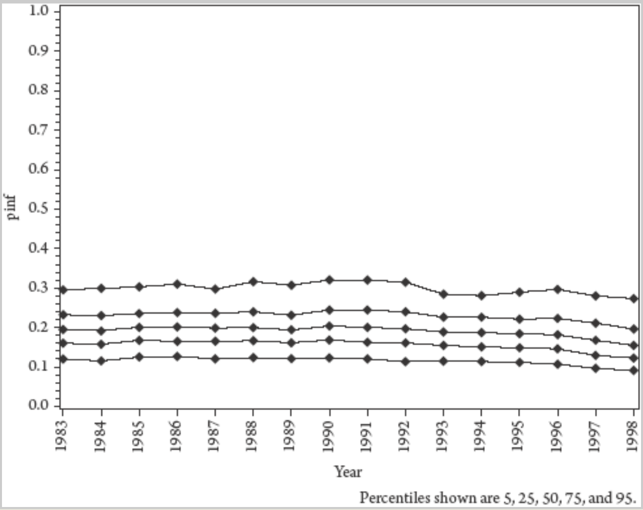


Figure 5.5. Estimated median PIN for selected trading volume deciles

(Source: panel A of figure 3 in Easley et al., 2002, p. 2204)

another application, showing that the PIN is higher for stocks with less analyst coverage, which may be the reason why the spread widens when analysts stop following a stock (as found by Ellul and Panayides, 2011).[12]

5.5. Further Reading

This chapter is actually only an introduction to some empirical techniques in the field of market microstructure. For a systematic treatment, see Hasbrouck (2007).

5.6. Exercises

1. The information effect of orders.

You have a record of transaction prices for all trades on March 26, 2001, for AGF–a French stock listed on Euronext. There were 519 transactions (trading in Euronext is continuous from 9:00 **(p.185)** a.m. to 5:25 p.m.). The index t denotes the t^{th} transaction. The data set contains the following time series (the data are available in two different formats Ch5_AGF_data.xls or Ch5_AGF_data.dta in the companion website of the book): (i) transaction prices ("traprice"), (ii) the best ask price posted just before the transaction ("ask"), (iii) the best bid price posted just before the transaction ("bid"), (iv) the size of transactions ("tradesize"), and (v) the direction of each trade ("tradedir").

a. Estimate equation (5.23) and interpret the findings.
b. Estimate a VAR model such as equation (5.25) (accounting only for trade direction, not size, for comparability with the findings in **a**). **Note**: Be careful with the timing convention: m_t is the midquote before the t^{th} transaction in the model.
c. Describe the average dynamics of the midquote for AGF after a sell order and infer the estimate of $\alpha_{10}(1)$.
d. How do you explain the difference between the measure of adverse selection obtained in (a) and in (b)?

2. Empirical specification of price impact regressions.

Suppose that the midprice at time t, m_t, is determined by the best estimate of the stock's fundamental value based on the public information available, namely $\mu_{t-1} + \varepsilon_t$, minus a term reflecting the net inventory position z_t of market makers at time t:

$$m_t = \mu_{t-1} + \epsilon_t - \beta z_t,$$

where the inventory at time t is related to the order flow at time $t - 1$ by the identity:

$$z_t = z_{t-1} - q_{t-1}.$$

When market makers receive an order q_t at time t, they update their estimate of the fundamental value $\mu_{t-1} + \varepsilon_t$ to also reflect the innovation in order flow, $q_t - \mathrm{E}[q_t|\Omega_t]$:

$$\mu_t = \mu_{t-1} + \lambda\{q_t - E[q_t|\Omega_t]\} + \epsilon_t.$$

Finally, the order flow is generated by the following AR(2) process:

$$q_t = \phi_1 q_{t-1} + \phi_2 q_{t-2} + \eta_t,$$

where η_t is a zero-mean error term uncorrelated with all information at time t (i.e., all variables in the information set Ω_t).

a. Define the unexpected component of the order flow, $q_t - \mathrm{E}[q_t|\Omega_t]$, as a function of the current and past values of the order flow.

b. Determine the change in the best estimate of the fundamental value, $\Delta\mu_t$

(p.186) **c.** Determine the change in the midprice, Δm_t, as a function only of past values of the order flow.

d. If you estimate the equation for Δm_t obtained at point (c) jointly with the order flow process assumed in equation $q_t = \phi_1 q_{t-1} + \phi_2 q_{t-2} + \eta_t$, can you identify the parameters of the model, namely, β, λ, ϕ_1, and ϕ_2? If so, explain how you would infer their values, denoting the coefficients of first three lags of the order flow in the equation obtained at point (c) by b_0, b_1, and b_2.

e. Do we actually have over-identifying restrictions that can be tested? Specifically, does the above model imply a testable restriction on b_1/b_2?

3. Price impact estimation.

This exercise is based on data collected by Richard Lyons,[13] used these data to estimate a model of trading with inventory effects (see Lyons, 1995). The data are stored in the Excel file Ch5_ex3_data.xls and in the Stata data file Ch5_ex3_data.dta available in the companion website for the book. The data set covers one week of trading (August 3–7, 1992) of one dealer, in the Deutsche Mark/dollar market, at a major New York investment bank. The data set has two components:

1. A time series of all prices and quantities for all **direct**, **incoming** transactions in which the dealer was involved. **"Direct"** means that the transactions result from direct bilateral negotiations between the dealer and another dealer (in contrast to brokered transactions). **"Incoming"** means that the dealer does not initiate the

transaction (that is, he acts as liquidity supplier).
2. A time series of the dealer's inventory. This inventory is observed at the time of each incoming transaction.

There are 843 observations for the week. Quotes in the Deutsche Mark/dollar market were firm up to $10 million. The minimal price increment in this market is .0001 DM and is referred to as "one pip." Lyons (1995) reports that the median spread quoted by the dealer is three pips. The Excel file includes four time series:

1. **price:** transaction prices (p_t).
2. **inventories:** the dealer's inventory (I_t) in million U.S. dollars. I_t is the dealer's inventory just before the t^{th} transaction.
3. **tradesize:** the trade sizes (Q_t) in million U.S. dollars. Trades are signed according to the position taken by the liquidity demander. Purchases (i.e., dealer's sales) are positive, and sales (i.e., dealer's purchases) are negative.
(p.187) 4. **signedtr**: an indicator variable (d_t) that is equal to +1 for buy orders (sales by the dealer) and −1 for sell orders (purchases for the dealer).

Overnight observations are suppressed since microstructure effects explain only intraday price dynamics, not overnight changes.

a. Plot the time series of dealer's inventory at the time of each incoming order in the sample. How do you interpret the graph? Is there evidence of mean reversion in inventories?
b. Use the data to estimate equation (5.24). How do you interpret the findings?
c. Propose an estimate of the cost of trading $50 million with the dealer.

4. PIN estimation.

This exercise uses data from the German Stock Exchange (a subset of the data set used in Grammig, Schiereck, and Theissen, 2001).[14] The data set gives, for one stock (BVM) traded on the German Stock Exchange, the number of buy orders and sell orders from June 2 to July 31, 1997 (forty-two days). As usual, the orders are signed according to the position taken by liquidity demanders (i.e., an order is signed positively when the trade initiator is a buyer and conversely). The stock trades on two markets that operate in parallel: a floor market and an electronic trading system. The main difference between the two systems is relative anonymity: a floor market is less anonymous as traders on the floor negotiate prices one-on-one. The data are stored in the Excel file Ch5_ex4_data.xls and Ch5_ex4_data.dta available on the companion website for the book. This file contains four time series:

1. **buy_f:** The number of buy orders executed in the floor market on each day t, $t \in \{1,\ldots,42\}$.
2. **sell_f:** The number of sell orders executed in the floor market on each day t, $t \in \{1,\ldots,42\}$.
3. **buy_e:** The number of buy orders executed in the electronic market on each

day t, $t \in \{1,\ldots, 42\}$.
4. **sell_e:** The number of sell orders executed in the electronic market on each day t, $t \in \{1,\ldots, 42\}$.

a. Consider the tree describing the order arrival process in figure 5.3. We modify it to account for the possibility that traders can choose to trade in either the floor market or the electronic market. Specifically, we suppose that informed investors trade at rate μ_F in the floor market and at rate μ_E in the electronic market, while uninformed investors buy and sell at rates ε_{bj} and ε_{sj} in market $j \in \{E, F\}$. On these assumptions, **(p.188)** what is the likelihood that this trade is informed, conditional on a trade taking place in the floor market? Conditional on a trade taking place in the electronic market? Call these likelihoods PIN_F and PIN_E.
b. Using the series of buy and sell orders executed in each system, propose and implement a methodology to estimate PIN_F and PIN_E.
(**Note:** Estimation by maximum likelihood may not converge if the initial values for the parameters are not well chosen. For this you must calibrate the initial values of the parameters to estimate, so that at least the average number of buy and sell orders per day on each market implied by the Poisson distributions match the actual averages in the data.)
c. Are informed traders more likely to trade in the anonymous market? Is adverse selection greater in the anonymous market?

5. Decomposition of the bid-ask spread.

Using the data for exercise 1 (Ch5_AGF_data.xls or Ch5_AGF_data.dta available in the companion website for the book.) and following Glosten and Harris (1988), regress Δp_t on d_t, q_t, Δd_t, and Δq_t.

a. For a transaction of very small size, what is the bid-ask spread and what proportion is attributable to adverse selection versus order-processing cost?
b. Use an F-test to see whether the data support the restriction (imposed by Glosten and Harris, 1988) that the coefficients of Δq_t and d_t must both be zero.
c. Use an F-test to see whether the trade size effects (as measured by the coefficients of q_t and Δq_t) are jointly significant over and above the impact of the buy/sell indicators d_t and Δd_t.
d. Relate your answers to the spread measures calculated in the exercises of Chapter 2.

6. Bid-ask spread and PIN.

Consider the order arrival process described in figure 5.3 of section 5.4. Using equation (3.8) in Chapter 3, derive the ask and bid prices for the first transaction of the day.

7. Method-of-moments estimator for PIN.

For the PIN model as described in Section 5.4:

a. Derive the first and second moments of the daily buy and sell transactions, that is the expectations E[B] and E[S], the variances var[B] and var[S] and the covariance cov[B,S], as a function of the five underlying parameters of the model (α, θ, ϵ_i, ϵ_b, ϵ_s).
b. Use these five equations to express PIN as well as the underlying parameters of the model in terms of these moments.

Notes:

(1.) For instance, institutional investors trade larger volumes than retail investors. As they trade more frequently, they have more bargaining power (see Bernhardt et al., 2005). This effect implies that the bid-ask spread may decrease with trade size, a conclusion at odds with the prediction of asymmetric information models but consistent with empirical observations in bond markets (see Green, Hollifield, and Schuerhoff 2007) and some equity markets (e.g., Bernhardt etal.,2005).

(2.) The specification of the regression is a problem in itself as the omission of a single relevant explanatory variable can bias the estimates. For this reason, theoretical models of market microstructure are needed, such as those developed in Chapter 3. They provide a starting point to think about economically meaningful specifications of the relationships between returns and order flow.

(3.) Errors in measurement of the explanatory variables can lead to bias and inconsistent estimates of the coefficients in the classical linear regression model.

(4.) Our notation differs from that used by Glosten and Harris (1988). They denote by z_0 and z_1 the coefficients that we denote by λ_0 and λ_1, and by c_0 and c_1 the coefficients that we denote by γ_0 and γ_1.

(5.) In section 3.5.2 of Chapter 3, $\mu_t = \mu_{t-1} + \epsilon_t$ because $\lambda = 0$, since we were assuming the order flow to be uninformative. In this particular case, equation (3.54) in Chapter 3 coincides with (5.10) above.

(6.) Our notation differs from that of Huang and Stoll (1997). In their table 5, α is the fraction of the spread due to adverse selection cost ($\frac{\lambda}{\lambda+\gamma+\beta}$ in our notation), and β is the fraction of the spread due to order-processing cost ($\frac{\gamma}{\lambda+\gamma+\beta}$ in our notation). We also use a different timing convention for trades, but this does not affect the interpretation of the findings.

(7.) This illustrates a problem that often confronts empirical researchers: the data report trades—not the orders that generate them, but the predictions of the theory refer to orders. So the researcher has to infer which sequences of consecutive orders are likely to stem from a single order.

(8.) Our notation and timing conventions are different from Hasbrouck (1991).

(9.) To see how $\alpha_t(1)$ is computed, consider the calculation of $\alpha_2(1)$. First we compute q_1

using the equation for the dynamics of q_t: we obtain $q_1 = -0.486 \times q_0 = -0.486$. Substituting this in the equation for the dynamics of Δm_t, we obtain $\Delta m_2 = 1.998 \times q_1 + 0.096 \times q_0 = -1.998 \times 0.486 + 0.096 = -0.875$. Hence, $\alpha_2(1) = \Delta m_1 + \Delta m_2 = 1.998 - 0.875 = 1.123$.

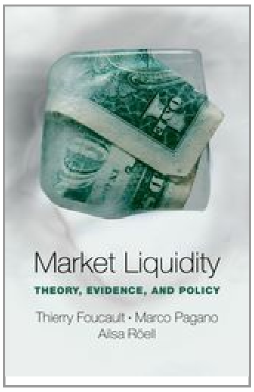
(10.) This uncertainty implies that the end value of the security has three possible realizations instead of two as in the model of Chapter 3.

(11.) In order to estimate the PIN, one must know the daily number of buy and sell market orders over the relevant time period. As explained above, some data sets do not identify the direction of market orders, and in these cases, the number of buy and sell orders must be inferred from other variables. This leads to measurement errors that can bias the PIN measure (see Boehmer, Grammig, and Theissen, 2006).

(12.) Other applications of this methodology include: Easley et al., 1996a, b; Easley, O'Hara, and Paperman, 1998; Easley et al., 2001; Easley et al.,2002; or Heidle and Huang, 2002.

(13.) The data are available at http://faculty.haas.berkeley.edu/lyons/index.html.

(14.) We thank Erik Theissen for providing the data used here.



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Abstract and Keywords

This chapter discusses limit order book (LOB) markets with continuous trading. It begins in Section 6.2, which analyzes a static model of the optimal bidding strategies for limit order traders. Section 6.3 applies the model to study various issues regarding the design of limit order markets: the impact of tick size, the role of priority rules, and the role of designated liquidity suppliers. Understanding the choice between market and limit orders is important, as the viability of limit order markets depends critically on traders choosing both types of order. Thus, Section 6.4 considers a model in which traders can choose between them. The models this chapter develops are useful in analyzing data on trades and quotes. The data are basically of two types: snapshots of LOBs at various points in time and flows of orders (market and limit) over a period of time. The final sections provide suggestions for further reading and exercises.

Keywords: limit order markets, continuous trading, bidding strategies, limit order traders

Learning Objectives:

- • Why limit order markets require separate analysis
- • Adverse selection and market depth in limit order markets
- • Effects of trading rules on market liquidity in limit order markets
- • Determinants of the choice between market and limit orders

6.1. Introduction

Trading in centralized markets increasingly occurs in continuous time, via an electronic LOB, as Chapter 2 describes. In fact nearly all the major equity markets (the NYSE-Euronext, the LSE, Deutsche Börse, etc.) are now continuous limit order markets, sometimes in combination with other trading mechanisms such as a dealer market. Many other instruments (bonds, currencies, derivatives) are also traded in continuous limit order markets. For instance, LIFFE uses this mechanism to trade derivatives, and MTS to trade bonds.

Limit order markets deserve specific analysis for a number of reasons. First, their trading rules differ from those in call markets or dealership markets, analyzed in Chapter 4. Further, they have no strict dichotomy between the traders who “make” the market by posting quotes and those who hit these quotes; Any participants can carry out the trades desired by submitting limit orders, market orders, or a combination. Thus, the provision of liquidity does **(p.192)**

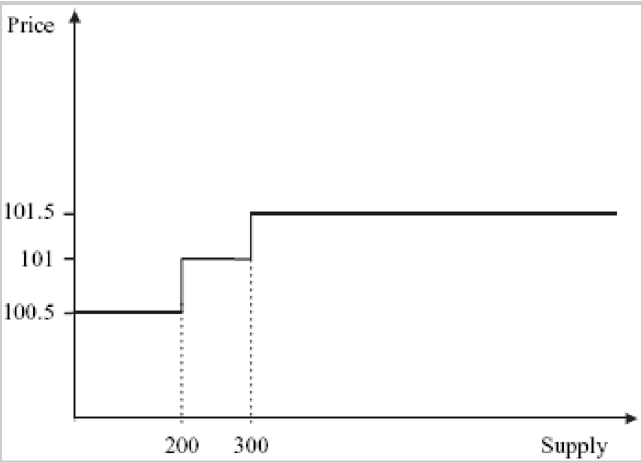


Figure 6.1. A limit order book

not rest exclusively with a designated group of market makers. As a result, in these markets, liquidity depends on how agents trade off the costs and benefits of limit orders and market orders. In this chapter we analyze the implications of these features for market liquidity.

Orders are not matched in the same way in continuous limit order markets as in call auctions. The limit order market has a *discriminatory auction*: all limit orders filled in a given transaction are executed at their own posted price. In contrast, the call market is a *uniform auction*: all traders involved in a given transaction receive or pay the same price.

The following example illustrates the difference. One hundred shares are offered for sale at A_1 = \$100.50, two hundred additional shares are offered at A_2 = \$101, and an unlimited number of shares is offered for sale at A_3 = \$101.50 (figure 6.1).[1]

A buy market order for three hundred shares arrives. It triggers the execution of the sell limit orders placed at \$100.50 and \$101. In the call market, all sell limit orders eligible for execution execute at \$101, the marginal execution price for the market order. The buyer's payment is therefore \$30,300. But in the continuous limit order market, the buy market order "walks up" the book and each limit order executesat its own price. That is, one hundred shares are bought at \$100.5 and two hundred shares at \$101. The buyer then pays \$30,250.

Clearly, for a given LOB, traders submitting market orders get a better deal in the discriminatory auction. However, participants will typically use different **(p.193)** order placement strategies in the two types of market since execution prices are determined differently. Chapter 4 analyzes the behavior of investors in a call market. Here, their behavior in continuous limit order markets is examined.

Section 6.2 analyzes a static model of the optimal bidding strategies for limit order traders. Section 6.3 applies the model to study various issues regarding the design of limit order markets: the impact of tick size, the role of priority rules and the role of designated liquidity suppliers. Chapter 7 uses the model also to study the liquidity effect of competition between trading platforms (see section 7.4.2 in Chapter 7).

Understanding the choice between market and limit orders is important, as the viability of limit order markets depends critically on traders choosing both types of order; so section 6.4 considers a model in which traders can choose between them. In making this choice, investors trade off price improvement against the risks of non-execution and being picked off. Ultimately, it is the determinants of these risks that shape investors' choice between market and limit orders. One determinant is the volatility of the asset's value, which accordingly affects the relative proportions of market and limit orders.

The models this chapter develops are useful in analyzing data on trades and quotes. The data are basically of two types: snapshots of LOBs at various points in time and flows of orders (market and limit) over a period of time. Snapshots of the LOB give such data as the size of the bid-ask spread and the cumulative number of shares offered up to a given ask or bid price. That is, they describe the state of the LOB at a given point in time. The framework presented in section 6.2 helps to explain why this state may vary from security to security or over time. Flow data provide information on the terms of completed trades and give insights into the drivers of trading aggressiveness (market orders are more aggressive than limit orders, buy limit orders with high prices are more aggressive than with low prices, etc.). An important driver, of course, is the state of the book, which itself is influenced by the flow of orders. Dynamic models in which traders can choose between market and limit orders offer insight into these interactions between the flow of limit and market orders and the state of the LOB (section 6.4).

6.2. A Model of the Limit Order Book (Lob)

6.2.1 The Market Environment

We consider a limit order market for a risky security with final value $v = \mu + \varepsilon$, where $E(\varepsilon) = 0$. In period 0, traders submit limit orders. Then, in period 1, a market order arrives and executes against the limit orders in the book. In **(p.194)**

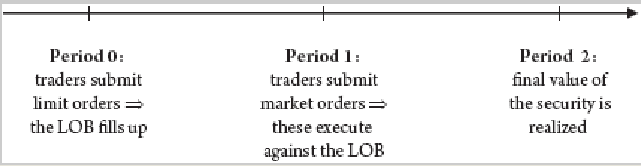


Figure 6.2. Timing

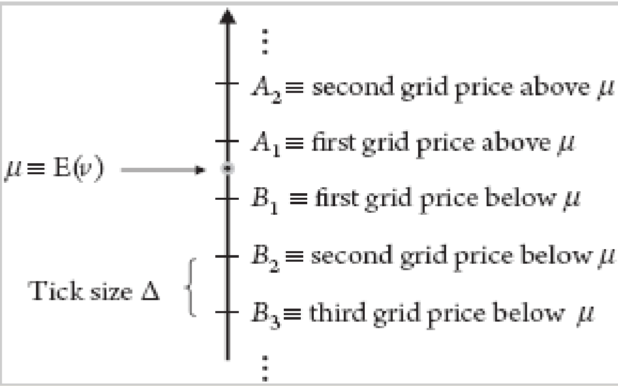


Figure 6.3. The grid of possible quotes in the limit order book

period 2, the value of the security and traders' payoffs are realized. The time line is given in figure 6.2.

The limit orders are positioned on a price grid. The distance between two consecutive prices on the grid, the tick size, is denoted by Δ. In figure 6.3, we denote by A_j the j^{th} price on this grid above the expected value μ of the security, and by B_j the j^{th} price on this grid below μ.

In this model, traders never submit sell limit orders at a price lower or buy limit orders at a price higher than μ. For this reason, we refer to $\{B_k\}_{k=1}^{k=\infty}$ as the set of bid prices and to $\{A_k\}_{k=1}^{k=\infty}$ as the set of ask prices. Let yk be the number of shares offered at price A_k and Y_k be the cumulative depth at price A_k (i.e., the total number of shares offered at price A_k or less). Figure 6.4 represents the ask side of the LOB.

In period 1, a trader submits a market order of size q. A buy market order executes against sell limit orders until it is fully served. (Symmetrically, a sell market order executes against buy limit orders.) As usual, the size of the market order is signed according to its direction: $q \langle 0$ for sell and $q \rangle 0$ for buy. Market orders execute against the book at the limit order prices. Suppose a trader submits a buy market order for q shares such that: $Y_1 \langle q \langle Y_2$. His total payment is then $Y_1 \cdot A_1 + (q - Y_1) \cdot A_2$.

The market order submitted in period 1 can be a buy or a sell with equal probability. Its size is unknown to the limit order traders when they submit their orders in period 0. The probability distribution of the market order size is **(p.195)**

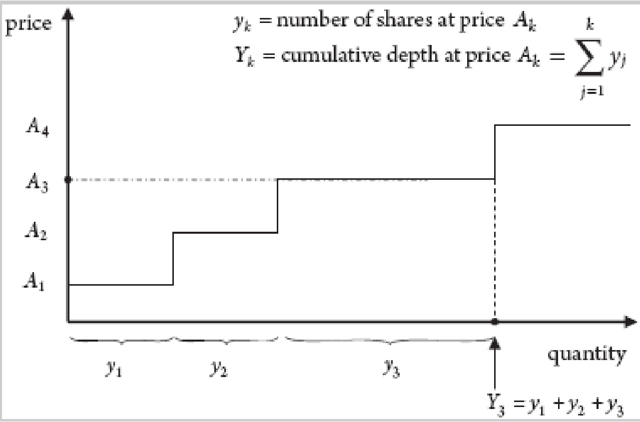


Figure 6.4. The ask side of the limit order book

denoted by$f(\cdot)$ and the cumulative probability distribution by $F(\cdot)$ In some of the examples below, we assume that the size of market orders is exponentially distributed, that is, (6.1)

$$f(q) = \frac{1}{2}\theta_e - \theta\,|q|\,.$$

In this case, buy and sell market orders arrive with equal probability, since $\Pr(q\rangle 0) = \Pr(q\langle 0) = \frac{1}{2}$. And for this distribution the expected size of a market order, $E(|q|)$, can be shown to be equal to $\frac{1}{\theta}$.

At time 0, limit orders are submitted by a continuum of risk-neutral traders who arrive sequentially. They fill the book up to the point where there are no expected profit opportunities: a competitive equilibrium is reached when (i) there is no price at which adding a limit order is profitable (the “no entry” condition) and (ii) there is no price at which cancelling a limit order is profitable (the “no exit” condition).

Last, we assume a time-priority rule for tie-breaking.[2] This means that two limit orders at the same price are executed in the order in which they have been submitted. For instance, the last share offered at price A_k (the marginal share at this price) executes if and only if the next market order is a buy order that exceeds the number of shares offered up to price A_k, that is $q \rangle Y_k$. Hence, the execution probability of the marginal share offered at price A_k is

(6.2)

$$P(Y_k) \equiv \Pr(q \geq Y_k) = 1 - F(Y_k).$$

This execution probability declines with cumulative depth at price A_k. As the total number of shares offered up to a given price increases, it becomes less and **(p.196)** less likely that the next market order will be large enough to trigger execution of the marginal share. This is akin to a queuing situation in which, as the queue lengthens, the last person has a smaller chance of being served because the object for sale is in unknown but limited supply. The execution probability for the marginal buy limit order at price B_k is obtained in a similar way and is equal to $F(Y_k)$, where $|Y_k|$ is the cumulative quantity demanded at bid price B_k.

This provides a simple framework for analyzing many interesting issues relating to limit order markets. However, two inherent limitations should be noted at the outset.

First, the model says nothing about how the competitive equilibrium is reached. The underlying idea is that the market is sufficiently competitive so that there are no strictly profitable opportunities for adding limit orders to the book. This is an interesting benchmark, but in the real world competition among limit order traders is likely to be imperfect. The competitive equilibrium should therefore be viewed as the limit of the equilibrium obtained when the number of market participants is finite.

Second, in this model, traders are classed in either of two groups: (i) those who submit limit orders (supply liquidity) and (ii) those who submit market orders (demand liquidity). They cannot switch from one group to the other. We defer the analysis of how traders choose between a market order or a limit order to section 6.4.

6.2.2 Execution Probability and Order Submission Cost

To illustrate the notion of competitive equilibrium, we first consider a simple case in which no trader has private information about the final value of the security. Market orders are exogenous, and competing traders place limit orders at a display cost of C per share. This cost represents the time required to submit and monitor the order, and any entry fee charged to traders submitting limit orders, whether or not they are filled.

This specification of the model illustrates the importance of execution probabilities for limit orders and shows how order submission costs affect cumulative depth. The mechanics here also prepare the ground for the more complex case in which market order traders may be informed (section 6.2.3). We consider the determination of cumulative depth on the ask side only, leaving the symmetric case of the bid side to the reader.

The zero-profit condition.

We write the expected profit on the “marginal” unit offered at price A_k when cumulative depth at this price is Y_k. The trader offering this unit is called the *marginal trader* at price A_k.

(p.197) In case of execution, the realized profit on the marginal unit is $A_k - v - C$. In case of non-execution, it is $- C$: the submission cost is paid but the order does not execute. Consequently, the expected profit, $\Pi_k(Y_k)$, on the marginal unit is:

(6.3)

$$\Pi_k\left(Y_k\right) = P\left(Y_k\right)\left[A_k - \mathrm{E}\left(v | q \geq Y_k\right)\right] - C.$$

Since we assume that market order traders have no private information, the order flow at time 1 is independent of the value of the security. This means that $E(v \mid q \geq Y_k) = E(v) = \mu$. Hence,

(6.4)

$$\Pi_k\left(Y_k\right) = P\left(Y_k\right)\left(A_k - \mu\right) - C.$$

The probability of execution for a sell limit order cannot be greater than one-half, as a buy market order is submitted with probability $\frac{1}{2}$. Hence, the expected profit on a marginal limit order at an ask price below $2C + \mu$ is negative, so that no sell limit orders are submitted below $A^* = 2C + \mu$.

What is the cumulative depth Y_k of the order book at price $A_k \geq A^*$? Consider a trader contemplating a limit order for an infinitesimal quantity at price A_k. Given the time-priority rule, this order goes to the back of queue Y_k and its execution probability is therefore $P(Y_k)$. The trader faces a trade-off between the revenue in case of execution of the limit order, which is $A_k - \mu$ per share, and the order display cost C. The optimal point along this trade-off depends on the order's execution probability. If the queue Y_k at price A_k is short enough, then the execution probability is high enough so that the trader can expect a positive profit, that is,

$$\Pi_k\left(Y_k\right) = P\left(Y_k\right)\left(A_k - \mu\right) - C \geq 0.$$

Otherwise, it is unprofitable for the trader to add a limit order to the queue at price A_k.

Thus, the no-entry/no-exit condition for a competitive equilibrium holds when the cumulative depth at price A_k solves

(6.5)

$$\Pi_k\left(Y_k\right) = 0,$$

that is,

(6.6)

$$P\left(Y_k\right) = \frac{C}{A_k - \mu},$$

(6.7)

$$\text{or } A_k = \mu + \frac{C}{P(Y_k)}.$$

(p.198) Example 1.

Consider the case in which the distribution of market order size is exponential, as defined in equation (6.1). In this case:

$$P(Y_k) = \frac{1}{2} e^{-\theta Y_k}.$$

Thus, solving equation (6.6) for Y_k, we obtain the cumulative depth at price $A_k \geq A^*$:

$$Y_k = \frac{1}{\theta} \text{In} \left(\frac{A_k - \mu}{2C} \right).$$

The determinants of cumulative depth.

In the model, the cumulative depth at price A_k depends on the order submission cost and the probability distribution of market order size. This is clearly shown in example 1. The cumulative depth increases with *1/θ,* the average order size, because people anticipate "the queue to go faster." Further, cumulative depth at each price is decreasing in the order display cost C. This is a direct implication of the fact that the execution probability of the marginal limit order at a given price decreases as the cumulative depth at that price diminishes (see equation (6.6)). Intuitively, the queue at or below price A_k is shorter when the cost of joining the line is higher. This means that an order entry fee for limit orders impairs the depth of the LOB at every price.

Empirically, the level of trading activity varies across stocks. For instance, firms with a large market value of equity, known as large caps (LC), are typically more active, in the sense that trades are more frequent and larger than for small caps (SC). This means that the likelihood of a market order larger than any given size is greater for large caps, i.e. $P_{LC}(Y) \rangle P_{SC}(Y)$ for all $Y \rangle 0$. Thus, by equation (6.6), the cumulative depth in the large-cap market in equilibrium will be greater at all prices. In this sense, liquidity demand (active trading) begets liquidity supply, as it improves the chance of execution for limit order traders and so induces them to submit more orders.

6.2.3 Limit Order Trading with Informed Investors

We now explore how adverse selection affects the LOB. We assume that market orders are submitted either by uninformed traders or by informed traders (those who know the true value of the security, $v = \mu + \varepsilon$). Limit order traders do not observe ε but are aware that market orders may come from informed traders. This model is originally due to Glosten (1994).

Exposure to informed trading has two distinct sources. First, informed traders may have private information not yet available to the market, as Chapters 3 and 4 suggest. Second, as is noted in the introduction to this chapter, the arrival of **(p.199)** public information exposes limit order traders to the risk of being "picked off" if they are slow in updating their prices to reflect the new consensus value. In both cases, the actions of the informed traders create a positive correlation between the order flow and the change in the estimate of the true value, which is the essence of the adverse selection problem.

The very fact that a limit order is executed itself contains information. It reveals the direction of the market order and it shows that it is large enough to trigger execution of the limit order. This information is value-relevant, since the size and direction of the market order are correlated with the true value. So when placing a limit order and choosing one's bid, one must take this information spillover into account. Recall that, by equation (6.3), the expected profit on the marginal unit offered at price A_k is

(6.8)

$$\Pi_k(Y_k) = P(Y_k)\,(A_k - \mathrm{E}(v|q \geq Y_k)) - C,$$

where the upper tail conditional expectation, $E(v \mid q \geq Y_k)$, is the estimate of the value of the security conditional on execution of the marginal unit at price A_k.

Intuitively, with informed trading this upper tail expectation increases with Y_k, the cumulative quantity at each price. Consider an informed trader's optimal strategy when he knows that the value of the security is $v \rangle A_k$. If he is not wealth-constrained, his demand is unlimited at any price below v, zero at any price above v. Thus, he will definitely sweep up all sell limit orders in the book at prices at or below A_k. In contrast, an uninformed trader only hits the limit orders needed to fill his desired order. Thus, as the queue of limit orders at price A_k lengthens, it becomes more and more likely that the market order triggering execution of the marginal order is from an informed investor who knows that the limit order is "stale," i.e., that $v \rangle A_k$. As a consequence, the marginal trader's valuation conditional on execution gets larger as Y_k increases. The next example illustrates this point.

Example 2.

The final value of the security can be either low ($v^L =/\mu - \sigma$) or high ($v^H = \mu + \sigma$), with equal probability. The market order trader is either informed with probability π or uninformed with probability $1 - \pi$. The uninformed trader is a buyer or a seller with equal probability, with a desired trade size that is either small (qS) or large (qL) with probabilities φ and $1 - \varphi$, respectively. Let $A(qS)$ and $A(qL)$ be the ask prices at which cumulative depth on the ask side is equal to qS and qL shares, respectively. Moreover, assume that $A(qL) \langle v^H$.[3] Thus, if the informed investor knows that the value of the security is v^H, he sweeps all limit orders up to price $A(qL)$, at least.

Let I and U denote the event that the market order is submitted by the informed and the uninformed traders, respectively. Given the order submission **(p.200)** strategy of liquidity demanders, we have:

$$\Pr(I|q \geq Y) = \frac{\frac{\pi}{2}}{\frac{\pi}{2} + \frac{1-\pi}{2}} = \pi \ \text{ for } Y \leq qs,$$

$$\Pr(Iq \geq Y) = \frac{\frac{\pi}{2}}{\frac{\pi}{2} + \frac{(1-\pi)(1-\phi)}{2}} = \frac{\pi}{\pi + (1-\pi)(1-\phi)} \text{ for } qs \left\langle Y \leq qL. \right.$$

The second expression is greater than the first: a trade no larger than qS conveys no information about the identity of the person placing the market order, but a trade larger than qS is more likely to come from an informed trader. Notice that:

(6.9)

$$\begin{aligned} \mathrm{E}\left(v|q \geq Y\right) &= \Pr(U|q \geq Y)\mu + \Pr(I|q \geq Y)\left(\mu + \sigma\right) \\ &= \mu + \Pr(I|q \geq Y)\sigma, \end{aligned}$$

so that $E(v \mid q \geq Y)$ is weakly increasing in Y because a limit order far back in the queue is more likely to be hit by an informed investor ($Pr(I \mid q \geq Y)$ increases in Y). In particular,
(6.10)

$$\mathrm{E}\left(v|q \geq qs\right) = \mu + \pi\sigma,$$

and
(6.11)

$$\mathrm{E}\left(v|q \geq qL\right) = \mu + \frac{\pi\sigma}{\pi + (1-\pi)(1-\phi)}.$$

Let us now go back to deriving the equilibrium LOB. Since $E(v \mid q \geq Y_k)$ increases with Y_k, the expected profit on the marginal order at a given price decreases along with cumulative depth at this price. As in the previous section, a competitive equilibrium is obtained when cumulative depth Y_k at each price A_k is such that the following zero profit condition is satisfied:

(6.12)

$$\Pi_k\left(Y_k\right) = 0 \text{ if } \Pi_k\left(Y_{k-1}\right)\rangle 0, \text{ and } Y_k = Y_{k-1} \text{ if } \Pi_k\left(Y_{k-1}\right) \leq 0,$$

meaning that the marginal unit must yield no profit if offering less were to yield positive profits; otherwise, that unit must not be offered. From equation (6.8), the zero profit condition is equivalent to:
(6.13)

$$A_k = \mathrm{E}\left(v|q \geq Y_k\right) + \frac{C}{P(Y_k)} \text{ if } Y_k \Big\rangle Y_{k-1},$$

(6.14)

$$A_k \left\langle \mathrm{E}\left(v|q \geq Y_{k-1}\right) + \frac{C}{P(Y_{k-1})} \, \text{if}\, Y_k \right\rangle Y_{k-1}.$$

In the remainder of this section, in order to isolate the impact of asymmetric information on the equilibrium price schedule, we assume that the order submission cost C is zero.

(p.201) The example below illustrates, in a simple way, some important and general properties of the limit order market with informed trading.

Example 1, continued.

Assume again, as in example 1, that the market order size has an exponential distribution $\left(f(q) = \frac{1}{2}\theta e^{-\theta|q|}\right)$ and that the expected value of the security conditional on the market order size is linear:

(6.15)

$$\mathrm{E}\left(v|q = x\right) = \mu + \lambda x.$$

Equation (6.15) is the "updating rule" that specifies how liquidity suppliers (limit orders traders) would update their estimate of the asset value if they knew the total size of the market order. The parameter λ is therefore a measure of the informativeness of the order flow. In this example, we take the updating rule and the order size distribution as primitives. This parametric specification of the model is a useful shortcut for analyzing the properties of the LOB with asymmetric information and has proved useful empirically. Later on, we consider a richer setting in which the updating rule and the distribution of order size are endogenous.

Using the law of iterated expectations and the updating rule, we obtain

$$\begin{aligned} \mathrm{E}\left(v|q \geq x\right) &= \mathrm{E}\left(\mathrm{E}\left(v|q = x\right)|q \geq x\right) \\ &= \mu + \lambda \mathrm{E}\left(q|q \geq x\right) \\ &= \mu + \lambda \frac{\int_x^{\infty} q\theta e^{-\theta_q} dq}{\int_x^{\infty} \theta e^{-\theta_q} dq} \\ &= \mu + \frac{\lambda}{\theta} + \lambda x, \quad \text{for}\, x \geq 0. \end{aligned}$$

Consider first the case of zero tick size (i.e., prices are not required to take discrete values on a grid). In this case, from equation (6.13) the equilibrium price schedule on the ask side of the book is

$$A\left(Y\right) = E\left(v|q \geq Y\right) = \mu + \frac{\lambda}{\theta} + \lambda Y.$$

This price schedule is shown as the upward-sloping line in figure 6.5. Note that the relationship between bid prices and cumulative depth at each price is the mirror image of the ask side, since the probability distribution for market order sizes is symmetric around zero.

When the tick is not zero, so that prices must be at discrete intervals (no less than ten cents, say), the cumulative depth at each price on the ask side can be **(p.202)**

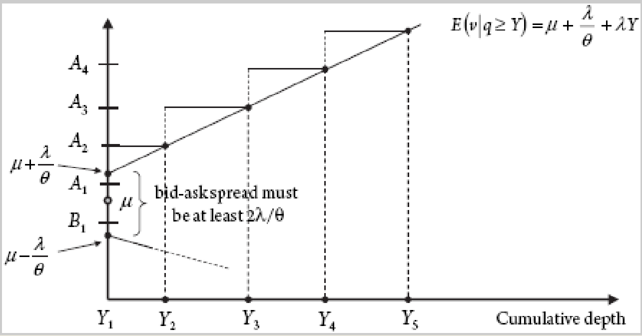


Figure 6.5. Equilibrium limit order book in the exponential case

derived using equations (6.13) and (6.14). This yields
(6.16)

$$Y_k = 0 \text{ for } A_k \leq \mu + \frac{\lambda}{\theta},$$

(6.17)

$$Y_k = \frac{A_k - \left(\mu + \frac{\lambda}{\theta}\right)}{\lambda} \text{ for } A_k \Big\rangle \mu + \frac{\lambda}{\theta}.$$

Graphically, the ask side of the book in this case is represented by the step function in figure 6.5. The equilibrium has two interesting properties:

- • The limit order market features a non-zero bid-ask spread even for very small orders. That is, even market orders for an infinitesimal quantity get executed at a premium or discount relative to the expected value of the security. This feature is not an effect of the tick size, since it persists even when the tick is zero, in which case the spread for a very small order is $A(0) - B(0) = \frac{2\lambda}{\theta} \Big\rangle 0$. This property does not obtain in the models presented in Chapter *4,* such as that of Kyle (1985), in which the spread on an order of size q is given by $2\lambda q$ and goes to zero as the order size goes to zero. The reason is that in the limit order market, liquidity suppliers cannot make their quotes contingent on the total size of market orders. As a result, the limit orders at the top of the book are more exposed to adverse selection. To see this, let us take the sell limit order with the most competitive ask price. It will execute against market buy orders of every size, as it takes precedence over all the other limit orders. So in choosing his offer, the trader at the top of the book must take account of the possibility that his offer can be hit both by very small market orders (which convey very little information) and by very large ones (which are strongly informative). **(p.203)** For this reason, his valuation conditional on execution is strictly larger than his prior estimate of the asset value μ.[4]

- • The depth of the LOB is decreasing in the informativeness of order flow. As X increases, the LOB thins out (i.e., the cumulative depth at each price diminishes). The intuition should by now be standard for the reader: the more informative the order flow, the more exposed liquidity suppliers are to adverse selection, so they bid less aggressively (see Chapter 4).

In this example, we have taken the distribution of market order sizes and the updating rule as given. This simplifies the analysis but is not entirely satisfactory, as these functions ultimately depend on the informed trader's order placement strategy, which itself depends on the offers available in the book. Thus, in equilibrium the informed investor's order placement strategy and the LOB are determined *jointly.* We now provide a new example in which this is the case, to show the robustness of the conclusions obtained so far (see also exercise 1).

Example 2, continued.

Consider example 2 again and assume that the tick size is zero. In this case, an equilibrium is a situation such that if the investor arriving at time 1 is informed, he submits an order of arbitrary size larger than qL, and the LOB displays the following offers:

(6.18)

$$A(Y)\mathrm{E}(v|q \geq Y) = \begin{cases} \mu + \pi\sigma & \text{for } Y \leq qs, \\ \mu + \dfrac{\pi}{\pi + (1-\pi)(1-\phi)}\sigma & \text{for } qs\langle Y \leq qL, \\ \mu + \sigma & \text{for } Y\rangle qL. \end{cases}$$

(6.19)

$$B(Y)\mathrm{E}(v|q \leq -Y) = \begin{cases} \mu - \pi\sigma & \text{for } Y \leq qs, \\ \mu - \dfrac{\pi}{\pi + (1-\pi)(1-\phi)}\sigma & \text{for } qs\langle Y \leq qL, \\ \mu - \sigma & \text{for } Y\rangle qL. \end{cases}$$

To verify that this is an equilibrium, we must show that the informed investor's behavior is optimal, given the offers in the book, and that the LOB is in a competitive equilibrium, given the behavior of the informed investor.

The first step is straightforward. Suppose that the investor arriving at time 1 is informed and learns that the value of the security is high. He can buy up to qL shares at a price strictly below the value $v^H = \mu + \sigma$. Thus, buying at least qL shares is optimal for this informed investor. Additional shares are purchased exactly at v^H. Hence, the informed trader is willing to buy any number of shares over and above qL.

Given this behavior, it follows immediately that $E(v \mid q \geq Y)$ is given by equation (6.9) for $Y \leq qL$. Moreover, only the informed trader submits a buy **(p.204)** market order larger than qL and he does so upon learning that the true value is v^H. That is,

$$\mathrm{E}\left(v|q \geq Y\right) = v^H = \mu + \sigma \, \text{for} \, Y \geq qL.$$

Thus, in a competitive equilibrium, ask prices in the book are as given by equation (6.18). The analysis for the bid side is symmetric.

This example confirms the robustness of the conclusions drawn from the previous example. First, the book becomes thinner as the exposure to informed trading (π) heightens. And even though the informed investor never submits a small market order, these orders execute at a markup relative to μ. Again, this reflects the fact that execution of limit orders at the top of the book may be against small orders (with no information content) or large orders (which are informative), leading to a non-zero bid-ask spread for small trades, also known as the "market touch" ($A(q) - B(q) = 2\pi\sigma$ for $0 \leq q \leq qS$).

6.3. The Design of Limit Order Book Markets

We now use the model developed in the previous section to address market design issues: how liquidity and the distribution of trading gains among traders are affected by tick size (section 6.3.1), secondary priority rules (section 6.3.2), and designated market makers like the NYSE specialist (section 6.3.3).

6.3.1 Tick Size

Market organizers or regulators usually set a tick size, which varies across exchanges and across stocks, as it often is a function of the stock price. On the NYSE, it was $\$\frac{1}{8}$ for stocks with prices over one dollar until June 1997, when, under regulatory pressure, it was reduced to $\$\frac{1}{16}$ and finally, in 2000, to one cent. Decimalization was imposed on Nasdaq and AMEX as well. This change was very controversial and the appropriate size of the tick is still debated (see section 7.5.1 on new regulatory developments in Chapter 7).

It may seem surprising that investors, exchanges and intermediaries care about tick size. After all, it is often very small compared to stock prices. Yet as the model shows, the tick determines liquidity suppliers' expected profits: the number of shares offered at each price is such that the expected profit on the last share (the marginal limit order) is just zero. But inframarginal limit orders (those ahead of the marginal order in the queue at a given price) obtain strictly positive expected profits *if* the tick size is strictly positive. Consider for instance the trader at the head of the queue of limit orders at price A_k. His expected **(p.205)** profit on the first share offered is

(6.20)

$$\Pi_k(Y_{k-1}) = (1 - F(Y_{k-1}))\,(A_k - \mathrm{E}(v|q \geq Y_{k-1})) - C,$$

because this order is filled if and only if the size of the market order exceeds the cumulative depth offered at price A_k-1. In a competitive equilibrium, the zero profit condition on the marginal share at price A_{k-1} requires
(6.21)

$$A_{k-1} = \mathrm{E}(v|q \geq Y_{k-1}) + \frac{C}{(1 - F(Y_{k-1}))}.$$

Equations (6.20) and (6.21) imply:

$$\Pi_k(Y_{k-1}) = (1 - F(Y_{k-1}))\,(A_k - A_{k-1}) = (1 - F(Y_{k-1}))\,\Delta\rangle 0.$$

Thus, the expected profit on the “first” share sold at A_k is proportional to the tick size, while that on additional shares offered at A_k is necessarily lower, because the probability of execution is lower. In the extreme case of tick size nil, limit order traders get zero expected profits. Otherwise, their aggregate expected profit is strictly positive and bounded by the tick size.

The tick size is a source of profit for liquidity providers because it hinders competition: it stops them from bidding the security price up or down to their marginal valuation. As we have just seen, the marginal valuation for the first share offered at price A_k is A_{k-1}, which is less than A_k if the tick size is positive. Traders could earn a strictly positive expected profit by undercutting this offer by an infinitesimal amount, but given the tick, they cannot. If they undercut a quote, they must do so by at least Δ. In equilibrium, the depth at each price is just enough to make undercutting unprofitable.

Figure 6.6 shows the effects of cutting the tick in half from Δ to $\frac{\Delta}{2}$. The reduction creates new eligible prices on the grid, and as a result, the spread

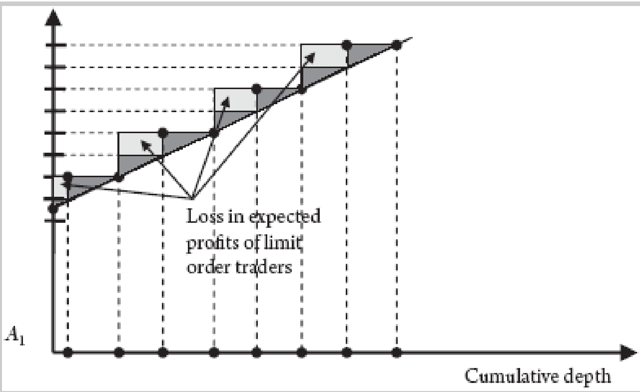


Figure 6.6. Effect of a reduction in the tick size

(p.206) tends to fall. The number of shares posted at any given price on the old grid is reduced as well. Intuitively, some of the shares formerly offered at any price A_k are now offered at price $A_k - \frac{\Delta}{2}$. But the smaller tick does not affect the cumulative depth at a given price on the old grid, because cumulative depth at any price must adjust so that the valuation of the marginal seller just equals the price. If the valuation is unaffected by the change in the tick size (as assumed here), then the cumulative depth *at a given price* is independent of the tick size. Thus, lowering the tick size lowers the total trading cost for market orders.

Consider a buy market order of size q. Its total purchase cost is given by the area under the step function $A(q)$, mapping cumulative depth and quotes in equilibrium. Clearly this area becomes smaller when the tick size is reduced. The decrease in trading costs for liquidity demanders reflects the fact that the smaller tick intensifies competition among liquidity suppliers. For instance, the offers posted at price $A_k - \frac{\Delta}{2}$ compete away some of the profits at price A_k on the old grid and so reduce total trading costs.

To sum up, a decrease in tick size has the following effects:

(i) Reduces expected profits of liquidity suppliers and trading costs for liquidity demanders.
(ii) Reduces the number of shares offered at each price, leaving cumulative depth at a given price unchanged.
(iii) Reduces the bid-ask spread.

These points explain why reducing the tick size is such a controversial measure. It clearly hurts the "sell-side" firms that supply liquidity (e.g., firms with market-making activities), and it benefits the "buy side" that demands liquidity. Not surprisingly, the two sides disagree over reducing the tick.

Several empirical studies have analyzed the effect of changes in tick size on market liquidity, generally finding either that it narrows the bid-ask spread or else has no effect.[5] Goldstein and Kavajecz (2000) studied the effect of the reduction in the tick size from \$ $\frac{1}{\$}$ to \$ $\frac{1}{16}$ on the NYSE in 1997. Interestingly, their data enable them to study the resulting change in the cumulative depth. As predicted, they found a reduction in trading costs for small orders, but for larger orders the evidence was less clear cut, they even found an increase in the trading costs of large orders for low-price stocks.

(p.207) Overall, their findings paint a more complex picture than is suggested by the simple model analyzed in this section. This is not surprising, since the model ignores several possible side-effects. First, as tick size is a compensation for liquidity providers, reducing it may prompt them to exit the market and so reduce total liquidity provision. Moreover, limit order traders may pay less attention to the LOB as the reduction in the tick size diminishes the gain from being at the head of the queue. Or they may choose to improve non-competitive offers in the book by a smaller amount rather than directly post a competitive offer (since the expected profit obtained at the competitive offer is smaller). In both cases, it will take longer for the LOB to be replenished after a trade, and market orders will execute more frequently against non-competitive offers.[6] Lastly, if the return to limit order trading is lower, traders might use market orders more frequently. Again, this effect could decrease liquidity when the tick size is reduced.

6.3.2 Priority Rules

The LOB can be seen as a queue. Priority rules determine the sequence in which the orders in the queue are “served.” Price priority comes first. That is, limit orders at a given price are executed before those at worse prices. Secondary priority rules determine the sequence of execution of multiple limit orders at any given price. The most common is time priority. That is, limit orders at the same limit price are eligible for execution in the sequence in which they were submitted. But there are also other rules. For instance, the electronic futures markets for the leading short-term interest rate in the United States (Eurodollar), Europe (Euribor), and the United Kingdom (Short Sterling) and for the two-year U.S. Treasury Note future have a pro-rata allocation rule.

To see how this rule operates, assume that there are two sell limit orders tied at the best ask price. The first offers one thousand shares, the second four thousand shares. Suppose a buy market order for four thousand shares arrives. Under the pro-rata rule, the first limit order serves 20 percent of the incoming order (800 shares) and the second serves the remaining 80 percent (3,200 shares). Other tie-breaking rules can also be considered. For instance, the market order could be assigned randomly among all the limit orders at the same price.

Priority rules affect traders’ bidding behavior and thereby market liquidity. To see this, we compare the LOB with time priority and that with pro-rata **(p.208)** allocation. Let Y_k^r be the cumulative depth at price A_k under the pro-rata allocation rule. Using equation (6.8), the aggregate expected profit of all the limit orders posted at price A_k is

(6.22)

$$\int_{Y_{k-1}^r}^{Y_k^r} \Pi_k(y)\,dy = \int_{Y_{k-1}^r}^{Y_k^r} \{P(y)\,[A_k - \mathrm{E}(v|q \geq y)] - C\}\,dy.$$

Under the pro-rata rule, this expected profit is shared among all the investors with a limit order at price A_k in proportion to the size of each one’s order. Thus, a competitive equilibrium is reached when the cumulative depth at each price in the LOB satisfies:

(6.23)

$$\int_{Y_{k-1}^r}^{Y_k^r} \Pi_k(y)\,dy = 0 \forall k.$$

Otherwise, it would be profitable either to add limit orders to the book (to get a fraction of the aggregate expected profit at price A_k) or to withdraw limit orders from the book (if the aggregate expected profit is negative).

As $\Pi_k(\cdot)$ is a decreasing function, the zero profit condition (6.23) implies that $\Pi_k\left(Y_k^r\right) \langle 0$. But recall that under time priority, the cumulative depth at price A_k, which we denote here by Y_k^t, satisfies $\Pi_k\left(Y_k^t\right)=0$. Hence,

(6.24)

$$Y_k^r \rangle Y_k^t.$$

Thus, the market is always deeper under pro-rata allocation, holding the distribution of market order flow constant (see exercise 3).

This result assumes that the probability distribution of the market order size does not change when the priority rule is changed. Actually, liquidity demanders are likely to submit larger orders when the book is deeper. In section 6.2.2 such a shift in the distribution of uninformed market orders results in larger cumulative depth. Thus, with an endogenous distribution for market order size, the positive effect of the pro-rata allocation rule on market depth would be even stronger. However, the effect is less clear-cut with informed trading, since informed traders' strategies and the depth of the LOB are jointly determined (see section 6.2.3). Thus, the net liquidity effect of a change in the priority rule (or, more generally, any change in market design) ultimately requires an analysis of the reaction of both limit order traders and informed traders. Exercise 2 provides an example.

Last, one may wonder why time priority is used in real markets if the prorata allocation rule results in a more liquid LOB. As with tick size, a broader perspective is required. In a dynamic setting, time priority rewards liquidity suppliers who are quick to replenish the book when it is depleted. This effect is not considered in the model. Also, exchanges may want to guarantee some minimal profit for limit order traders to encourage limit orders. For instance, **(p.209)** some exchanges give rebates to limit order traders in case of execution, which shows that they see the profitability of limit orders as critical to their success. Indeed, if the profitability of a limit order is too low compared to its cost or the potential gain from alternative strategies (e.g., placing market orders), the liquidity of the LOB could dry up, endangering the viability of the limit order market.

6.3.3 Hybrid LOB Markets

In limit order markets, trading platforms sometimes assign one or several market makers to a stock. In this case, liquidity is provided both by the LOB and by the designated market makers' offers. In some exchanges (like Euronext), the designated market makers simply post additional limit orders in the LOB, under the constraint that their spread not exceed a threshold set by contract with the exchange or the issuer.[7] In other exchanges, like the NYSE or the Frankfurt Stock Exchange, designated market makers can make offers contingent on the exact size of incoming market orders. That is, instead of providing liquidity ex ante (before observing the size of the incoming market orders) like regular limit order traders, they can also do so ex post (after observing the size of the orders).

The NYSE is a good example of this arrangement. Liquidity is provided by limit order traders, floor brokers, and the specialist in the stock. The specialist and the floor brokers are physically present on the floor of the NYSE. The specialist acts as a market maker since he can fill incoming market orders from his own portfolio. Floor brokers transmit orders from their clients to the specialist for execution against limit orders or the specialist's own quotes. Sometimes, they also trade for their own account. The participation rate of both the specialist and floor brokers has declined sharply in recent years, as traders off the floor increasingly bypass floor traders by routing limit or market orders electronically.

The key question is whether a hybrid structure combining a specialist and a LOB improves market liquidity over the pure limit order market. To study this, we extend the model developed in section 6.2.3 as described in figure 6.7. After the arrival of a market order, one market maker–the "specialist"–chooses a "stop-out price" $p^S(q)$ at which he is willing to execute all or part of the market order. This price depends on the direction and size of the market order, since unlike limit order traders the specialist sets his stop-out price *after* the market order arrives. **(p.210)**

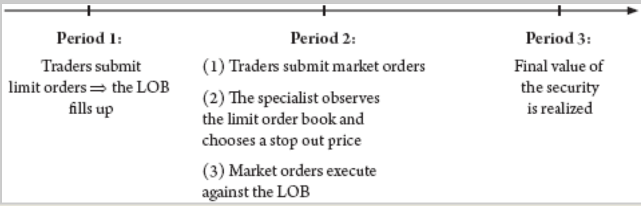


Figure 6.7. Timing with a specialist

By exchange regulations, the specialist yields both price and time priority to the book. Say the LOB displays one thousand shares at $A_1 = 100$ and six hundred shares at $A_2 = 100.1$. A buy market order for two thousand shares arrives and the specialist decides to "stop" the order out at$p^S(2000) = 100.1$. In this case, the buy order executes first against the limit orders at $A_1 = 100$ and $A_2 = 100.1$, after which the specialist fills the order for the remaining four hundred shares at$p^S = 100.1$.

At first glance, it seems that the specialist's intervention could only enhance market liquidity since he expands that offered by the LOB. But this thesis neglects the impact of the specialist on the bids of limit order traders. In the presence of asymmetric information, the specialist's ability to condition his intervention on the exact size of the market order allows him to cream-skim orders with little information content (small orders). This last-mover advantage reinforces the exposure of limit orders to informed trading and so impairs liquidity.

To see this, let us revisit example 2 in section 6.2.3. As noted, in the equilibrium described in this example, small market orders (of size qS) are submitted only by uninformed investors. The specialist can exploit this in the following way. When a small order arrives, he stops it out by slightly undercutting the prices offered by limit order traders for an order of this size (that is, at $A(qS)$ or $B(qS)$ depending on the direction of the order); when a large order arrives, he refrains from trading. This strategy yields a positive expected profit amounting to

$$\frac{(1-\pi)\phi}{2}\left(A(qs)-\mu\right)+\frac{(1-\pi)\phi}{2}\left(\mu-B(qs)\right)=\ (1-\pi)\pi\phi\sigma\Bigg\rangle 0.$$

Intuitively, by cherry-picking the small orders, the specialist captures the compensation required by limit order traders for their exposure to informed trading without being exposed himself. The specialist can do this because, unlike limit order traders, he can condition his intervention on the exact size of the market order.

(p.211) But then the equilibrium described in example 2 unravels. Indeed, in this case, limit orders at the top of the book execute if and only if a market order for at least qL shares arrives. Hence, they no longer break even at price $A(qS)$.

Instead, the equilibrium of the hybrid market can be readily determined as follows. If the investor arriving at time 1 is informed, he submits an order of at least qL shares and the LOB displays the following prices:

(6.25)

$$A^h(Y)=\begin{cases}\mu+\dfrac{\pi}{\pi+\ (1-\pi)\ (1-\phi)}\sigma & \text{for } Y\leq qL, \\ \mu+\sigma & \text{for } Y\rangle qL,\end{cases}$$

(6.26)

$$B^h(Y)=\begin{cases}\mu-\dfrac{\pi}{\pi+\ (1-\pi)\ (1-\phi)}\sigma & \text{for } Y\leq qL, \\ \mu-\sigma & \text{for } Y\rangle qL.\end{cases}$$

Moreover, at time $t = 1$, the specialist stops small market orders out by slightly improving on the best quotes in the book ($A(qL)$ or $B(qL)$ depending on whether the market order is a buy or a sell). If a large market order arrives, the specialist stays put.

In equilibrium the difference between the quotes at the top of the book is thus now:

(6.27)

$$A^h(0)-B^h(0)=\frac{2\pi\sigma}{\pi+\ (1-\pi)\ (1-\phi)}\Bigg\rangle 2\pi\sigma.$$

The quoted spread in the hybrid market is strictly larger than in the pure limit order market. Indeed, the specialist's intervention has a perverse effect. It increases limit order traders' exposure to informed trading. As the spread on large orders is identical in the two market structures, all liquidity demanders are either unaffected or worse off in the hybrid limit order market: in the pure limit order market, small and large market buy orders execute wholly or at least partially against the limit orders at at $A(qS)$, while in the hybrid, all market orders execute at a higher price, namely $A(qL)$.

To sum up, the hybrid market has the drawback of creating a last-mover advantage for the specialist, aggravating limit order traders' adverse selection problem, and so reducing the liquidity of the LOB. In our analysis, only the specialist benefits from the last-mover edge, which dramatically increases trading costs for small market orders since the specialist can charge very high prices for them. In reality, on the NYSE, the specialist faces competition from floor brokers. Exercise 4 shows that competition among multiple specialists (or between the specialist and floor brokers) alleviates the problem raised here, but does not eliminate it. Such competition reduces trading costs for small market orders compared to the case of the single specialist, but trading costs for large market orders remain higher in the hybrid than in the pure limit order market. **(p.212)**

Box 6.1 Flash Orders

The analysis in this section is relevant to the recent controversy over flash orders in U.S. equity markets. With the development of algorithmic trading, some electronic trading platforms allowed investors to submit flash orders (or "step up orders"). These are almost always market orders. In the United States, market orders must be executed at the NBBO or national best bid and offer price (i.e., the best in all competing trading venues for a stock). So a platform that receives a buy market order and does not have the best ask price, must reroute the buy order to the platform that posts the best ask (see Chapter7, section 7.5.1). If it is a flash order, though, before doing so the platform will show ("flash") the order to its participants at the national best offer for flash orders to buy, and the national best bid for flash orders to sell. Market participants that receive the flashed order information then have a very brief period (generally less than a second) to respond with their own order to execute against the flashed order at a price that matches the nationwide best quote. As the permitted response time is fleeting, only automated trading systems (algorithmic traders) have a chance to execute the flashed order. Flashed orders are a way for a platform to avoid rerouting their order flow to competing platforms. But in effect they give a last-mover advantage to the traders who can actually respond to flashed orders. The model described in this section suggests that this advantage may reduce limit order traders' incentives to post good quotes.[8] In fact, this problem was pointed out by a number of market participants. For instance, after Nasdaq and BATS requested authorization from the SEC for flash orders, Morgan Stanley said the proposed rule changes "will provide a material disincentive to publicly display limit orders on exchanges, thereby impairing price discovery."[9] To fix this problem, the SEC is now considering a "trade at" rule, which would prohibit a trading venue (e.g., an ECN) from executing a trade at the best offer or bid prices unless it was already quoting these prices when the order was received.

The analysis to this point paints a gloomy picture of hybrid systems. In fairness, let us also note some of their potential advantages. Glosten (1989) **(p.213)** shows that a market with a monopolist specialist can achieve better risk sharing in the presence of asymmetric information. More generally, specialists (or designated market makers) may help to guarantee a minimum level of liquidity when there are sudden liquidity shortages. Goldstein and Kavajecz (2004) study the contribution of each source of liquidity on the NYSEto total liquidity during an episode of high volatility, 27–28 October, 1997. On October 27, the Dow Jones Industrial Average (DJIA) fell by 5.54 percent, regaining 3.37 percent the next day. For the index's constituent stocks, Goldstein and Kavajecz (2004) find a sharp decline in the liquidity provided by limit orders on 28 October: the difference between the best ask price and the best bid price in the LOB for these stocks was six cents at the close on October 27 but rose to about three dollars the next day. And cumulative depth throughout the book was significantly less. Nevertheless, the quoted spreads were not significantly larger and quoted depth not significantly smaller on October 28 than on the previous day. This indicates that on the 28th, the specialists and floor brokers replaced the LOB as the main source of liquidity provision. Overall, these findings suggest that a limit order market might be more vulnerable than a hybrid market to liquidity risk (a drain in liquidity at the time of large swings in market value).

Moreover, in our analysis, limit order traders obtain zero expected profit. This implicitly assumes that there are many liquidity suppliers, but in reality, the participation of limit order traders in some stocks may be limited, resulting in noncompetitive offers. In this case, the intervention of designated market makers or even just the threat of it can oblige limit order traders to post better offers.

Finally, the problems associated with the last-mover advantage for designated market makers do not arise in systems in which market makers undertake to provide liquidity (like Euronext). Here designated market makers can help to smooth price movements by guaranteeing a minimal level of liquidity, as section 10.5.2 of Chapter 10 demonstrates.

6.4. The Make or Take Decision in Lob Markets

A key feature of limit order markets is that traders can choose whether to act as makers or takers (either post or hit a limit order). These markets operate well when some investors are makers and others are takers. As we shall see here, the make or take decision hinges on the trade-off between price improvement and exposure to non-execution and pick-off risk. In section 6.4.1, we formalize this trade-off. In sections 6.4.2 and 6.4.3, we study how the risk of non-execution and the risk of being picked off affect the bid-ask spread and its relationship with volatility. In section 6.4.5, we analyze how the state of the LOB (the number of shares offered on the bid and ask sides) affects traders' choices between market and limit orders.

(p.214) 6.4.1 Risk of Being Picked Off and Risk of Non-Execution

We consider the market for a security. Its final payoff v_T is realized at time T and is equal to

(6.28)

$$vT = \mu_0 + \sum_{t=1}^{t=T} \varepsilon_t,$$

where ε_t is a zero-mean innovation of which market participants are informed at the beginning of period t; ε_t is equal to $+\sigma$ ("good news") or $-\sigma$ ("bad news") with equal probability. The final payoff time, T, is random. Specifically, in each period, there is a probability $1 - \tau$ that this payoff is realized and the market for the asset is then closed. The parameter τ is an inverse measure of the participants' eagerness to clinch the trade: the further τ is below 1, the more impatient they are.

At time t, if the asset has not yet paid off, the expected value of the security— its "fair value"—is:

(6.29)

$$v_t = \mathrm{E}_t\left(vT\right) = \mu_0 + \sum_{k=1}^{k=t} \varepsilon_k = v_{t-1} + \varepsilon_t.$$

Thus, σ is the per-period volatility of the security's fair value.

After the realization of each ε_t, a new trader comes to the market. Traders have different valuations of the stock. The utility of trader i if he trades $q \in \{+1, 0, -1\}$ shares at price p is:

$$U\left(q, y_{i,p}\right) = q\left(v_T + Y_i - p\right).$$

Parameter y_i is specific to each trader; it represents his "private value" from trading. It takes one of two values, $+L$ or $-L$, with equal probability. Intuitively, traders with a large gain from holding the security ($y_i = +L$) should buy from those with a low valuation ($y_i = -L$).[10]

The trader arriving at time t observes v_t and the state of the LOB. Given this information and his private valuation, the trader must choose whether to buy **(p.215)** or sell, and whether to place a limit order or a market order. The market order executes immediately at the best quotes available.

In this setting, traders submitting limit orders are exposed to both the risk of being picked off and the risk of non-execution. To see this, consider a trader who submits, at time t, a buy limit order at bid price B valid for only one period. If the trader is of type y_i, his expected utility with this order is

(6.30)

$$\mathrm{E}_t\left[U\left(q_{t+1}, y_i, B\right)\right] = \mathrm{E}_t\left[\left(v_T + y_i - B\right) \cdot q_{t+1}\right],$$

where $q_{t+1} = 1$ if the order executes and $q_{t+1} = 0$ otherwise. Let $P_t(B)$ be the execution probability. Then we can rewrite (6.30) as:
(6.31)

$$\begin{aligned} \mathrm{E}_t\left[U\left(q_{t+1}, y_i, B\right)\right] &= \mathrm{E}_t\left[\left(v_T - v_t + y_i - B\right) \cdot q_{t+1}\right] \\ &= \left[\left(v_t + y_i - B\right) + \mathrm{E}\left(\varepsilon_{t+1} \middle| q_{t+1} = 1\right)\right] P_t(B). \end{aligned}$$

The first component $(v_t + y_i - B)$ is what the trader would gain with certain execution of his limit order if execution were uncorrelated with changes in the expected value of the security, so that $E(\varepsilon_{t+1} \mid q_{t+1} = 1) = E(\varepsilon_{t+1}) = 0$. His expected utility is less than this, however, because there is both a risk of non-execution $(P_t(B) < 1)$ and of being picked off (the trader's buy limit order is more likely to be filled when $\varepsilon_{t+1} < 0$ than when $\varepsilon_{t+1} > 0$, so that $E(\varepsilon_{t+1} \mid q_{t+1} = 1) \leq 0$).

To illustrate these effects, consider a numerical example with $\sigma = 0.7$ and $L = 1$. A trader arrives at time t when $v_t = 100$. The LOB at time t has a single sell limit order at $A_t = 100.1$. If the trader submits a buy market order, his expected utility is:

$$\mathrm{E}_t\left[U(1, 1, 100.1)\right] = 101 - 100.1 = 0.9.$$

Now, suppose that instead he submits a buy limit order at $B = 99.9$. The expected utility from this order depends on whether it executes and on the evolution of the fair value of the security. There are four possible events at time $t + 1$:

1. Public information is good ($\varepsilon_{t+1} = +0.7$) and the trader arriving at time $t + 1$ submits a sell market order. The expected utility of the trader who arrived at time t is:

$$\mathrm{E}_{t+1}\left[U(1, 1, 99.9)\right] = 100.7 + 1 - 99.9 = 1.8.$$

2. Public information is good ($\varepsilon_{t+1} = +0.7$) and the trader arriving at time $t + 1$ submits a limit order. In this case the trader who arrived at time t does not trade, obtaining expected utility of E_{t+1} $[U(0, 1, 99.9)] = 0$.
(p.216) 3. Public information is bad ($\varepsilon_{t+1} = -0.7$) and the trader arriving at time $t + 1$ submits a sell market order. The expected utility of the trader who arrived at time t is then:

$$\mathrm{E}_{t+1}\left[U(1, 1, 99.9)\right] = 99.3 + 1 - 99.9 = 0.4.$$

4. Public information is bad ($\varepsilon_{t+1} = -0.7$) and the trader arriving at time $t + 1$ submits a limit order. The trader who arrived at time t does not trade and gets zero.

Let φ be the probability of an increase in the value conditional on execution, that is, the probability of event 1 relative to the total probability of execution (events 1 and 3). Then, at time t, the expected utility from the limit order is

$$\begin{aligned}\mathrm{E}_t\left[U\left(q_{t+1},1,99.9\right)\right] &= \left\{(v_t+y_i-B)+\mathrm{E}_t\left(\varepsilon_{t+1}|q_{t+1}=1\right)\right\}P_t(B)\\ &= \{1.1+[0.7\varphi-0.7(1-\varphi)]\}P_t(B)\\ &= [1.1+0.7(2\varphi-1)]P_t(B)= (0.4+1.4\varphi)P_t(B),\end{aligned}$$

If φ ‹ 0.5, we have $E(\varepsilon_{t+1} \mid q_{t+1} = 1)$ ‹ 0 because execution is more likely if the fair value goes down than if it goes up. This effect clearly reduces the expected gain from the limit order. Thus, the lower is φ, the higher is the investor's risk of being picked off.

The limit order at 99.9 dominates a market order if:

$$\mathrm{E}_t\left[U\left(q_{t+1},1,99.9\right)\right] \geq 0.9.$$

Thus, the decision on order type depends on φ and P_t. For instance, if P_t is large and φ is close to 1/2, then a limit order at 99.9 dominates the market order. But suppose instead that the limit order is only executed when the news is bad at time $t + 1$ and that the next trader has a low private valuation: $P_t = 0.25$ and φ =0. In this case, the trader's expected utility with a limit order at 99.9 is:

$$\mathrm{E}_t U\left(q_{t+1},y_i,B)\right) = (1.1-0.7).0.25=0.1,$$

implying that he would be better off with a market order.

Of course, there may be another limit order at a different price that yields a larger expected utility, so that eventually the trader will submit a limit order. Finding the optimal limit order requires knowledge of the execution probability and the probability φ of good news conditional on execution associated with each possible limit order. These probabilities cannot be exogenously specified, since they depend on future traders' anticipated order choices in each possible future state of the market (defined at any given time by the type of trader arriving, the fair value of the security, and the offers present in the LOB). In a **(p.217)** rational expectations equilibrium, the probabilities must be consistent with the order submission strategies that are used in equilibrium.

For this reason, it is difficult to solve for the equilibrium when traders choose between market and limit orders.[11] In particular, the number of possible future states is very large if limit orders can stay on the LOB for many periods. To simplify the analysis, let's assume that all limit orders expire after one period, as in the last example. In sections 6.4.2 and 6.4.3, we show how traders' optimal order submission strategies can be derived in this case. As we shall see, this analysis affords insight into the impact of execution risk and volatility on liquidity in limit order markets.

Box 6.2 Soesbandits

The risk of being picked off stems from the fact that traders do not monitor their limit orders continuously and/or react too slowly to the arrival of value-relevant news.

Hence, they may be too slow in updating their quotes when new information emerges. Market makers too are exposed to this risk. The controversy in the 1990s over Nasdaq's small order execution system (SOES) provides a good illustration. SOES was introduced by Nasdaq to provide automatic execution of small orders (say, less than five hundred shares) against dealers' quotes. Hence SOES also allowed traders who were quick in reacting to news to pick off Nasdaq dealers who were slow to adjust their quotes. By contrast, for larger orders, dealers had to first confirm that they were willing to execute, which gave them time to first check that their quotes were in line with current available information (e.g., other dealers' quotes). Thus, although initially intended for retail investors, SOES quickly became a trading tool for professional traders. As they were inflicting losses on market makers by picking off their quotes, the latter used to call them "SOES bandits," and maintained that the practice forced them to quote larger spreads.

6.4.2 Bid-Ask Spreads and Execution Risk

We first solve for the equilibrium of the limit order market in the simplest case, when the security is riskless ($\sigma = 0$), so traders are exposed only to non-execution risk. An example would be a risk-free security, such as a high-grade **(p.218)** government bond. In fact, bonds are increasingly traded in electronic limit order markets (e.g., eSpeed or BrokerTec are electronic limit order markets for on-the-run U.S. treasuries).

If a trader of type y_i trades q shares at price p, he obtains:

$$U(q, y_i, p) = q\left(\mu_0 + y_i - p\right).$$

For instance, suppose the trader sells one share (i.e., $q = -1$). This gives a payoff equal to $p - (\mu_0 + y_i)$, which is positive if and only if $p \rangle \mu_0 + y_i$ i.e., if the sale price is higher than the trader's valuation of the asset. As this valuation is always greater than $\mu_0 - L$, no trader will submit a sell order if the transaction price is below $\mu_0 - L$. By symmetry, no trader submits a buy order at a price above $\mu_0 + L$. Therefore, when $\sigma = 0$, we can restrict our attention to the case in which quotes are in the range $[\mu_0 - L, \mu_0 + L]$. For prices in this range, traders of type $+L$ submit only buy orders and those of type $-L$ submit only sell orders. Hence, in what follows, we study only the optimal buy orders for traders of type $+L$ and sell orders for traders of type $-L$.

Let $\widehat{A_t}$ be the maximum ask price that a trader of type $+L$ arriving at time t is willing to take using a buy market order. The trader's strategy is:

$$\text{submit a buy matket order if } A_t \leq \widehat{A_t},$$
$$\text{submit a buy limit order at price } B_t^* \text{ if } A_t \Big\rangle \widehat{A_t}.$$

Similarly, let $\widehat{B_t}$ be the minimal bid price that a trader of type $-L$ arriving at time t is willing to take with a sell market order. The strategy is:

$$\text{submit a sell matket order if } B_t \geq \widehat{B_t},$$
$$\text{submit a sell limit order at price } A_t^* \text{ if } B_t \Big\langle \widehat{B_t}.$$

Suppose that a (high-valuation) investor submits a limit order to buy at bid price B at time t and recall that the successor trader is a low-valuation type with probability $\tau/2$. Given the above strategies for the successor at time $t + 1$, the execution probability P_t of the limit order is:

$$P_t(B) \quad = 0 \text{ if } B \Big\langle \widehat{B}_{t+1},$$
$$P_t(B) \quad = \frac{\tau}{2} \text{ if } B \geq \widehat{B}_{t+1},$$

Hence, if the trader opts for a buy limit order, the optimal bid price is $B_t^* = \widehat{B}_{t+1}$, because a lower bid will not execute, and a higher bid yields no improvement in execution probability. Thus, the highest expected utility from a buy limit order is

$$\mathrm{E}_t \left[U \left(q_{t+1}, +L, \widehat{B}_{t+1} \right) \right] = \frac{\tau}{2} \left(\mu_0 + L - \widehat{B}_{t+1} \right).$$

(p.219) Therefore, the cut-off ask price $\widehat{A}_t$ that makes the buyer indifferent between a buy market order and a buy limit order solves:

(6.32)

$$\mu_0 + L - \widehat{A}_t = \frac{\tau}{2} \left(\mu_0 + L - \widehat{B}_{t+1} \right)$$

The argument for a low-valuation seller is analogous. The seller's optimal ask price is $A_t^* = \widehat{A}_{t+1}$, and the cut-off bid price $\widehat{B}_t$ that makes the seller indifferent between a sell market order and a sell limit order solves:

(6.33)

$$\widehat{B_t} - \left(\mu_0 - L \right) = \frac{\tau}{2} \left(\widehat{A}_{t+1} - \left(\mu_0 - L \right) \right).$$

As the parameters (τ, μ_0, L) are time-invariant, traders face exactly the same problem in each period. Hence, it is natural to focus on steady-state equilibria in which traders choose strategies that do not depend on time, that is:

(6.34)

$$\widehat{B_t} = \widehat{B} \text{ and } \widehat{A_t} = \widehat{A} \quad \forall t.$$

We can now easily compute traders' equilibrium bids, offers and cut-off prices by solving equations (6.32), (6.33), and (6.34). We obtain:

(6.35)

$$A^* = \widehat{A} = \mu_0 - L + \frac{4L}{2+\tau},$$

(6.36)

$$B^* = \widehat{B} = \mu_0 + L - \frac{4L}{2+\tau}.$$

As τ increases, execution risk decreases. There are two possible reasons why a limit order may not be taken: either trading is terminated (probability $1 - \tau$), or the next period's trader's private valuation is the same as the current trader's (probability $\tau/2$), so there is a non-execution probability $1 - \tau/2$. Thus, in this model parameter τ measures execution risk, which is a key determinant of the bid-ask spread since:[12]

$$A^* - B^* = 2\left(\frac{2-\tau}{2+\tau}\right)L.$$

The bid-ask spread is decreasing in the parameter τ, and thus increasing in execution risk. Indeed, as τ rises, the cost of not trading immediately upon arrival with a market order becomes smaller because the likelihood of a limit order being filled increases. Thus, in equilibrium, limit orders must be more aggressive if they are to attract counterparties; accordingly ask and bid prices become more aggressive as τ gets larger $\left(\frac{\partial A_t^*}{\partial \tau} \leq 0 \text{ and } \frac{\partial B_t^*}{\partial \tau} \geq 0\right)$.

(p.220) This effect of execution risk on traders' order placement strategies can explain the fact that in real-world markets, bid-ask spreads widen at the end of the trading day. Indeed, as the end of the day nears, the likelihood that limit orders will remain unfilled gets higher. This prospect increases market makers' power, as traders are willing to pay greater price concessions to avoid losing a trading opportunity.

6.4.3 Bid-Ask Spreads and Volatility

Now we turn to the more complex case in which the security is risky so that $\sigma \rangle 0$. Let $\widehat{B}_{t+1}\left(y_{t+1}, v_{t+1}\right)$ be traders' sell cut-off price at time $t+1$. Intuitively, $\widehat{B}_{t+1}(\cdot)$ increases with v_{t+1} and y_{t+1}, other things equal. But it is not clear a priori whether $\widehat{B}_{t+1}(-L, v_t + \sigma)$ is higher or lower than $\widehat{B}_{t+1}(+L, v_t - \sigma)$. To fix things, suppose that

(6.37)

$$\widehat{B}_{t+1}(-L, v_t - \sigma) \left\langle \widehat{B}_{t+1}(-L, v_t + \sigma) \right\langle \widehat{B}_{t+1}(+L, v_t - \sigma) \left\langle \widehat{B}_{t+1}(+L, v_t + \sigma) \right.$$

Under this conjecture, which will be verified in equilibrium, figure 6.8 shows the execution probability of a buy limit order at price B submitted at time t. For example, a buy limit order with a low bid price in the interval $\left[\widehat{B}_{t+1}(-L, v_t - \sigma), \widehat{B}_{t+1}(-L, v_t + \sigma)\right)$

executes if and only if the next trader is of type $-L$ and $\varepsilon_{t+1} = -\sigma$. This happens with probability $\frac{\tau}{4}$.

A trader who places a buy limit will necessarily lose money if it is hit by traders with a high private valuation, because the latter are better informed when they

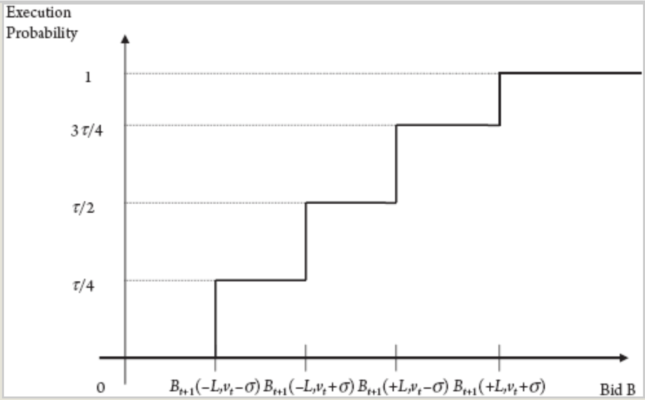


Figure 6.8. Bid price and execution probability

(p.221) place their market orders (they observe v_{t+1} before making their decision) and always value the security at least as high as the first trader. Thus, if these counterparties are willing to sell at the limit price, this price necessarily exceeds the trader's own valuation. This argument rules out any buy limit orders at prices of $\widehat{B}_{t+1}\left(+L, v_t - \sigma\right)$ or higher, because the resulting increased execution probability is achieved solely by attracting high-private-valuation counterparties in addition to those who would be willing to take a lower bid.

Thus a trader of type $+L$ who places a buy limit order must choose one of only two prices: a conservative low price B_l with a low execution probability of $\frac{\tau}{4}$

$$B_l \equiv \widehat{B}_{t+1}\left(-L, v_t - \sigma\right),$$

and a more aggressive price B_h with a greater execution probability $\frac{\tau}{2}$,

$$B_h \equiv \widehat{B}_{t+1}\left(-L, v_t + \sigma\right).$$

Intuitively, the trader faces a trade-off between paying less and having a higher probability of execution. The optimal choice depends on volatility. In equilibrium, traders choose the low-execution-risk strategy if volatility is sufficiently low, so that the utility loss when the asset value drops remains small. Specifically, in equilibrium, traders of type $+L$ adopt the following bidding strategies:

$$\text{post a buy limit order at } B_h = v_t + L - \frac{2\left(2L - \sigma\right)}{2 + \tau} if \sigma \leq \widehat{\sigma},$$

$$\text{post a buy limit order at } B_l = v_t + L - \sigma - \frac{8L}{4 + \tau} if \sigma \Big\rangle \widehat{\sigma},$$

where the threshold volatility $\widehat{\sigma}$ is defined by:

$$\widehat{\sigma} = \frac{4L}{4+\tau}.$$

Similarly, traders of type $-L$ post the following quotes in equilibrium:

$$\text{post a sell limit order at } A_h = v_t - L + \frac{2\,(2L-\sigma)}{2+\tau} \text{ if } \sigma \leq \widehat{\sigma},$$
$$\text{post a sell limit order at } A_l = v_t - L + \sigma + \frac{8L}{4+\tau} \text{ if } \sigma \Big\rangle \widehat{\sigma},$$

If $\sigma = 0$, the expressions for bid and ask prices are identical to those obtained in section 6.4.2. The method of proof is the same as that used for this simpler case, as explained in the Appendix to this chapter. **(p.222)**

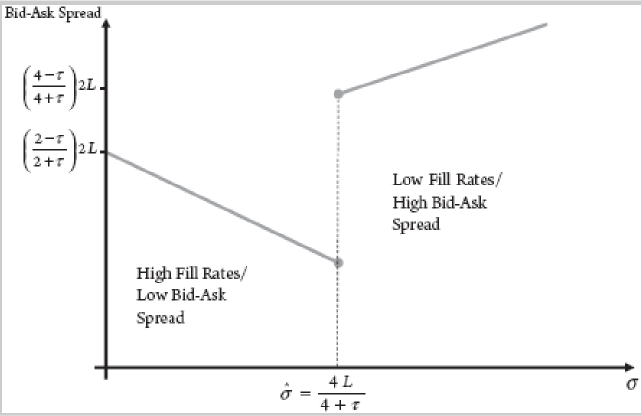


Figure 6.9. Impact of volatility on the bid-ask spread

The bid-ask spread in equilibrium is:

$$A_h - B_h \quad = 2\frac{2-\tau}{2+\tau}L - \frac{4}{2+\tau}\sigma \text{ if } \sigma \leq \widehat{\sigma},$$
$$A_i - B_l \quad = 2\frac{4-\tau}{4+\tau}L + 2\sigma \text{ if } \sigma \Big\rangle \widehat{\sigma}.$$

Figure 6.9 shows the spread as a function of volatility, σ. Interestingly, in contrast to most other models of bid-ask spreads, the effect of volatility here is non-monotonic. The reason is that an increase in σ raises the cost of being picked off, but also reduces the market power of those who submit limit orders.

To see this, recall that when $\sigma \leq \widehat{\sigma}$, investors optimally choose limit orders with a high execution probability, i.e. that execute whether the asset value goes down or up. Now an increase in volatility raises the lowest price (e.g., $\widehat{B}_{t+1}\left(-L, v_t + \sigma\right)$)required by investors to sell when the value goes up. As a result, if a buyer wants to retain a high execution probability, he must submit limit orders at increasingly high prices as volatility increases.

By symmetry, a seller must submit limit orders at increasingly low prices. In other words, an increase in σ reduces the market power of those submitting limit orders. Thus, when $\sigma \leq \widehat{\sigma}$ the bid-ask spread declines as volatility increases, and as a result, limit order investors obtain less and less surplus.

At $\sigma = \widehat{\sigma}$, this surplus is so small that it becomes more profitable for limit order traders to submit less aggressive quotes at the cost of a lower execution probability. As noted earlier, such quotes only execute when there is bad news. Hence, they are shaded by the amount of the adverse movement that triggers their execution, so when $\sigma \rangle \widehat{\sigma}$ the spread is increasing in σ.

When $\sigma \leq \widehat{\sigma}$, limit orders have a higher execution probability than when $\sigma \rangle \widehat{\sigma}$. There are more market orders relative to limit orders, and the volume **(p.223)** of trading is higher. This means that markets where limit orders are relatively more prevalent ($\left(\sigma \rangle \widehat{\sigma}\right)$) are not necessarily more liquid. Indeed, such intensive use of limit orders may simply reflect the fact that traders find it too costly to submit a market order, as in the model for $\sigma \rangle \widehat{\sigma}$.

6.4.4 Indexed Limit Orders, Monitoring, and Algorithmic Trading

Pegged limit orders.

The risk of being picked off makes the market more illiquid. One solution to this problem is to peg limit orders to changes in the fair value of the security. That is, traders' limit orders can be automatically "repriced" when there is a change in the fair value of the security, and so protected against pick-off risk. The equilibrium is therefore identical to the case in which $\sigma = 0$.[13]

In practice, some trading platforms (e.g., Chi-X in Europe and the International Securities Exchange in the United States) allow indexed limit orders. Since market organizers do not know the fair value of the security, they peg the prices of indexed limit orders to the price of some other, related security (e.g., an index) or to the quotes posted for the same security on other platforms.[14] But the protection offered by this form of indexing is imperfect and does not fully eliminate the risk of limit orders being picked off.

Monitoring limit orders.

Traders can also mitigate pick-off risk by monitoring. For instance, if bad news arrives a buyer can quickly cancel his limit order and resubmit it at a lower price, and similarly in case of good news he can resubmit it at a higher price to keep a high execution probability. If this monitoring is costless, then we are back to the case where $\sigma = 0$. In practice, however, monitoring is not costless (it takes work) and trading platforms sometimes charge a fee for order cancellations.

The cost of monitoring limit orders has declined considerably in recent years, as traders increasingly automate their routing decisions to get better protection against pick-off and non-execution risk. Traders also invest to reduce trading latency, the time it takes for their messages (e.g., a request to cancel an order) to be processed by the exchange. For instance, they pay fees to have their servers close to trading platforms (a practice known as colocation). However, automation can also be used to pick off stale limit orders more quickly. Thus, **(p.224)** its net effect on liquidity depends on whether it is a greater help to those who seek protection against the risk of being picked off or to those who want to pick off stale quotes.[15] Hendershott, Jones, and Menkveld (2011) show empirically that the development of algorithmic trading coincides with an improvement in liquidity and a much higher rate of cancellations. This suggests that algorithmic trading may have helped liquidity providers to protect themselves better against the risk of being picked off.

Box 6.3 Limit Orders As Free Options

There is an interesting analogy between options and limit orders. For instance, submitting a sell limit order is similar to writing an American call option: it enables investors to buy a security at a “strike” price equal to the ask price of the limit order.[16] Market participants want to exercise these options when they are “in the money,” that is, when the price attached to a sell limit order is below the fair value of the security.

The options implicit in limit orders are “free”, since the investors who submit them receive no payment for “writing” (selling) them. Yet, these options can be profitable because, unlike actual options, limit orders sometimes execute even when they are “out the money,” simply because some investors (liquidity traders) need to buy the security (say, to hedge), even in the absence of solid information about its value. Limit order traders must therefore price their orders to balance the expected loss when the implied option goes into the money and is exercised against the expected gain when it is exercised out of the money.

This balancing act is very similar to that of market makers who compensate for the cost of trading with better informed investors with the revenue from dealing with liquidity traders, as Chapter 3 analyzes (see Copeland and Galai, 1983). There are some significant differences, though: the exposure to pick-off risk also depends on how fast the trader can access the market and how intensively he monitors his quotes. Thus, the liquidity effect of pick-off risk is linked to investment decisions in routing and monitoring technologies. Moreover, competition between traders seeking to pick off limit orders forces them to race to do so as soon as the option is even slightly in the money, thus endogenously forcing accelerated exercise of the option and limiting its potential value.

(p.225) 6.4.5 Order Flow and the State of the LOB

In the previous sections we have analyzed how execution risk and pick-off risk affect the bid-ask spread, the composition of the order flow, and order aggressiveness. However, the model remains too simple to illuminate the relationship between the depth of the LOB and order flow. In particular, it does not allow us to study how variations in depth affect the aggressiveness of traders' orders. Yet intuitively, the depth of the book on both sides should affect order placement strategies.

Consider a buyer. If a large number of shares are offered at the best bid, then the likelihood of quick execution with a limit order is low. In this case, the buyer is more likely to submit a limit order that improves upon the best bid price, or even a market order. That is, an increase in depth on the buy side of the book makes aggressive buy orders more likely.

Similarly, sellers are more likely to submit a market order when the LOB is deep at the best ask price. Hence, when the number of shares offered at the best ask price increases, the execution probability of buy limit orders improves, other things equal. That is, an increase in the depth on the sell side of the LOB makes aggressive buy orders less likely.

In this section, we modify the model developed above to formalize these intuitions. Suppose there is no uncertainty about the time at which the asset pays off, and that we count time backward. That is, $t = 1$ denotes the last period of the trading day, $t = 2$ the penultimate period, and so on. For simplicity, consider only the last three periods. The results generalize to an arbitrary number of periods.

To simplify the analysis further, we assume that traders must position their limit orders on a grid with tick size Δ. The first quote on the grid below the fair value is $b = \mu_0 - \frac{\Delta}{2}$, the first above it is $a = \mu_0 + \frac{\Delta}{2}$. The tick size is such that

(6.38)

$$\frac{\Delta}{2} \Big\langle L \Big\langle \Delta.$$

Thus, investors of type $+L$ never trade at a price above a and investors of type $-L$ never trade below b. Submitting a limit order with a price outside the range $[b, a]$ is never optimal, since it has zero execution probability. As we shall see, several buy limit orders will queue at price b, several sell limit orders at a. Time priority is used to determine the sequence in which they are filled.

In addition, a market maker stands constantly ready to sell shares at a and buy shares at b. This market maker does not have priority of execution over limit orders placed at the same price. Thus, arriving traders face a simple choice. Consider a trader of type $+L$ for instance. Since he can always trade immediately at price a using a market order, his choice boils down to a market order filled at price a or a limit order to buy at b.

(p.226) If a trader of type $+L$ submits a buy market order at time t, he obtains

$$U(1,L,a) = L - \frac{\varDelta}{2}.$$

If he submits a buy limit order instead, he obtains

$$U(1,L,b) = P_{bt}(n_{bt},n_{at}) \left(L + \frac{\varDelta}{2}\right),$$

where $P_{bt}(n_{bt}, n_{at})$ is the execution probability of a buy limit order submitted at time t when the book has n_{bt} buy limit orders (each for one share) at price b and n_{at} sell limit orders (each for one share) at price a (not counting the shares offered by the market maker).

Thus, submitting a buy limit order is optimal if and only if

(6.39)

$$P_{bt}(n_{bt},n_{at}) \Bigg\rangle \frac{2L-\varDelta}{2L+\varDelta}.$$

We obtain the same decision rule for a seller. That is, a trader of type $-L$ optimally submits a sell limit order at price a if and only if

(6.40)

$$P_{st}(n_{bt},n_{at}) \Bigg\rangle \frac{2L-\varDelta}{2L+\varDelta},$$

where $P_{st}(n_{bt}, n_{at})$ denotes the execution of a sell limit order at price a at time t when the state of the book is $\{n_{bt}, n_{at}\}$.

Hence, the decision for a limit order depends on execution probability, which varies over time, insofar as non-execution gets more likely as the end of the trading day approaches, and the state of the book varies.

Equilibrium.

In equilibrium, the execution probability of a limit order at a given time in each state of the LOB depends on future traders' order placement decisions. In turn, these decisions depend on execution probabilities. At time $t = 1$, however, execution probabilities are zero for limit orders, since the trader arriving at this time is the last. Thus, we can solve for traders' optimal actions at each time by "backward induction": first, we derive traders' optimal decision at time $t = 1$; we obtain limit order execution probabilities at time $t = 2$, and using conditions (6.39) and (6.40), we infer traders' optimal orders in each state of the book. We can then proceed to time $t = 3$.

Time t = 1.

If the trader arriving at time $t = 1$ were to place a limit order, it would certainly not execute, since he is the last trader of the day. Thus, the trader submits a buy market order if he is type $+L$ and a sell market order if he is type $-L$.

Time t = 2.

Suppose that the trader arriving at $t = 2$ is a buyer ($y_i = +L$). Given the equilibrium actions of the trader arriving at time $t = 1$, the execution **(p.227)** probabilities are

$$
\begin{aligned}
P_{b2}(0, n_{a2}) &= \frac{1}{2}, \\
P_{b2}(n_{b2}, n_{b2}) &= 0 \text{ if } n_{b2} \geq 1.
\end{aligned}
$$

Notice that $\frac{2L-\Delta}{2L+\Delta} < \frac{1}{2}$ since $L < \Delta$. Thus, if $n_{b2} = 0$, condition (6.39) is satisfied and the investor optimally submits a buy limit order; otherwise, a buy market order. The optimal strategy of a seller is symmetric: a sell market order if $n_{a2} \geq 1$ and a sell limit order if $n_{a2} = 0$.

Time t = 3.

Finally, we derive the optimal strategy of the trader arriving at time $t = 3$. Again, suppose that this trader is a buyer. Given traders' optimal actions at times $t = 1$ and $t = 2$, we have:

$$
\begin{aligned}
P_{b3}(0, n_{a3}) &= \frac{3}{4} \text{ if } n_{a3} \geq 1, \\
P_{b3}(0, n_{a3}) &= \frac{1}{2} \text{ if } n_{a3} = 0, \\
P_{b3}(1, n_{a3}) &= \frac{1}{4} \text{ if } n_{a3} \geq 1, \\
P_{b3}(1, n_{a3}) &= 0 \text{ if } n_{a3} = 0, \\
P_{b3}(n_{b3}, n_{a3}) &= 0 \text{ if } n_{b3} \geq 2.
\end{aligned}
$$

Thus, the buyer arriving at time $t = 3$ submits a buy limit order at b if $n_{b3} = 0$. If $n_{b3} = 1$ and $n_{a3} > 1$, the optimal order placement strategy depends on L/Δ. If $\frac{L}{\Delta} \leq \frac{5}{8}$, then a buy limit order is optimal since its execution probability (1/4) is such that condition (6.39) is satisfied. Otherwise, a buy market order is optimal. In other states of the LOB, a trader of type $+L$ submits a buy market order, since a limit order has zero execution probability. The strategy of a seller is symmetric. Table 6.1 summarizes the optimal action for a buyer arriving at time $t = 3$ for each possible state of the book and the execution probability of a buy limit order in each state, when $\frac{L}{\Delta} \leq \frac{5}{8}$. The table shows the optimal order for a buyer arriving at time $t = 3$ ("lo" denoting limit order and "mo" market order) and the execution probabilities of a buy limit order at this time for each possible state of the LOB.

Order flow and state of the book.

The equilibrium obtained in this example has several interesting properties (see table 6.1). First, at each time the execution probability of a limit order decreases as the number of shares offered on that side increases (e.g., at time 3, $P_{b3}\left(0,1\right)=\frac{3}{4}\rangle P_{b3}\left(1,1\right)=\frac{1}{4}$), and it increases along with the number of shares on the opposite side (e.g., $P_{b3}\left(0,1\right)=\frac{3}{4}\rangle P_{b3}\left(0,0\right)=\frac{1}{2}$). These properties extend to any time. For instance, at all times, $P_{bt}(n_{bt}, n_{at})$ decreases in n_{bt} and increases in n_{at}. **(p.228)**

Table 6.1. Optimal Order Submission Strategy and Execution Probabilities for a Buyer At *t* = 3

nb3:	0	1	≥2
na3:			
0	(lo, $\frac{1}{2}$)	(mo,0)	(mo,0)
1	(lo,$\frac{3}{4}$)	(lo, $\frac{1}{4}$)	(mo,0)
2	(lo,$\frac{3}{4}$)	(lo,$\frac{1}{4}$)	(mo,0)
3	(lo,$\frac{3}{4}$)	(lo,$\frac{1}{4}$)	(mo,0)
≥4	(lo,$\frac{3}{4}$)	(lo,$\frac{1}{4}$)	(mo,0)

The first property follows from the fact that limit orders queuing at the same price execute in the sequence in which they were submitted (time priority). Hence traders are more likely to submit market orders when the LOB on their side is deeper. But this means that a limit order has a better chance of execution when the book on the other side is deeper, which is the second property. Thus, at any given time, a buy limit order is more likely when the book on the sell side is deep and less likely when the buy side is deep. Several empirical studies (e.g., Biais, Hillion, and Spatt, 1995; Griffiths et al., 2000; Ranaldo, 2004) confirm this prediction. Using data from the Swiss Stock Exchange, Ranaldo (2004) finds that traders submit more aggressive orders (higher-priced limit orders or market orders) when the LOB is deeper on their side and less aggressive orders when it is deeper on the opposite side. In the same vein, in a pioneering study of the Paris Bourse, an electronic limit order market, Biais, Hillion, and Spatt (1995) observe that: "Improvements in the best quote are especially frequent when the depth at the quotes is already large. This reflects the trade-off between the execution probability and price" (p. 1657).

These properties of execution probability imply that successive orders submitted to the market are not independent, even though traders' types (buyer/seller) are not serially correlated. To see this, let $I_{bt}=1$ if a buy limit order is submitted at time t, and $I_{bt}=0$ otherwise. Similarly $I_{st}=1$ if a sell limit order is submitted at time t, and $I_{s}t=0$ otherwise. In equilibrium, we have:

$$\Pr(I_{k2}=1|I_{k3}=1)\leq\Pr(I_{k2}=1|I_{k3}=0)\text{ for }k\in\ \{b,s\}.$$

(p.229) That is, the likelihood of two consecutive limit orders in the same direction (e.g., buy-buy) is smaller than that of two limit orders in opposite directions (e.g., buy-sell).[17] In other words, the model predicts that the direction of limit orders should be negatively serially correlated.[18] Suppose that a buy limit order arrives at time t. This order increases the depth at the best bid price, making it less attractive for the next trader to also submit a buy limit order and more attractive to submit a sell limit order.

The model also implies a positive serial correlation in the direction of trades (remember that this is defined as the direction of the market order side). The intuition is as follows. Suppose that a buy market order arrives at time t. The depth of the book on the ask side decreases by one share. As a consequence, the execution probability for an investor submitting a sell limit order improves. Sell limit orders become more attractive and the likelihood that the next investor will submit a sell market order consequently declines. Hence, the likelihood of observing a buy market order at time $t-1$ is greater given a buy market order at time t than given a sell market order. For instance, suppose that one share is offered on each side of the book at time 3 ($n_{b3} = n_{a3} = 1$) and a sell market order now arrives. Then the likelihood of another sell market order at time 2 is $\frac{1}{2}$ in equilibrium. If instead a buy market order hits the market at time 3 then the likelihood of a sell market order at time 2 is nil. Thus, the likelihood of two consecutive transactions in the same direction is greater than that of two transactions in opposite directions.

These properties imply that variations in the state of the LOB can generate time-series dependence in the order flow endogenously, even though traders' types (buyer/seller) are not correlated over time. In fact, in limit order markets orders are serially correlated. For instance, the likelihood of an order of a given type at a given time (say, a buy market order) is greater following the arrival of an order of the same type–which is partly consistent with the model presented here, where two consecutive transactions are more likely to be in the same direction than in opposite directions. But the model presented here does not explain why, empirically, a buy (sell) limit order is more likely after another buy (sell) limit order. This effect may stem from the fact that traders often split large orders into small consecutive trades, a strategy that is outside the model considered in this section.

(p.230) 6.5. Further Reading

Garman (1976) and Cohen et al. (1981) provide the first formal analyses of dynamic limit order markets. But they specify traders' beliefs on the execution probabilities of their limit orders exogenously.

Glosten (1994) develops the first equilibrium model of trading in a limit order market. His model is similar to that developed in section 6.2.3 but he assumes that a pro-rata allocation rule is used to determine how limit orders tied at the same price are executed (as in section 6.3.2). He also assumes that the number of traders submitting limit orders is infinite. Biais, Martimort, and Rochet (2000) relax this assumption and characterize the equilibrium of the limit order market when there is imperfect competition among limit order traders.

Using data from the Stockholm Stock Exchange, Sandas (2001) tests the specification of the model considered in example 1 (in its more elaborate version with asymmetric information). He tests whether snapshots of LOBs collected at the time of each transaction satisfy the restrictions imposed on cumulative depth and quotes by equations (6.16) and (6.17). He rejects the model, principally because the LOB predicted by the model appears too deep relative to the actual LOBs that he observes. One possible explanation is that competition among limit order traders is imperfect, so that the zero profit conditions that characterize the competitive equilibrium are not satisfied in reality.

In the model considered in section 6.2.3, informed investors are assumed to submit market orders. Bloomfield, O'Hara, and Saar (2005) run laboratory experiments in which informed and uninformed participants trade in a limit order market. They find that informed traders sometimes use limit orders. Kaniel and Liu (2006) study the conditions under which an informed investor is better off using a limit order.

Seppi (1997) introduces the model of limit order trading without asymmetric information considered in section 6.2. He extends this model to account for adverse selection and analyzes the last-mover advantage of the specialist (see section 6.3.3). Parlour and Seppi (2003) use this model to study competition between a hybrid and a pure limit order market. In this framework, Foucault and Menkveld (2008) analyze competition between two pure limit order markets (see section 7.4.2 of Chapter 7).

Seppi (1997), Viswanathan and Wang (2002), and Back and Baruch (2007) compare limit order markets and uniform price auctions. Back and Baruch (2007) show that a dynamic uniform auction in which informed investors can split their market orders over time always has an equilibrium that is identical to that obtained in Glosten (1994).

(p.231) Dynamic models of limit order trading have been developed by Foucault (1995); Parlour (1998); Foucault (1999); Goettler, Parlour and Rajan (2005); Foucault, Kadan and Kandel (2005); Van Achter (2006); Rosu (2009); and Large (2009). The model presented in section 6.4.1 is based on Foucault (1999). Foucault, Kadan and Kandel (2005) and Rosu (2009) study the role of impatience and time to execution. Parlour (1998) offers a systematic equilibrium treatment of the relationships between the depth of the LOB and the aggressiveness of order submission, as discussed in section 6.4.5. Goettler, Parlour, and Rajan (2005) develop a more general version of the framework considered in section 6.4.1, allowing traders to submit multiple limit orders when they arrive, and to stay for more than one period, and drawing traders' private valuations from a continuous distribution. They develop an algorithm to find a stationary Markov-perfect equilibrium of the limit order market numerically and run it to derive various comparative statics. In particular, they consider the effect of reducing tick size. They show that a smaller tick reduces limit order traders' surpluses but increases total investor surplus. Goettler et al. (2009) apply the same methodology to analyze the decision to acquire information in a limit order market.

A natural question is whether order choices that are observed in reality can be explained by the trade-offs described in section 6.4.1. Hollifield, Miller, and Sandas (2004) test this, using a more general version of that framework. Their test consists in estimating execution probabilities and pick-off risks for limit orders. Then they check whether observed limit orders maximize expected payoffs.[19] They do not reject the model when they test it for buy and sell orders separately.

Hollifield et al. (2006) use this framework to measure the efficiency of a limit order market empirically (the Vancouver Stock Exchange). They find that the limit order market is quite efficient, in that about 90 percent of the maximum gain from trades is realized, according to their estimates. Interestingly, unexecuted limit orders constitute the main source of inefficiency.

There is also a rich empirical literature on the determinants of order submission choices (see, for instance, Griffiths et al., 2000; Ellul et al. 2007; Ranaldo, 2004) and the references therein) and the cost and benefits of using market and limit orders (see Harris and Hasbrouck, 1996 and Handa and Schwartz, 1996). See Parlour and Seppi (2008) for a detailed survey of the literature on limit order markets.

(p.232) 6.6. Appendix

Here we briefly sketch how the equilibrium of the model of section 6.4.3 can be derived when $\sigma > 0$. As for $\sigma = 0$, we consider steady state equilibria in which investors' strategies do not depend on time. Assume first that $\sigma \leq \widehat{\sigma}$. In this case, in equilibrium, buyers and sellers post only limit orders with high execution probability, that is: $B^* = B_h = \widehat{B}(-L, v_t + \sigma)$ and $A^* = A_h = \widehat{A}(+L, v_t - \sigma)$. Thus, traders' cutoff prices are

(6.41)

$$\mu_0 + L - \widehat{A}(+L, v_t) = \frac{\tau}{2}\left(\mu_0 + L - \widehat{B}(-L, v_t + \sigma)\right),$$

(6.42)

$$\widehat{B}(-L, v_t) - \left(\mu_0 - L\right) = \frac{\tau}{2}\left(\widehat{A}(+L, v_t - \sigma) - \left(\mu_0 - L\right)\right).$$

Further, it must be that the expected payoff with aggressive (high-execution-probability) orders is greater than that obtained with conservative (low execution probability) limit orders. Thus the following inequalities must hold:

(6.43)

$$\frac{\tau}{2}\left(\mu_0 + L - \widehat{B}(-L, v_t + \sigma)\right) \geq \frac{\tau}{4}\left(\mu_0 + L - \widehat{B}(-L, v_t - \sigma)\right)$$

(6.44)

$$\frac{\tau}{2}\left(\widehat{A}(+L, v_t - \sigma) - \left(\mu_0 - L\right)\right) \geq \frac{\tau}{4}\left(\widehat{A}(+L, v_t + \sigma) - \left(\mu_0 - L\right)\right)$$

Investors' equilibrium cutoff prices are obtained by solving the system of equations (6.41) and (6.42). This yields closed-form solutions for B_h and A_h. Armed with these solutions for the cut-off prices, we can verify that conditions (6.37), (6.43), (6.44) are satisfied if and only if $\sigma \leq \widehat{\sigma}$ The derivation of the equilibrium when $\sigma \rangle \widehat{\sigma}$ follows similar steps; see Foucault (1999) for a full analysis.

6.7. Exercises

1. Deriving a competitive LOB.

Consider the model developed in section 6.2.3. We make the following parametric assumptions:

1. The trader who arrives in period 1 knows the final value of the security v with probability π. Otherwise, he is uninformed.
2. If the trader who arrives in period 1 is uninformed, he buys or sells (with equal probability) a number of shares x that has an exponential distribution with parameter θ. That is, the size distribution of the market order submitted by an uninformed trader arriving in period 2 is $f(x) = \frac{1}{2}\theta e^{-\theta|x|}$.
(p.233) 3. The final value of the security in period 2 has the following probability distribution:

$$g(v) = \frac{1}{2\sigma}\exp\left(-\frac{|v-\mu|}{\sigma}\right).$$

4. The tick size is nil ($\Delta = 0$).

Assumption 3 implies that σ is the mean absolute deviation of v and $E(v \mid v \geq z) = z + \sigma$.

a. Let $Y(A)$ be the cumulative depth up to ask price A in the book and A^* be the lowest ask price in the LOB. Show that when $v \geq A^*$, the optimal strategy of the informed trader is to buy $Y(v)$ shares.
b. Using this observation and the zero profit condition (6.13), show that in equilibrium:

$$Y(A) = \frac{1}{\theta}\left[\text{In}\left(\frac{1-\pi}{\pi}\right) + \text{In}\left(\frac{A-\mu}{\sigma}\right) + \frac{A-\mu}{\sigma}\right] \text{ if } A \Big\rangle A^*.$$

c. Show that the book becomes thinner on the ask side when (i) π increases or (ii) σ increases. What is the economic intuition for this result?

2. Time priority vs. random tie-breaking rule.

Consider example 2 in section 6.2.3 but assume that the tick size Δ is strictly positive, such that

$$A_1 = \mu + \Delta\langle\mu + \sigma\langle\mu + 2\Delta.$$

Time priority is enforced as in the baseline model of section 6.2.3.

a. Explain why the LOB will feature at least qL shares offered at price A_2.
b. Let Y_1 be the number of shares offered at price A_1. Define $r = \sigma/\Delta$. Observe that $r \in [1,2]$. Using the assumptions regarding the order flow at time *1,* show that:

(a) $Y_1 = 0$ iff $\frac{(r-1)\pi}{1-\pi} \geq 1$.
(b) $Y_1 = q_S$ iff $\frac{(r-1)\pi}{1-\pi} \in [1-\phi, 1)$.
(c) $Y_1 = q_L$ iff $\frac{(r-1)\pi}{1-\pi} \in [0, 1-\phi)$.

c. Why does Y_1 decrease with π?
d. Now assume that $\frac{(r-1)\pi}{1-\pi} \in [1-\Phi, 1)$ and suppose that time priority is not enforced any more. Instead, if two traders post a limit order at price A_1, then the offer that is executed first is determined randomly. Specifically, the limit order posted by trader $j \in \{1,2\}$ is executed first with probability 0.5. Let Y_1^j be the number of shares offered by trader j **(p.234)** $\in \{1,2\}$ at price A_1. Explain why, in equilibrium, Y_1^1 and Y_1^2 must satisfy the following conditions for $j = 1$ and $j = 2$:

$$\left(A_1 - \mathrm{E}\left(v \middle| q \geq Y_1^j\right)\right) \mathrm{pr}\left(q \geq Y_1^j\right) + \left(A_1 - \mathrm{E}\left(v \middle| q \geq Y_1^1 + Y_1^2\right)\right)$$
$$\mathrm{pr}\left(q \geq \left(Y_1^1 + Y_1^2\right)\right) \leq 0$$

with a strict inequality if $Y_1^1 = Y_1^2 = 0$.
e. Suppose that $qL = 2q_S$. Deduce that $Y_1^1 = Y_1^2 = q_S$ form an equilibrium if:

$$\frac{(r-1)\pi}{1-\pi} \in \left[(1-\phi), \frac{1+(1-\phi)}{2}\right],$$

when the "random" tie-breaking rule is used.
f. Why is cumulative depth greater when the random tie-breaking rule is used for $\frac{(r-1)\pi}{1-\pi} \in \left[1-\phi, \frac{1+(1-\phi)}{2}\right]$?

3. Time priority vs. pro-rata allocation.

Consider the model developed in section 6.2.2 and suppose $C < A_1 - v_0$. The size of the incoming market order (in absolute value) has a uniform distribution on $[0, Q]$, that is,

$$F(q) = \frac{q}{Q}.$$

a. Show that in this case the cumulative depth at price A_k is:

$$Y_k = Q\left(1 - \frac{C}{A_k - \mu}\right), \forall k.$$

b. Now suppose that instead of time priority, a pro-rata allocation rule is used, as described in section 6.3.2. Further assume that $A_1 - \mu \rangle 2C$. Then show that the cumulative depth at price A_1 is $Y_1^r = \frac{(A_1 - \mu)Q}{2C}$.
c. Why does the pro-rata allocation rule yield greater cumulative depth at all ask prices?

4. Competition among specialists and liquidity.

Consider the model of section 6.3.3 and assume that two specialists can stop out a market order. When a market order arrives, they post a stop-out price at which they are ready to fill. The specialist with the more competitive price executes the order. If there is a tie, the order is split equally between the two specialists.

a. Show that if

$$\frac{q_L}{q_s} \Bigg\rangle 1 + \frac{\pi}{(1-\pi)\,(1-\phi)},$$

(p.235) then, in equilibrium, the offers in the LOB are as described by equations (6.25) and (6.26), and the specialists stop out the small orders at a price (bid or ask) equal to μ.
b. In this case, do the specialists improve liquidity?

5. Make/take fees and bid-ask spreads.

Consider the model of section 6.4.1 with $\sigma = 0$ and $\tau = 1$. As Chapter 7 explains, trading platforms often charge different fees for market and limit orders. Let f_{mo} be the fee per share paid by a market order placer and f_{lo} the fee per share for a limit order placer when the limit order executes (there is no entry fee for limit orders). Finally let f be the total fee earned by the platform on each trade, $f = f_{mo} + f_{lo}$.

a. Compute bid and ask quotes in equilibrium.
b. Show that the bid-ask spread decreases in f_{mo} and increases in f_{lo}. Explain.
c. Trading platforms often subsidize traders who submit limit orders. That is, they set $f_{lo} \langle 0$ and $f_{mo} \rangle 0$, maintaining that this practice ultimately helps to narrow the spread and benefits traders submitting market orders. Holding the total trading fee fixed, is this argument correct?

Notes:

(1.) Formally, the LOB on the ask side corresponds to the following supply function:

$$\$\,(p) = \begin{cases} 0 & \text{for}\, p \langle \$100.50, \\ 100 & \text{for} \$100.50 \le p \langle \$101, \\ 300 & \text{for} \$101 \le p \langle 101.50, \\ \infty & \text{for} \$101.50 \le p. \end{cases}$$

(2.) Other tie-breaking rules are considered in section 6.3.2.

(3.) This will in fact be the case in equilibrium.

(4.) The trader's estimate of the value of the security conditional on execution is $E(v|q \geq 0) = \mu + \frac{\lambda}{\theta} \rangle \mu.$

(5.) For instance, Bacidore (1997), Ahn, Bae, and Kalok (1998), and Porter and Weaver (1997) studied the impact of the reduction in the tick size on the Toronto Stock Exchange in April 1996. They found a reduction of 17.27 percent in the quoted spread and of 27.52 percent in the quoted depth at the best price (depending on study and sample). The reduction in depth is not surprising, given the reduction in the spread.

(6.) This effect is analyzed in Cordella and Foucault (1999) and Foucault, Kadan, and Kandel (2005). Both studies conclude that the tick size that minimizes trading costs for liquidity demanders is strictly positive.

(7.) In compensation, the designated market makers receive a payment from the issuer or are exempted from the exchange's trading fees. We discuss further the role of designated market makers further in Chapter 10, section 10.5.2.

(8.) Skjeltorp, Sojli, and Tham (2011) study empirically the impact of Nasdaq decision to introduce and then remove the flash order function. In contrast with the argument developed here, their findings suggest that flash orders improve market quality.

(9.) SEC release no. 34 – 60684: "Elimination of Flash Order Exception from Rule 602 of Regulation NMS."

(10.) We do not model the reasons why traders have different valuations y_i. Several stories come to mind. One is that traders have a long or short position in the security or in another asset whose payoff is correlated with v_T. In this case, traders with a long position value a purchase of the security less than traders with a short position and conversely, as explained in Chapter 3. Or traders may have different discount factors or face different tax treatment and hence have different valuations for identical payoffs. For instance, for individual investors in the United States, interest on municipal bonds is tax-exempt while payouts of other securities are not; in contrast, pension funds are income tax-exempt irrespective of the securities they hold. Thus, other things being equal, individuals value munis more than tax-exempt institutions. Last, one could interpret y_i as reflecting differences of opinion among traders about the final payoff.

(11.) Another reason why the problem is difficult is that investors submitting limit orders may also want to revise them as market conditions change.

(12.) The structure of the model is such that the limit order book features either a buy limit order at price B^* or a sell limit order at price A^* but not both orders at once. However, as all buy market orders execute at A^* and all sell orders at B^*, it is natural to define the bid-ask spread as $A^* - B^*$.

(13.) As figure 6.9 shows, authorizing pegged limit orders does not necessarily minimize the spread. Indeed, in the model, suppressing limit order traders' exposure to changes in the asset value can increase their market power, as explained at the end of the previous section.

(14.) For instance, limit orders in a given stock traded on the International Securities Exchange can be pegged to the best bid and ask prices for this stock across all markets. Pegged limit orders are then automatically repriced every time there is a change in the best bid or ask price, so that their limit price remains at a fixed distance from the best price.

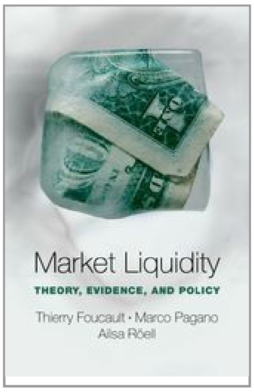
(15.) See Foucault, Röell, and Sandas (2003) for an analysis.

(16.) A buy limit order, by symmetry, is similar to an American put option.

(17.) Here, we just consider dates $t = 3$ and $t = 2$ since the model has only three dates. But this property of the model is more general.

(18.) For instance, suppose that $n_{b3} = 0$ and $n_{a3} = 0$. If a sell limit order is submitted at time 3, the likelihood of observing a buy limit order at time $t = 2$ is $\frac{1}{2}$. In contrast, if a buy limit order is submitted at time 3, the likelihood of observing a buy limit order at time 2 is zero.

(19.) They show that this can be checked by testing a simple monotonicity condition, using the estimates obtained for execution probabilities and pick-off risk.



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Market Fragmentation

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

When a security trades in multiple venues, orders submitted to the different venues do not necessarily contribute jointly to price formation. In this case, the market for the security is said to be "fragmented." Market fragmentation raises several concerns that have been at the forefront of recent regulatory debates, in the United States and in Europe. Market fragmentation may lead to excessive "price dispersion," meaning that the same security may trade at different prices at the same instant. Another concern is that market fragmentation could raise trading costs for investors compared to centralized trading. This chapter discusses the costs of fragmentation, liquidity externalities, and the benefits of fragmentation. It also describes two recent changes in the United States (RegNMS) and Europe (MiFID) designed precisely to organize competition among trading platforms. The final sections provide suggestions for further reading and

exercises.

Keywords: securities trading, fragmentation, liquidity, securities regulation, RegNMS, MiFID

Learning Objectives:

- • Market fragmentation
- • Costs and benefits of market fragmentation
- • How these costs and benefits relate to liquidity externalities
- • Implications for regulation of competition among trading platforms

7.1. Introduction

It is common for a security to trade in several venues. For instance, large companies often have their primary listing in their home country and a secondary listing abroad (the Italian energy company ENI, for instance, is listed not only on the Milan Stock Exchange but also on the NYSE). Moreover, several trading platforms permit trading of securities listed elsewhere. For instance, in the United States, stocks listed on Nasdaq or the NYSE can be traded on a myriad of other trading platforms: BATS, DirectEdge, NYSE Arca, and more. These platforms compete fiercely to attract order flow, and the market share of incumbent exchanges has declined steadily in the United States.

Table 7.1 shows the market shares of the main trading venues for U.S. equities in 2009: NYSE and Nasdaq have market shares of less than 20 percent each, and electronic communication networks (such as BATS and Direct Edge), which operate electronic limit order markets, capture about 20 percent. Trading is also **(p.237)**

Table 7.1. Market Fragmentation in the United States, 2009

Trading Venue	Market Share
NYSE	14.7%
NYSE Arca	13.20%
NASDAQ	19.4%
NASDAQ OMXBX	3.3%
BATS	9.5%
ECN:2Direct Edge	9.8%
ECN3:Others	1%
Other Registered Exchange	3.7%
Broker-Dealer Internalization (more than 200)	17.5%
Dark Pools (about 32)	7.9%

Source: Securities and Exchange Commission (2010), Concept release on equity market structure. Release 34-61358. File No. S7-02-10.

conducted in regional exchanges and venues known as “dark pools” (crossing networks with minimal transparency requirements). Last, a relatively large fraction of U.S. trading (17.5 percent) is “internalized,” executed directly in house by broker-dealer firms.

The same phenomenon has been observed in the European Union (E.U.) since 2007 with the implementation of the Market in Financial Instruments Directive (MiFID), a new regulatory framework for European Union securities markets. Prior to MiFID, each member state could require all trading in domestically listed stocks to be executed on the national market. MiFID abolished this “concentration rule,” triggering the entry of new trading platforms (Chi-X, BATS Europe, Turquoise, etc.). As a consequence, the market shares of the incumbent markets (Euronext, LSE, Deutsche Börse, etc.) have declined.[1] Table 7.2 shows that, as of February 2012, Euronext retained only 60.5 percent of the trading activity in the CAC40 stocks, and the LSE retained only 54.77 percent of trading in the FTSE100 stocks.

When a security trades in multiple venues, orders submitted to the different venues do not necessarily contribute jointly to price formation. In this case, the market for the security is said to be “fragmented.” For instance, an aggressively priced sell limit order for Royal Dutch/Shell on the NYSE does not have priority of execution over a simultaneous, higher one in Amsterdam. Hence, the NYSE **(p.238)**

Table 7.2. Market Fragmentation in Europe, 2011

Trading Platforms	Market Share	
	CAC40 Stocks	FTSE100 Stocks
Euronext	60.5%	n.a.
LSE	n.a.	54.77%
Chi-X	24.93%	30.43%
Turquoise	5.67%	7.93%
BATS Europe	4.31%	6.66%

Source: Fidessa at http://fragmentation.fidessa.com.

and Amsterdam markets for Royal Dutch/Shell operate independently. OTC markets like bonds and foreign exchange markets, are generally highly fragmented since they have no mechanisms for consolidating quotes and they execute orders according to price and other, secondary priority rules.

Market fragmentation raises several concerns that have been at the forefront of recent regulatory debates, in the United States and in Europe. Market fragmentation may lead to excessive “price dispersion,” meaning that the same security may trade at different prices at the same instant. Worse, outright arbitrage opportunities—the sign of a dysfunctional market—can arise: for instance, the best offer price in one platform may be below the best bid price in another. Such situations of “locked” or “crossed” markets are not at all uncommon.[2] In principle, arbitrageurs should exploit these inefficiencies, which should make them both rare and fleeting. If a security trades at two different prices, arbitrageurs can theoretically make a certain profit by buying in one market and reselling in the other. However, there are many reasons why they may decide not to intervene if price differences are small. For instance, for a cross-listed stock, investment or short-sales restrictions may raise the cost of building an arbitrage portfolio. More generally, arbitrageurs may not monitor the market continuously and so may react late to transient price divergences.

Another concern is that market fragmentation could raise trading costs for investors compared to centralized trading for at least three reasons:

(i) Fragmentation heightens informed investors' ability to exploit their information, allowing them to trade on various platforms and thereby make it harder for other traders to detect them and infer their **(p.239)** information (section 7.2.1). Moreover, by hindering interactions among investors located in different trading venues, fragmentation reduces the scope for risk sharing among investors (section 7.2.2). Fragmentation also gives local market power to liquidity suppliers in each trading platform, since they do not have to compete with counterparts operating on another platform (section 7.2.3).
(ii) In a fragmented market, investors have to conduct a search for the best price since quotes are not centralized. This search is costly for several reasons. For instance, it takes time to find the intermediary that posts the best price. In electronic trading platforms (as in European and U.S. equities markets), the search can be automated by using "smart order routing systems" that scan the platforms for the best price. But these technologies are costly. Search costs also exacerbate agency problems between investors and brokers, as the brokers bear the cost of search while the benefit accrues to their client: fragmentation increases the risk that brokers will not shop around systematically for the best possible price (section 7.2.4), which may in turn reduce the incentives for liquidity providers to post good quotes (section 7.4.3).
(iii) Fragmentation prevents investors from taking full advantage of the "thick market externalities" (also called "liquidity externalities") that arise from the fact that each additional market participant reduces the trading fees or increases the liquidity of all other traders. Economies of scale are one source of thick market externalities: in the past, the fixed cost of running a market was high,[3] so that platforms with large volumes could spread this cost over many traders and so charge lower fees. Thus, concentrating trading in a single market place produced efficiency gains. In today's securities markets, this argument for concentration is less compelling since automation has considerably shrunk the fixed-cost component of running a market. Another positive externality of market participation is that it makes it more likely to find a trading partner quickly enough: this is easier if the market attracts many traders. Hence, by joining a platform, an investor indirectly increases other market participants' likelihood of trading and so makes them better off. In section 7.3, we explore other mechanisms for these externalities, such as reduced adverse selection and improved risk-sharing.

(p.240) Fragmentation does not have only drawbacks, however. It can benefit investors in various ways. First, competition among platforms tends to reduce fees and foster innovation in trading technology. The recent evolution of US and European equities markets shows the relevance of this argument very well. The entry of new trading platforms (such as BATS in the United States or Chi-X in Europe) drove trading fees sharply down. In the United States, competition has also prompted the NYSE and Nasdaq to overhaul their trading systems. A few years ago, the NYSE was still predominantly a floor market and Nasdaq was a dealer market; both now have electronic LOBs at their core. Moreover, to serve the trading needs of some clients (in particular, algorithmic traders), all trading platforms strive to reduce their "latency" period, which is the time it takes to receive and send messages (such as orders or information on standing quotes).

Another benefit of intermarket competition is that different trading needs of investors can be best served by different trading mechanisms. For instance, unlike retail investors, institutional investors often execute very large trades, which can have a significant price impact in transparent electronic limit order markets. They accordingly seek to arrange their trades OTC or in dark pools. Section 7.4 explores these benefits of market fragmentation in greater detail.

In the light of these costs and benefits, ideally one would want to organize securities markets so as to promote intermarket competition while allowing interactions between orders placed in different markets. One approach is to let market forces decide on the level of fragmentation. But, it is unlikely that the socially optimal amount of market fragmentation will arise spontaneously. For instance, liquidity externalities act as a barrier to entry, conferring market power on the incumbent markets. Conversely, strategic considerations by trading platforms may lead to excessive fragmentation. For instance, one trading platform might not allow a competitor to access its quotes.[4]

For this reason, market fragmentation is high on the regulatory agenda. In section 7.5 we describe two recent changes in the United States (RegNMS) and Europe (MiFID) designed precisely to organize competition among trading platforms. For instance, the SEC describes the main objectives of Regulation NMS (the regulatory framework under which the U.S. equities market has operated since 2005) as follows:

> "The NMS is premised on promoting fair competition among individual markets, while at the same time assuring that all of these markets are **(p.241)** linked together, through facilities and rules, in a unified system that promotes interaction among the orders of buyers and sellers in a particular NMS stock […] Accordingly, the Commission […] has sought to avoid the extremes of: (1) isolated markets that trade an NMS stock without regard to trading in other markets and thereby fragment the competition among buyers and sellers in that stock; and (2) a totally centralized system that loses the benefits of vigorous competition and innovation among individual markets. Achieving this objective and striking the proper balance clearly can be a difficult task." (Regulation NMS, SEC Release No. 34-51808)

One way to reduce fragmentation is to link markets together so that trades for a given security always occur at the best possible price. And this is the approach taken in Regulation NMS. The so-called order protection rule (or trade-through rule) obliges a market center such as an electronic communication network to reroute marketable limit orders to the trading platform that posts the best price when the order is submitted. But this approach has proven very controversial. In particular, it gives price of execution priority over speed of execution, whereas, for some investors, the latter might actually be more important. In section 7.4.3, we discuss the effects of a trade-through rule on market liquidity.

7.2. The Costs of Fragmentation

In this section, we analyze three channels through which market fragmentation can reduce liquidity compared to centralized trading:

(i) asymmetric information (section 7.2.1),
(ii) risk bearing capacity (section 7.2.2),
(iii) market power (section 7.2.3).

In all three cases, fragmentation is a source of illiquidity because investors cannot (or do not) access all the possible trading venues for a security: if they could, each platform would simply be a different portal to the same single market. But in practice, multimarket trading is marked by technical hurdles (e.g., differences in time zones or in clearing and settlement systems) and institutional barriers (e.g., divergent tax treatments of groups of investors). Investors may also be unable to split their orders optimally across markets, because this requires costly investment in technology or because their brokers lack the incentive to do so (see section 7.2.4).

(p.242) 7.2.1 Information Effects

We first analyze the effect of fragmentation in a multimarket version of the model presented in Chapter 4 (Kyle, 1985). We assume that the security is traded in two markets, A and B, and that liquidity traders are captive customers of one of the two markets. Let u_A and u_B denote their liquidity demand in markets A and B, and assume that these variables are independently and normally distributed with zero mean and variances σ_A^2 and σ_B^2. As in Chapter 4, the informed trader knows the final value of the security, v, which is normally distributed with mean μ and variance σ_v^2. Unlike the liquidity traders, he can trade in both markets, placing an order x_A in market A and x_B in market B. This structure captures the idea that traders with superior information tend to coincide with sophisticated market professionals (for instance, hedge funds), whose greater trading activity gives them an economic interest in securing access to several markets.

In each market, order imbalances are absorbed by risk-neutral market makers. In market A, market makers set the price p_A conditional on the order flow $u_A + x_A$. The same process occurs in market B. As in Chapter 4, we seek an equilibrium in which market makers post linear price schedules in each market:

(7.1)

$$p_A = \mu + \lambda_A (\mu_A + x_A), \quad p_B = \mu + \lambda_B (u_B + x_B).$$

The optimal order placement strategy for the informed trader (his choice of x_A and x_B) maximizes his expected profits from trading in the two markets:

(7.2)

$$\mathrm{E}\left[x_A (v - p_A) + x_B (v - p_B) |v\right].$$

Substituting into this expression the prices p_A and p_B from equations (7.1), the first-order conditions of the informed trader with respect to x_A and x_B yield the following order placement strategy:

$$x_A = \frac{v - \mu}{2\lambda_A}, \qquad x_B = \frac{v - \mu}{2\lambda_B}.$$

Hence, the expression for the informed trader's strategy in each market is the same as in Chapter 4 for a single market. In particular, note that the optimal order size in one market does not depend on the depth of the other. This is because in this setting, an order placed in one market has no impact on the price in the other. Indeed, the informed trader submits orders *simultaneously* in the two markets, so that in market A the price cannot be immediately adjusted in response to concomitant orders in market B, or vice versa.[5]

(p.243) Proceeding as in Chapter 4, we can then compute the equilibrium values of λ_A and λ_B:

(7.3)

$$\lambda_A = \frac{\sigma_v}{2\sigma_A}, \qquad \lambda_B = \frac{\sigma_v}{2\sigma_B}.$$

The corresponding equilibrium prices in the two markets are:

(7.4)

$$p_A = \mu + \frac{1}{2}(v - \mu) + \frac{\sigma_v}{2\sigma_A} u_A, \qquad p_B = \mu + \frac{1}{2}(v - \mu) + \frac{\sigma_v}{2\sigma_B} \mu_B.$$

Thus, equilibrium prices differ and the two markets are not perfectly integrated. At least in the very short run, such pricing differences are in fact often observed for securities that trade in different markets, even when their trading hours overlap. For instance, Biais and Martinez (2004) document significant price discrepancies between the home and foreign prices for French stocks cross-listed in Frankfurt and for German stocks cross-listed in Paris. Similar evidence is reported by Hupperets and Menkveld (2002) for Dutch stocks cross-listed on the NYSE and by Eun and Sabherwal (2003) for Canadian stocks cross-listed in the U.S. Such transient discrepancies suggest that continuous arbitrage is impossible due to the limitations of the trading mechanisms available to market professionals, which prevent them from stepping in to cross offsetting orders in the two markets.

Now consider the depth λ and the price that would prevail in equilibrium if the two markets were consolidated (see Chapter 4):

(7.5)

$$\lambda = \frac{\sigma_v}{2\sqrt{\sigma_A^2 + \sigma_B^2}}, \qquad p = \mu + \frac{1}{2}(v - \mu) + \frac{\sigma_v}{2\sqrt{\sigma_A^2 + \sigma_B^2}}(u_A + u_B).$$

Thus, the market is deeper when it is not fragmented since $\lambda < \min\{\lambda_A, \lambda_B\}$. The reason is that consolidation increases overall liquidity demand, as measured by $\mathrm{var}(u_A + u_B)$. This may not hold when liquidity demands in each market are negatively correlated; see Exercise 1. This effect reduces the informativeness of order flow, and market orders' price impacts are smaller when trading is consolidated in a single market.

Thus, fragmentation redistributes the trading gains from liquidity traders to informed traders. In the fragmented environment, the informed trader's profits are on average

(7.6)

$$E[x_A(v - p_A) + x_B(v - p_B)] = \frac{\sigma_v \sigma_A}{2} + \frac{\sigma_v \sigma_B}{2} = \frac{\sigma_v \sqrt{\sigma_A^2 + \sigma_B^2 + 2\sigma_A\sigma_B}}{2},$$

(p.244) whereas in a consolidated market they are

(7.7)

$$\mathrm{E}[x(v - p)] = \frac{\sigma_v \sqrt{\sigma_A^2 + \sigma_B^2}}{2},$$

where x is the order placed by the insider and p is the equilibrium price in the consolidated market (see Chapter 4). Comparing these two expressions, it is clear that the informed trader's expected profits are larger in the fragmented market. Since market makers have zero expected profits, the profits of informed traders are equal to the losses of the liquidity traders. Hence, the increased profits of the informed trader in the fragmented market come at the expense of liquidity traders.

Dealers in one market can better forecast the asset payoff by observing the price set by dealers in the other market. Hence, the price in one market will adjust after a transaction in the other. This process can explain the lead-lag relations that exist between returns of related or identical securities. To see how this works, suppose that after transactions in their respective markets, market makers in one observe the transaction price in the other. Market makers in both markets then adjust their estimate of the value of the security and set their new (long-run) midpricepl equal to:

$$pl = \mathrm{E}\left(v | p_A, p_B\right).$$

Using equation (7.1) and the fact that v, p_A, and p_B are normally distributed, we obtain the long-run price:

(7.8)

$$pl = \mu + \alpha_A (p_A - \mu) + \alpha_B (p_B - \mu),$$

with $\alpha_A = \alpha_B = 2/3$.[6] This equation shows that market makers' estimate of the fundamental value of the security depends on the informational content of the price changes, $p_j - \mu$, in the two markets. Here the coefficients on these price changes are identical because the informational content of both innovations is identical. In a more general model, the coefficients would differ, with the ratio $\frac{\alpha_j}{\alpha_A + \alpha_B}$ being a natural measure of the "share" of market j in price discovery. **(p.245)** For instance, if α_B is lower than α_A, this means that dealers consider trades (and therefore prices) in market B less informative than those in market A. Hence, market B's response to a price change in market A is stronger than the reverse. In this sense, market A "leads" market B, or contributes more to price discovery (its "share" of price discovery is higher). Using this intuition, researchers have developed methodologies to estimate the price discovery share of a given market for securities that trade on several markets (see Hasbrouck, 1995)[7].

One concern about market fragmentation is that it may harm price discovery. This is not the case in the situation posited here. Indeed, consider the variance of the asset payoff conditional on the realization of prices in market A and market B, that is, $\text{var}(v|p_A, p_B)$. This variance measures the uncertainty that still remains for market participants after the trading in markets A and B. The lower this variance, the more accurate the price discovery. Calculations show that:[8]

(7.9)

$$\text{var}(v|p_A, p_B) = \frac{\sigma_v^2}{3}.$$

Instead, when trading is concentrated in a single market, we have

$$\text{var}(v|p) = \frac{\sigma_v^2}{2}.$$

Hence, price discovery is better with two markets. The intuition behind this finding is simple: prices in the two markets provide different signals about the value of the security; as these signals are not perfectly correlated (because liquidity trades in each market are not perfectly correlated), they combine to produce a more accurate forecast than the single price signal in a single marketplace.

(p.246) 7.2.2 Risk-sharing Effects

The previous section examines the cost of market fragmentation consisting in trading costs due to breaking up total liquidity into separate pools. In that model, this effect damages liquidity traders but benefits informed traders to the same extent, since trading is modeled as a zero-sum game. Here, instead, we shall see that the reduction in liquidity caused by fragmentation can generate a net social loss (in the sense that gains may not fully offset losses), because it reduces the extent to which traders can share risks.[9]

As an illustration, consider the model of a competitive call market developed in Chapter 4 (section 4.3.1), where market makers are assumed to be risk averse, and suppose that the security is traded on two separate call markets. Let the number of market makers be K_A in market A and K_B in market B. For simplicity, we assume that they have no initial inventory ($Z = 0$) and are all equally risk averse, with coefficient $\bar{\rho}$.

Now consider an investor who wants to place a market order of size q on either of the two markets. Using equation (4.25) in Chapter 4 (section 4.3.1), the equilibrium price at which he will trade in market A is

$$p_A = \mu + \frac{\bar{\rho}}{K_A}\sigma_v^2\, q.$$

Hence, the expected price impact of his order is

$$\frac{\bar{\rho}}{K_A}\sigma_v^2\, |q|\,,$$

which shows that the liquidity of the market is inversely related to the number of dealers, K_A. Similarly, his expected price impact if he were to trade in market B would be

$$\frac{\bar{\rho}}{K_B}\sigma_v^2\, |q|\,.$$

If instead the two markets were to merge, the expected price impact on this consolidated trading platform would be

$$\frac{\bar{\rho}}{K_A + K_B}\sigma_v^2\, |q|\,.$$

Hence, the liquidity of the consolidated market is greater: by increasing the number of market makers, consolidation improves the risk-bearing capacity of the market, and thereby reduces the compensation required by market **(p.247)** makers to absorb risky inventories. As in the model of the previous section, consolidation again improves liquidity but for a different reason.

Market fragmentation not only impairs liquidity but also diminishes the opportunities for risk sharing among market participants. To see this point, consider a situation where market makers' inventories are unbalanced, so that a reallocation among market makers is Pareto improving. If this reallocation could occur only by trading at competitive prices separately on each of the two markets, then trading would equalize market makers' inventories *within* each market. However, since the participants in the two markets will generally have different per-capita inventory endowments, their holdings after trading would not be equalized *across* the two markets. In that case, there will be further gains from trading if the two markets are merged, because there are further opportunities for risk sharing.

The costs of market fragmentation described so far hinge crucially on the assumption that customers cannot access both markets simultaneously, and thus split their orders across them. To illustrate, let us go back to the customer who wants to trade q shares with a market order. Assume now that he can split the order across the two markets by placing an order q_A in market A and an order q_B in market B so that $q_A + q_B = q$. In this case, he splits the order so as to minimize his total trading cost. As the reader can verify by solving exercise 2, the optimal order placement strategy is

(7.10)

$$q_A^* = \frac{K_A}{K_A + K_B} q, \quad q_B^* = \frac{K_B}{K_A + K_B} q.$$

The trader submits a relatively larger order in the deeper market. This strategy equalizes the execution price the two markets since

$$p_A\left(q_A^*\right) = p_B\left(q_B^*\right) = \mu + \frac{\bar{\rho}\sigma_v^2}{K_A + K_B} q.$$

Interestingly, the execution price is identical to the price that the customer would obtain if there were a single market. Hence, fragmentation is innocuous if traders can split their orders seamlessly between markets.

These findings suggest two ways of overcoming the costs of fragmentation. The first approach is to merge markets or require concentration of trading in a single market. The second approach consists in facilitating traders' access to all the markets in which a security is traded. Technological advances, such as data aggregators (which consolidate quotes in different markets) and smart order routing technologies (which optimally split orders between trading platforms), lower the cost of multimarket trading. Yet, obstacles remain such as differences in clearing and settlement systems across markets (see box 7.5), differences in transparency, different trading hours, and agency problems. **(p.248)**

Box 7.1 Stock Exchange Mergers and Liquidity

The analysis developed in this section implies that concentration of trading in a single market generates liquidity gains. Some empirical evidence on this point can be garnered from results on the mergers between stock exchanges. Arnold et al. (1999) study the effects of three successive mergers of regional U.S. stock exchanges in the 1940s and 1950s, which transformed them from venues for listing local securities to competitors for the order flow of NYSE-listed companies. The merged exchanges attracted a larger market share and their bid-ask spreads narrowed. A more recent example is the Euronext merger between September 2000 and November 2003: the French, Belgian, Dutch, and Portuguese stock exchanges merged into a single exchange called Euronext, with a single trading platform and a single clearing and settlement system. Padilla and Pagano (2006) find that this consolidation was associated with a significant increase of liquidity, as measured both by trading volume and bid-ask spreads. Following the consolidation, the average spreads on the securities included in the main indices of the Paris, Brussels, Amsterdam, and Lisbon exchanges fell by an estimated 16 percent to 21 percent, controlling for other determinants.

7.2.3 Competition among Liquidity Suppliers

Another cost of market fragmentation is that it hinders competition among liquidity providers, by weakening interactions between market makers active in different liquidity pools and thereby decreasing their incentives to offer good quotes. We illustrate this point below by extending the model analyzed in the previous section to allow for imperfect competition. Then, we analyze a further instance of fragmentation, when market makers can also internalize order execution, that is, execute them against their own inventory at market prices. This effectively turns each market maker into a “separate market.” We show that internalization further reduces competition.

Fragmentation and Market Power

When risk-averse market makers behave as imperfect competitors, the bid-ask spread for investors placing market orders is larger than under perfect competition, as was shown in section 4.3.2 of Chapter 4. As in the previous section, consider the price impact of a market order of size q on either one of two separate markets A and B, but now assume that each is imperfectly **(p.249)** competitive, so that in market A the equilibrium price is

$$p_A = \mu + \frac{K_A - 1}{K_A - 2} \frac{\bar{\rho}}{K_A} \sigma_v^2 q.$$

Thus, the price impact of an order of size $|q|$ on market A is

$$\frac{K_A - 1}{K_A - 2} \frac{\bar{\rho}}{K_A} \sigma_v^2 \, |q| \, ,$$

and similar in market B. If A and B were instead consolidated into a single market with $K_A + K_B$ active market makers, the expected price impact of an order of size $|q|$ would be

$$\frac{K_A + K_B - 1}{K_A + K_B - 2} \frac{\bar{\rho}}{K_A + K_B} \sigma_v^2 \, |q| \, .$$

This expression is lower than its analogue under fragmentation, because the consolidated market has not only greater risk-bearing capacity but also a lower markup. As before, for traders on market A (or market B), the price impact reduction to better risk-sharing is captured by the reduction of the second fraction in the expression from $-\rho/K_A$ to $-\rho/(K_A + K_B)$. The further reduction due to more aggressive competition is reflected in the lowering of the first fraction from $(K_A - 1)/(K_A - 2)$ to $(K_A + K_B - 1)/(K_A + K_B - 2)$.

Again, the negative effect of fragmentation on market liquidity would vanish if traders could split their orders between the two markets, or if all market makers were active on both markets simultaneously. In this case, order splitting effectively forces the two pools of market makers to compete against one another.

Internalization and Market Power

Even when trading is centralized, an intermediary can sometimes respond to a client's order in a dual capacity as both broker and dealer. He can choose to channel only a fraction of the order to the market (in his capacity as broker) and take the rest himself (in his capacity as dealer)—a practice known as internalization. In this case, so-called best execution rules require that the broker-dealer executes the internalized order at the current market price. This practice is common in the U.S; in Europe, where until recently it was largely banned by regulation, it is now allowed by the MiFID (which came into effect on November 1, 2007). The impact of internalization on market liquidity is controversial.

To analyze the effect of internalization, consider the following variant of the previous model. Assume that there is a single centralized market with K broker-dealers. One receives a buy market order of size q, and decides what **(p.250)** portion (if any) to internalize and what to execute on the main market. If he does internalize, he will refrain from bidding in the main market; otherwise, he would be competing against himself on the internalized portion.[10] Denote by q_I the number of shares internalized and by $q_M = q - q_I$ the remaining shares routed to the market. In this case, the portion of the order routed to the market executes at the following equilibrium price:

$$p(q_M) = \mu + \frac{K-2}{K-3}\frac{\bar{\rho}\sigma_v^2}{K-1} q_M,$$

which is the same expression as in the previous subsection under the assumption that only $K - 1$ markets are active.

Now, consider the broker-dealer's optimal strategy, assuming for simplicity that his initial inventory is zero (the case where it is not zero is left as an exercise). The expected utility from routing an order for q_M shares to the market is

$$\begin{aligned} \mathrm{E}[U(q_M)] &= \mathrm{E}[p(q_M) - \mu] q_I - \frac{\bar{\rho}\sigma_v^2}{2} q_I^2 \\ &= [\mathrm{E}(p(q_M)) - \mu](q - q_M) - \frac{\bar{\rho}\sigma_v^2}{2}(q - q_M)^2. \end{aligned}$$

The market maker faces a trade-off. On the one hand, he earns an expected margin equal to $\mathrm{E}(p(q_M)) - \mu$ on each additional share that he executes. On the other hand, by internalizing the execution of this additional share, he decreases the market impact of the remaining shares and thereby the price at which he eventually executes the internalized order (since he must match the market price). The best point along this trade-off is found by maximizing the previous expression. The solution to this maximization problem is to route a fraction τ of the order to the market:

$$q_M^* = \frac{(K-2) + (K-3)(K-1)}{2(K-2) + (K-3)(K-1)} q \equiv \tau q.$$

Thus, the market maker optimally internalizes the remaining fraction $1 - \tau$.

Substituting q_M^* into the expression for $p(q_M)$, we find that, with internalization, the execution price for a market order of size q is

$$p\left(q_M^*\right) = \mu + \frac{K-2}{K-3}\frac{\bar{\rho}\sigma_v^2}{K-1}\tau q.$$

(p.251) If instead internalization were prohibited, the market maker would have to route the entire order to the market and compete with the other dealers if he wished to be a counterparty to the order. As the previous section shows, the expected execution price for the order in this case would be

$$p_A = \mu + \frac{K-1}{K-2}\frac{\bar{\rho}\sigma_v^2}{K} q.$$

It is readily confirmed that under internalization the price impact of the order (i.e., $\frac{K-2}{K-3}\frac{\bar{\rho}\sigma_v^2}{K-1}\tau$) is greater than when internalization is banned, in which case the price impact is $\frac{K-1}{K-2}\frac{\bar{\rho}\sigma_v^2}{K}$. For instance, if $K = 4$ and $\bar{\rho}\sigma_v^2 = 10$, the price impact of an order is 4.76 per share when internalization is authorized compared to 3.75 when it is banned.

There are two reasons for this result. First, the dealer who internalizes the order does not bid for the portion of the order that he routes to the market. This effectively reduces competition for this order and works to increase the rents earned by all dealers. Second, the dealer strategically executes a significant fraction of the order on the market in order to amplify its market impact, thereby increasing his rent. He benefits from this “price manipulation” both by bearing less risk and by internalizing the order at a less competitive price. Indeed, absent internalization, the market maker would on average absorb a fraction $1/K$ of the order in equilibrium. Under internalization, he absorbs a fraction $1 - \tau$ of the order, which can be shown to be larger than $1/K$.

Notice that the illiquidity of the main market increases a broker-dealer’s ability to extract rents through internalization. Conversely, his scope for rent extraction is minimal if the main market is very liquid: when the number of market makers (K) is very large, τ tends to 1, so that the fraction of the order internalized tends to zero. In short, internalization impairs market quality, and the damage is greater when the main market is illiquid. Besides reinforcing dealers’ market power, internalization can also impact on liquidity in a context of asymmetric information, if internalizers skim off uninformed order flow (for a similar mechanism, see exercise 3). For a detailed discussion of this point, see section 8.3 of Chapter 8.

7.2.4 Fragmentation and the Broker-Client Relationship

When the market is fragmented, it is harder for investors to locate good prices and the proliferation of "dark pools," hidden orders on traditional exchanges or automatic trading systems and crossing networks of unknown depth, have compounded the problem.

Thus, in fragmented markets, finding the best trade is more costly and time consuming, a task investors often delegate to their brokers. This delegation **(p.252)** creates an agency problem. That is, brokers incur the costs of searching for and implementing the best execution strategy for their clients (which requires technological investment, expertise, and time) without necessarily being able to entirely charge this cost to these clients. As usual in any agency relationship, one should expect brokers to expend less effort in finding the most desirable execution strategy than would be optimal for clients.

Performance-based contracts cannot fully solve this incentive problem for two reasons. First, clients often lack the data and expertise to verify whether their broker did indeed use the best possible strategy to execute their order, especially when the market is fragmented. Second, a trading strategy that is optimal ex-ante may prove disastrous ex post, just because of bad luck. Say, a broker decides to split a client's buy orders into multiple lots to reduce the immediate impact of the order. For large orders, this strategy is often optimal *on average*, but the realized cost in any particular instance is random, as it depends on the evolution of prices over the period during which the order is executed. If prices have risen, the actual average execution price for the client's buy order might prove to be considerably higher than the client could have gotten by executing his order more aggressively. Yet, this poor performance does not indicate a lack of effort or expertise. This example simply shows that it is hard for clients to disentangle what depends on the broker's effort and what is due to bad luck. In this situation, writing incentive contracts is difficult, asone wants to penalize brokers for shirking but not for events beyond their control. To alleviate these agency problems, regulators often require brokers to comply with "best execution rules," which codify how clients' orders must be handled (see section 7.5).

Brokers' interests may also conflict with those of their clients because they sometimes receive monetary inducements for routing orders to specific markets or dealers. In particular, in the United States, dealers or exchanges can make payments to brokers who route specific orders to them (e.g., small orders or orders from retail investors). A study by the SEC (2000) investigated whether this practice, known as "payment for order flow", affects order-routing decisions in U.S. options markets. Until 1999, there was not much competition among U.S. options exchanges, stock options often being traded only on the exchange where they were listed. For instance, Dell computers options were listed on the Philadelphia Stock Exchange (PSX) and traded exclusively there. This situation changed in August 1999 when the Chicago Board Options Exchange (CBOE) and the

American Stock Exchange (AMEX) announced that they would start trading these options as well. The decision triggered a very fierce battle for market share. As a result, option exchanges started making payments for order flow. The SEC (2000) study shows that by August 2000, 78 percent of retail customers' orders were routed pursuant to payment for order flow arrangements. This study further found that payments for order flow have an **(p.253)** impact on routing decisions: the brokerage firms accepting the payments rerouted their customers' orders to the exchanges that paid for order flow (and away from those that did not) much more often than did other brokers (who were much more likely to direct their orders simply to the exchange with the largest market share in the option).

Battalio and Holden (2001) suggest that competition between brokers will force them to pass payments for order flow to their clients, via either smaller commissions or improved services; the evidence is mixed. The SEC (2000) reports that in the options markets "few firms are passing along the benefits of payment for options order flow onto their customers in the form of either reduced commissions or rebates." In contrast, Battalio, Jennings, and Selway (2001) find that the net cost of trading with brokers selling order flow to Knight Securities (a major Nasdaq dealer) is lower than the net cost of trading with sole brokers who are not selling order flow to Knight Securities. This suggests that brokers may pass at least part of their payments along to their clients.

In any case, payment for order flow could also results in transfers among the different categories of investors. Indeed, several studies suggest that payments for order flow are used to cream-skim orders from uninformed traders (in particular, those from retail investors). Executing these orders at best quotes is profitable since the bid-ask spread contains a compensation for adverse selection costs (see Chapter 3). However, if a dealer or an exchange attracts these orders by paying for them, it increases the likelihood that other dealers or exchanges will receive a larger fraction of informed orders and so be more exposed to adverse selection effect. As a result, the bid-ask spreads offered by these other dealers or exchanges will be wider with payment for order flow than without (see Röell 1990, and exercise 3 in this chapter; for empirical evidence, see Bessembinder and Kaufman (1997)). If this is the case, the investors whose brokers do not receive payments for order flow will ultimately be worse off, and even those whose brokers do receive payments might be worse off if the practice widens the spread enough. Again, there is conflicting evidence on this issue. For instance, Easley, Kiefer, and O'Hara (1996) find that orders routed pursuant to payments do show larger effective bid-ask spreads. In contrast, Battalio (1997); Battalio, Greene, and Jennings (1997); and Battalio, Greene, and Jennings (1998) find that the introduction of market makers who pay for order flow in equity markets is associated with a narrowing of bid-ask spreads.

7.3. Liquidity Externalities

As the previous sections observe, order splitting between different market segments can neutralize the potential damage done by market fragmentation. **(p.254)** Alternatively, trading naturally gravitates to a single marketplace because each trader benefits from the presence of others. Intuitively, as traders expect a market to be more active, they are more inclined to participate, and by patronizing a market, they reinforce other traders' incentive to join. This liquidity-begets-liquidity effect is a force that works naturally to consolidate trading. Here (section 7.3.1) we illustrate this point using the asymmetric information model set out earlier. Liquidity externalities carry two important implications. First, they imply that securities markets are subject to market tipping, situations in which order flow migrates swiftly from one platform to another. Second, they isolate incumbent markets from competition with strong entry barriers. We discuss these implications and illustrate them with empirical evidence in section 7.3.2.

7.3.1 Liquidity Begets Liquidity

The model developed in section 7.2.1 can be easily adjusted to capture the idea that liquidity begets liquidity, by assuming that some liquidity traders have discretion in choice of trading venue. Specifically, suppose that M liquidity traders may choose to participate either in market A or in market B. The liquidity demand of discretionary trader $k \in \{1,.., M\}$, denoted by u_k, is normally distributed with mean zero and variance σ_d^2. The other liquidity traders are locked into one of the two markets. The total liquidity demand of the "captive" liquidity traders in market A, denoted by u_{Ac}, is normally distributed with mean zero and variance σ_{Ac}^2. We use a similar notation for the total liquidity demand of those based in market B. Last, we assume that all liquidity demands are independently distributed and that $\sigma_{Ac}^2 \rangle \sigma_{Bc}^2$.

Suppose that M_A liquidity traders with discretion in their trading location choose market A and $M_B = M - M_A$ discretionary liquidity traders choose market B. Then equation (7.3) in section 7.2.1 yields the depth of market A, and of market B:

$$\lambda_A = \frac{\sigma_v}{2\sqrt{M_A \sigma_d^2 + \sigma_{Ac}^2}}, \qquad \lambda_B = \frac{\sigma_v}{2\sqrt{M_B \sigma_d^2 + \sigma_{Bc}^2}}.$$

Suppose that $M_A \sigma_d^2 + \sigma_{AC}^2 \Big\rangle M_B \sigma_d^2 + \sigma_{Bc}^2$. In this case, $\lambda_{\mathrm{A}} \langle \lambda_B$: market A is deeper than market B. A discretionary liquidity trader in market B can get lower trading costs by switching to the deeper market A. This relocation makes market A deeper still and so reinforces the incentive for liquidity traders to gravitate to it. Eventually, this "gravitational pull" or liquidity-begets-liquidity effect leads all the discretionary traders to cluster in market A. This situation is an equilibrium, since no trader now has an incentive to move his liquidity demand to market B.

(p.255) If $M_A \sigma_d^2 + \sigma_{AC}^2 \Big\langle M_B \sigma_d^2 + \sigma_{Bc}^2$, the same reasoning implies that trading will concentrate in market B. Such an equilibrium can only arise if $\sigma_{Ac}^2 - \sigma_{Bc}^2 \langle M \sigma_d^2$.

Last, if $M_A \sigma_d^2 + \sigma_{AC}^2 = M_B \sigma_d^2 + \sigma_{Bc}^2$, the two markets have exactly the same depth and discretionary liquidity traders are indifferent between them. Again, the condition for obtaining this third equilibrium can be satisfied only if $\sigma_{Ac}^2 - \sigma_{Bc}^2 \leq M \sigma_d^2$.

Thus, when $\sigma_{Ac}^2 - \sigma_{Bc}^2 \leq M \sigma_d^2$, there are three equilibria.[11] In two of them, discretionary liquidity traders cluster in one market, which is therefore the most liquid. In the third equilibrium, they split equally and the two markets have exactly the same liquidity. This equilibrium is fragile, however: defection by even a single trader from one market to the other tips the balance and leads to concentration of trading there-a phenomenon known as market tipping.

The evolution of trading volume for cross-listed securities is a good illustration of the gravitational pull effect. Halling et al. (2008) study the distribution of trading volume between the domestic and the foreign market for cross-listed stocks. They observe that foreign trading is active just after the cross-listing date but decreases dramatically over the next six months. In other words, trading activity “flows back” to the home market in the months after the cross-listing, which is consistent with the presence of liquidity externalities that induce the concentration of trading in a single marketplace.

Box 7.2 Tipping: The Case of Matif In 1998

The evolution of trading on the Matif, a French bond futures market, is a good illustration of how trading volume can shift quickly from one venue to another. Matif was an open outcry market since 1986. In 1998, the market organizers decided to switch to an electronic market. Floor brokers went on strike to oppose the switch; during the strike, trading migrated to a competing electronic trading system (Eurex) in Germany. After the strike, this trading never migrated back to Matif, despite several coordinated attempts by French banks to reactivate it. This episode can be seen as an instance of multiple equilibria in the model described above, where the closure of Matif–say, market B–induced discretionary liquidity traders to opt for Eurex–market A–independently of their expectations regarding other traders. In other words, the mere suspension of trading on market B permanently coordinated traders’ choices on a different equilibrium with all trading concentrated in market A.

(p.256) 7.3.2 Low-liquidity Traps

The gravitational pull effect is a force that naturally consolidates trading in securities markets. It implies that it is hard for a new trading platform to cut into the market of an incumbent, even if the platform has the potential to be more liquid.

For instance, in the previous model, trading can concentrate in market B even though liquidity traders with discretion would trade more cheaply if they all reconvened in market A. Indeed, if one liquidity trader expects the others to remain in market B, then he is better off staying there. This expectation is self-fulfilling and inefficiently locks traders into market B. All discretionary liquidity traders would benefit from switching to market A if they could do so collectively, but they individually have no incentive to do so. This coordination problem traps them in a low-liquidity equilibrium.

From the point of view of an exchange that wants to compete for the order flow, this translates into a powerful barrier to entry. To overcome it, entrant venues may disseminate information on their trading volumes to show that they attract some trading and make other potential participants more optimistic about their success. This is one reason why the new electronic platforms often publicize their volume statistics so aggressively. Another strategy is to organize meetings among prospective intermediaries ("users") to facilitate coordination. For instance, in the previous model, suppose that market A manages to attract at least $M_A \rangle \left(M\sigma_d^2 + \sigma_{AC}^2 - \sigma_{BC}^2 \right) / 2\sigma_d^2$ discretionary liquidity traders. In this case, the condition $M_A\sigma_d^2 + \sigma_{AC}^2 \rangle M_B\sigma_d^2 + \sigma_{Bc}^2$ is satisfied and the balance tips in favor of market A. There is a sharp decline of activity in market B, since all the remaining discretionary traders will also switch to market A. The decline thus becomes permanent, as trading is now concentrated in market A in equilibrium.

7.4. The Benefits of Fragmentation

Section 7.2 portrays the dark side of market fragmentation, and section 7.3 explains how liquidity externalities may lead to the concentration of trading in a single venue. Yet market fragmentation also has a benefit: it enhances "competition for order flow," by which markets try to attract buy and sell orders. This competition operates at two levels: between platforms and between liquidity providers operating within different platforms.

7.4.1 Curbing the Pricing Power of Exchanges

The concentration of trading on a single trading platform gives this venue monopoly power. Hence, in the absence of competition, a profit-maximizing **(p.257)**

Table 7.3. Fees Per Share in Cents Per Round Lot for Limit Orders (Make Fee) and Market Orders (Take Fee)

	NYSE Stocks		Other Stocks		NASDAQ Stocks	
	Make Fee	**Take Fee**	**Make Fee**	**Take Fee**	**Make Fee**	**Take Fee**
BATS	−24	25	−24	25	−24	25
EDGX	−25	30	−30	30	−25	30
LavaFlow	−24	27	−24	27	−24	27
Nasdaq	−20	30	−20	30	−20	30
NYSEArca	−23	30	−22	30	−23	30

Source: *Traders Magazine*, "Price of Liquidity," August 2009, p.45.

exchange will set its fees at the monopolistic level, constrained only by the elasticity of traders' demand for liquidity services.[12] Recent developments in trading fees in the United States and Europe suggest that this concern is realistic. In fact, the entry of new trading platforms in the United States (BATS) and in Europe (Chi-X or BATS Europe) triggered a cut in the fees charged by incumbent markets. As an example, box 7.3 chronicles the evolution of fees on Euronext in 2003 and 2004 when Deutsche Börse and then the LSE launched trading platforms that allowed brokers to trade Dutch blue chips listed on Euronext Amsterdam.

The escalation of intermarket competition has also led some trading platforms to offer "liquidity rebates" to brokers who submit limit orders. Table 7.3 indicates that in 2009 U.S. trading platforms paid brokers who submitted limit orders, but charged them a fee for market orders. The table shows that the revenue per share traded for the platforms is a fraction of a cent. For instance, BATS charges twenty-five cents per round lot (one hundred shares) to "takers" (investors submitting market orders) and rebates twenty-four cents to "makers" (those submitting limit orders). Thus, it earns 0.01 cents per share traded. On October 10, 2008, 838,488,549 shares of NYSE stocks were traded on BATS (about 9 percent of the day's total trading volume in these stocks).[13] Thus, limit order traders collectively received about $2.01 million in rebates from BATS, while market orders traders paid about $2.10 million in fees to BATS.

Liquidity rebates are similar to the payments for order flow discussed in section 7.2.4, but they go to liquidity suppliers, not demanders. There is one important difference, however. Payments for order flow are usually contingent **(p.258)** on some characteristics of the clients (e.g., retail orders), while liquidity rebates are not. Hence, their economic role is likely to be different. In both cases, their effect might depend on whether brokers pass their rebates along to their clients, or charge clients for the fees they pay to platforms. Exercise 5 in Chapter 6 considers the effects of make and take fees on the bid-ask spread.

Competition among trading platforms also forces them to install cutting-edge trading technologies. For instance, competition from electronic communication networks in the United States prompted Nasdaq and the NYSE to revamp their trading mechanisms. Another example is the "latency" war among platforms. Some high-frequency traders (e.g., GETCO, Optiver, etc.) specialize in automated market making and therefore play a critical role in the provision of liquidity to other market participants. To attract them, platforms have updated their trading systems to reduce latency, that is, the lag between an event on the platform (e.g., submission of an order) and the sending of a message about this event to market participants (e.g., "Your order has been executed."). As a consequence, the time interval between orders and trades is now around one millisecond.

Box 7.3 Rivalry Among Stock Exchanges

In May 2003, the Deutsche Börse and the LSE separately announced their intention to launch new trading platforms in the Dutch equity markets. At that time, trading in Dutch stocks was mainly concentrated on NSC, a platform operated by Euronext. Euronext reacted to this competitive threat by repeatedly reducing its fees in 2003 and 2004. For instance, in January 2004, Euronext set at e0.3 its order entry fee (in addition to execution fees). Then it halved this fee for limit orders on April 4, 2004, less than two months before the LSE launched its own trading platform for Dutch stocks, EuroSETS, on May 24, 2004. On that day, Euronext suspended the order entry fee on market orders until the end of July and even offered a rebate on total execution costs for these orders. Eventually, on January 31, 2005 Euronext announced that it would stop charging order entry fees on both market and limit orders and also cut its execution fees. EuroSETS's market share remained modest,[14] but the mere presence of this platform was sufficient to trigger a significant decrease in trading fees for Dutch stocks.

(p.259) 7.4.2 Sharper Competition among Liquidity Providers

Beside encouraging competition between platforms, market fragmentation can also intensify competition among liquidity providers, and thereby increase consolidated liquidity in comparison to trading concentrated in a single platform. At first glance, this possibility seems to contradict the analysis set out in section 7.2.2. There, however, it was assumed that liquidity demanders could not split their orders between markets and that liquidity providers could not operate simultaneously on several markets. These assumptions may not adequately describe today's electronic equities markets, where traders often have the technology to provide liquidity where it is most profitable or to demand liquidity where it is cheapest at any instant in time.

In this section we study the effects of market fragmentation when traders can split their orders freely in the context of platforms that use limit order markets, not call markets as in previous sections of this chapter. At the end of section 7.2.2, we showed that when platforms operate as call markets, fragmentation has no effect in the presence of order splitting. With limit order markets, this is not the case: here, the coexistence of multiple LOBs results in greater consolidated depth, that is, a greater number of shares posted at each price in the market.

To see this point, consider figure 7.1, which describes an extension of the model of limit order trading considered in section 6.2 of Chapter 6. At time 1, investors can submit their limit orders for a security in two limit order markets, the "incumbent" I and the "entrant" E. The payoff v of the security is realized at time 3 and $\mathrm{E}(v) = \mu$. The tick size, Δ, is identical in the two markets, and $A_1 = \mu + \Delta$ is the first ask price on the grid above the expected value of the security. For brevity, we focus on the determination of the number of shares, denoted by Y^I and Y^E, offered at price A_1 in each market. Considering other possible prices for limit orders delivers the same insights (see Foucault and Menkveld, 2008).

At time 2, a liquidity trader comes to buy or sell shares with equal probabilities. The cumulative probability distribution of the trade size q is denoted by

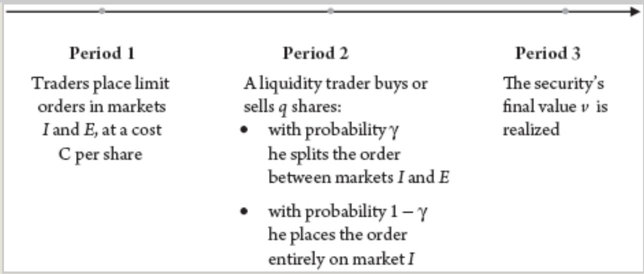


Figure 7.1. Competition between limit order markets: time line

(p.260) $F(q)$. For simplicity, we assume that there is no informed trading, but investors submitting limit orders pay an order submission cost C (equal in the two platforms for simplicity). As in section 6.2 of Chapter 6, this cost is the friction that limits the supply of shares at price A_1.

With probability $1 - \gamma$, the liquidity trader can access market I only. This possibility captures the fact that some brokers may ignore offers posted in some platforms (especially new ones) in order to achieve faster execution or to economize on search costs. Moreover, without an adequate routing technology, monitoring prices in both platforms and splitting orders is cumbersome and time-consuming. With probability γ, the liquidity trader can access both platforms. He then splits her order between the two platforms to minimize the total price impact.

To see how this splitting strategy works, suppose that the liquidity trader wishes to buy q shares of the security. If $q \geq Y^I + Y^E$, he optimally fills all limit orders posted at price A_1 in both trading platforms. The remainder of the order executes against limit order at higher prices. If $q < Y^I + Y^E$, the liquidity trader has several ways to execute the order against the limit orders standing at price A_1 in each market. He may give priority to market I, that is, first buy shares in market I and then consider market E if needed, or else may tap market E first and then buy more shares in market I if necessary. The liquidity trader is indifferent between these routing strategies since they result in the same total payment, $A_1 \times q$. Hence, we assume that the liquidity trader gives priority to market I with probability $\frac{1}{2}$.

As in section 6.2 of Chapter 6, the depth at price A_1 in market j (for $j = I, E$) is determined by a zero-profit condition for the marginal limit order posted at this price in that market. To derive this zero-profit condition, we must first compute the probabilities of execution of the marginal limit order placed at price A_1 in the two markets.

Consider the marginal limit order placed at price A_1 in market I. If the liquidity trader at time 1 only has access to I or gives priority to market I, the marginal limit order at the best ask price in market I executes if and only if $q > Y^I$. Thus, in this case, its execution probability is $1 - F(Y^I)$. If the investor arriving at time 1 can trade in both markets and gives priority to market E, the marginal sell limit order at price A_1 in market I executes if and only if $q > Y^I + Y^E$. In this case, therefore, its execution probability is equal to $1 - F(Y^I + Y^E)$. So the unconditional execution probability of the marginal limit order posted at price A_1 in market I is

(7.11)

$$P_I\left(Y^I,Y^E;\gamma\right)=\frac{1}{2}\left[\left(1-\gamma+\frac{\gamma}{2}\right)\left(1-F\left(Y^I\right)\right)+\frac{\gamma}{2}\left(1-F\left(Y^I+Y^E\right)\right)\right].$$

(p.261) Now consider the marginal limit order posted at price A_1 in market E. If the liquidity trader trades in both markets and gives priority to market E, this marginal limit order executes if and only if $q\Big\rangle Y_1^E$. If priority is instead given to market I, it executes if and only if $q\Big\rangle Y^I+Y_1^E$. In all other cases, this marginal limit order does not execute. Hence its unconditional execution probability is

(7.12)

$$P_E\left(Y^I,Y^E;\gamma\right)=\frac{\gamma}{4}\left[\left(1-F\left(Y^E\right)\right)+\left(1-F\left(Y^I+Y^E\right)\right)\right].$$

In a competitive equilibrium, the number of shares supplied in each market at price A_1 must be such that no limit order trader finds it profitable to expand the queue of limit orders at this price, in either market. Let $Y_1^{j*}(\gamma)$ be the number of shares offered in equilibrium at price A_1 in market j. Obviously, if $\gamma = 0$, market E attracts no order since there are no investors routing market orders to it. Thus, $Y^{I*}(0)$ is the number of shares offered at price A_1 when all trades take place in market I. As in section 6.2 of Chapter 6, if $Y_{I*}(0) \rangle 0$, it solves:

$$P_I\left(Y^{I*}(0),0;0\right)(A_1-\mu)=C,$$

that is, using equation (7.11) for $\gamma = 0$,
(7.13)

$$\frac{1}{2}\left[\left(1-F\left(Y^{I*}(0)\right)\right]=\frac{C}{\Delta}.$$

As $1 - F(Y^{I*}(0) \langle 1$, a necessary condition for the existence of an equilibrium with $Y^{I*}(0) \rangle 0$ is $2C \langle \Delta$. In the rest of this section, we assume that this condition is satisfied.

Now suppose that $\gamma \rangle 0$. If both markets attract limit orders at price A_1, then $Y^{I*}(\gamma) \rangle 0$ and $Y^{E*}(\gamma) \rangle 0$. In this case, the zero-profit conditions on the marginal limit order in the two markets require (from section 6.2 of Chapter 6):

(7.14)

$$P_I\left(Y^{I*},Y^{E*};\gamma\right)=\frac{C}{A_1-\mu}=\frac{C}{\Delta},$$

(7.15)

$$P_E\left(Y^{I*},Y^{E*};\gamma\right)=\frac{C}{A_1-\mu}=\frac{C}{\Delta}.$$

For a specific parameterization of the probability $F(\cdot)$, the equilibrium number of shares offered in each market at price A_1 can be computed explicitly by solving this system of equations (see exercise 4).

In any case, if the two markets coexist (i.e., both attract limit orders at price A_1), in equilibrium the number of shares offered in each at the best price must be such that their execution probabilities of the marginal limit orders are equal:

$$P_I\left(Y^{I*},Y^{E*};\gamma\right)=P_E\left(Y^{I*},Y^{E*};\gamma\right).$$

(p.262)

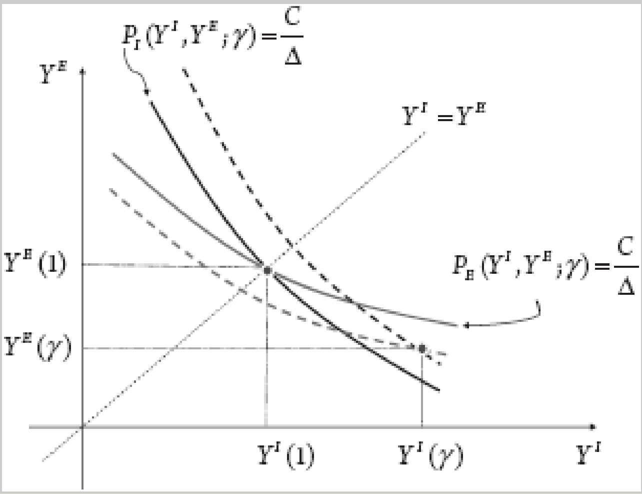


Figure 7.2. Zero-profit depth at the best quote in market *I* (black) and market E (grey)

This condition is intuitive; otherwise, the investor who placed the marginal limit order in the market with the smaller execution probability would have an incentive to cancel and resubmit in the other market at the same price. This would increase his expected profit, since the submission cost is the same.

Figure 7.2 illustrates how the equilibrium is determined, by plotting the number of shares Y^I and Y^I offered in the two markets at price A_1. The two solid lines in the figure are drawn for $\gamma = 1$, the case in which liquidity traders always access both platforms: the black curve P_I is the set of pairs (Y^I, Y^E) such that the execution probability of the marginal limit order in market I is equal to $\frac{c}{\Delta}$. Similarly, the grey curve P_E is the set of pairs $\left(Y^I,Y_1^E\right)$ such that the execution probability of the marginal limit order in market E is equal to $\frac{c}{\Delta}$. The equilibrium numbers of shares offered at price A_1 in each market are found at the intersection of the two curves. (The two dashed curves instead show how the equilibrium changes when γ decreases below 1, i.e., the case in which liquidity traders cannot always access both platforms, discussed in the next section.)

When $\gamma = 1$, the expressions for the execution probabilities in the two markets are symmetric. Hence, $Y^{I*}(1) = Y^{E*}(1)$, as shown in figure 7.2: the number of shares offered at price A_1 is the same in both platforms, so that the total number of shares offered at price A_1 (the consolidated depth at this price) is $2Y^{I*}(1)$. Why is there no concentration of trading in a single platform in this case? To answer this question, suppose that some shares were offered at price A_1 in market I but not yet in market E, and consider an investor who wants to place a limit order at price A_1. If it is placed in market I, the execution probability is $P_I\left(Y^I,0;1\right) = \frac{1}{4}\left(1 - F\left(Y^I\right)\right)$. If instead it is placed in market E, the execution probability is $P_E\left(Y^I,0;1\right) = \frac{1}{4}\left(2 - F\left(Y^I\right)\right)\rangle\frac{1}{4}\left(1 - F\left(Y^I\right)\right)$. Thus, the investor is better off submitting the limit order at price A_1 in market E, **(p.263)** as it yields a higher execution probability: doing this, the investor jumps the queue of limit orders already posted in market I at the same price, because time priority is not enforced across markets.

Intuitively, this “queue-jumping” possibility intensifies competition among limit order traders, as it allows latecomers, through competition, to reduce the rents of those who submitted their limit orders first.[15] As a result, the cumulative depth at price A_1 is greater when the two markets coexist, that is $Y^{E*}(1) + Y^{I*}(1)\ Y^{I*}(0)$. To see this, notice that the zero-profit conditions (7.14) and (7.15) imply

(7.16)

$$\frac{1}{4}\left[\left(1 - F\left(Y^*(1)\right)\right) + \left(1 - F\left(2Y^*(1)\right)\right)\right] = \frac{C}{\Delta},$$

where $Y^*(1) \stackrel{def}{=} Y^{I*}(1) = Y^{E*}(1)$. Equations (7.16) and (7.13) in turn imply that
(7.17)

$$F\left(Y^{I*}(0)\right) = \frac{F\left(Y^*(1)\right) + F\left(2Y^*(1)\right)}{2}.$$

As $F(x)$ increases with x, from equation (7.17) we obtain:

$$2Y^*(1)\Big\rangle Y^{I*}(0)\Big\rangle Y^*(1).$$

That is, when both markets are active the total number of shares offered at price A_1 (the consolidated depth) is greater, even though the number of shares offered at price A_1 in each market is smaller than when only market I is active. Hence, for traders who can submit market orders in both markets, the coexistence of two limit order markets improves liquidity at the best quotes.

This analysis shows that it can be misleading to consider the effect of market fragmentation on one platform only. For instance, suppose that initially market I operates alone and that $\gamma = 1$. In this case, the depth of market I at price A_1 is $Y^{I*}(0)$. Now suppose that platform E enters. We should observe a drop in the liquidity of market I since $Y^{I*}(0) > Y^{*}(1)$. Yet investors submitting market orders smaller than $2Y^{*}(1)$ and larger than $Y^{I*}(0)$ are better off, since they can execute at a better price. Investors who submit market orders smaller than $Y^{I*}(0)$ are indifferent, and those whose market orders are larger than $2Y^{*}(1)$ can be shown to be better off. Indeed, the improvement in liquidity at the best quote is propagated throughout the LOB via the channel just described for the depth at price A_1. Hence, despite the decrease in liquidity of market I, liquidity demanders are better off because consolidated depth increases at all prices.

As explained previously, the lack of time priority across markets is crucial to this result. The argument also relies on the fact that traders placing limit **(p.264)** orders at the head of the queue in each market earn positive expected profits. As explained in section 6.3.1 of Chapter 6, this is because the tick size is strictly positive; if it were nil, the number of competing platforms would have no effect on consolidated depth in the framework considered here.[16]

Box 7.4 Competition for Orders and Liquidity: Some Evidence

In 2004, the LSE launched a new limit order market (EuroSETS)to enable Dutch brokers to trade stocks listed on Euronext Amsterdam. Until then, these stocks were traded almost exclusively on NSC, a limit order market operated by Euronext. The launch of EuroSETS accordingly constitutes a good experiment to test whether intermarket competition enhances liquidity in an environment that resembles the one in this section (in particular NSC and EuroSETS are both limit order markets and their tick size is the same). Foucault and Menkveld (2008) analyze the effects of EuroSETS entry on consolidated depth, using snapshots of the LOBs in each market, and find that consolidated depth increased dramatically, as implied by the model presented in this section.

More evidence on the impact of fragmentation on the liquidity of limit order markets comes from Degryse, de Jong, and Van Kervel (2011), a study of a sample of fifty-two Dutch stocks (large and mid-cap) in 2006–09. This is an interesting period, since after 2007 the European market became significantly more fragmented owing to the implementation of MiFID (see section 7.5.2 below). The sample stocks are listed on Euronext Amsterdam and trade on Chi-X, Deutsche Börse, Turquoise, BATS trading, Nasdaq OMX and SIX Swiss Exchange. Degryse, de Jong, and Van Kervel (2011) measure market fragmentation using the Herfindahl index, a measure of the dispersion of the trading volume in a stock across the available trading platforms. Their data are very rich, covering LOBs of each trading platform for each stock, so they can study the relationship between their index of market fragmentation and measures of cumulative depth at each price point in all these books. They find a positive relationship between the market fragmentation and the consolidated liquidity of a stock, as predicted by the model.

(p.265) 7.4.3 Trade-throughs

Let us compare the case in which liquidity traders do not always have access to both the incumbent and the entrant market ($\gamma \langle 1$) with the previously analyzed case in which they do ($\gamma = 1$). A decrease in γ reduces the execution probability of the marginal limit order in market E, other things being equal. As a consequence, it becomes less profitable to submit a limit order in market E. But the execution probability of the marginal limit order in market I increases, so it becomes more profitable to submit a limit order there. Thus, in figure 7.2, the curve that shows the pairs (Y^I, Y^E) such that the execution probability of the marginal limit order in market I is equal to C/Δ shifts upward, while the other curve shifts downward, as shown by the dashed curves. The new equilibrium point is at the intersection of these two curves—clearly, in equilibrium,

$$Y^{I*}(\gamma)\Big\rangle Y^{*}(1)\Big\rangle Y^{E*}(\gamma) \text{ when } \gamma\Big\langle 1.$$

Hence, when $\gamma \langle 1$ the cumulative depth in market I is greater than when $\gamma = 1$, while that in market E is lower. More generally, the cumulative depth in market E diminishes as γ decreases, and there is a threshold γ^c below which no limit order can be profitably posted at price A_1. This threshold can be found by solving for the value of γ such that the solution to the system of equations (7.14) and (7.15) is $Y^{E*}(\gamma^c) = 0$. This yields $\gamma^c = \frac{4C}{\Delta+2C}$..[17] It can also be shown that if $\gamma \langle \min\{\gamma^c, 2/3\}$, then no limit order will be submitted in this market at any price. Intuitively, in this case the expected revenue from a limit order in market E is too low to cover the cost, because this order is unlikely to execute, due the paucity of investors who track limit orders in market E. Thus, to be active, a trading platform needs some *critical mass* of traders who pay attention to its quotes.

This feature forms a barrier to entry for a new platform. To see this, suppose that market E is a new trading platform. It will attract some trades if and only if γ is large enough. Actually, in reality, the value of γ is partly determined by investors' beliefs about the chances of success of the new market. Traders will indeed develop technologies to trade in both markets ("smart routers") if and only if they expect the new market to be liquid enough, so that the reduction in trading costs achieved by accessing both markets exceeds the additional cost of multimarket trading. But, as we have just seen, the liquidity of the entrant market depends critically on the fraction of traders who monitor quotes in both markets. This can lead to a self-fulfilling prophecy, in which traders expect the entrant market to be unsuccessful and so do not consider the offers on it, **(p.266)** and —precisely for this reason–market E fails to attract liquidity, thus confirming traders' expectations.

In other words, the entry of the new trading platform is blocked by a chicken-and-egg problem: to take off it needs a critical mass of traders following its quotes, but traders will pay attention only if they expect the platform's liquidity to be great enough. For example, this was the key hurdle for Tradepoint, a limit order platform that tried to draw order flow away from the LSE, as described by Tradepoint's CEO at the time, Nic Stuchfield:

> When I was CEO of Tradepoint (now virt-X), my team and I spent a considerable amount of effort "selling" the exchange to traders. However, although they all signed up as members, they did not use the market. One major reason was that access to the market was not connected to their trading systems. Even when better bids and offers appeared on our order book, the (momentarily) inferior prices available on the LSE were hit and lifted. Potential users simply could not see, nor easily access the market. If the Tradepoint terminal was at the end of the desk, it was not accessible. The solution [...] was to get Tradepoint integrated into the main order management systems [...]This proved to be easier negotiated than implemented [...] The traders had many other priorities and we could not demonstrate the required liquidity. Think chicken and egg again! (Stuchfield, 2003)

This problem can be alleviated if regulation bans violations of price priority between trading platforms—so-called "trade-throughs," in which a security trades at a worse price in one platform than in another at the same moment. For instance, suppose one hundred shares are offered for sale at A_1 = \$50 in markets I and E. The second best ask price is \$51, at which two hundred shares are offered in each market. Now consider a broker who wants to purchase two hundred shares and plans to execute this trade with a market order. Ignoring the offer in market E, the broker buys one hundred shares at \$50 and one hundred shares at \$51 in market I only. This routing decision is a "trade-through," since a buy order executes at \$51 while a sell limit order at \$50 is posted in market E.

Trade-throughs do occur in practice. In their study of competition between EuroSETS and NSC (see box 7.4), Foucault and Menkveld (2008) find that a significant fraction of market orders were executed in NSC (the incumbent) even when EuroSETS (the entrant) offered a better price. Similarly, trade-throughs occur in U.S. equities markets, albeit much less commonly as they are forbidden (see Hendershott and Jones (2005b)). In the United States, venues are required to reroute incoming market orders to the market posting the best **(p.267)** quote (provided that automatic execution at this quote is possible). This is the order protection rule or trade-through rule, designed specifically to prevent trade-throughs. The rationale is that trade-throughs discourage liquidity provision, as the SEC noted in its 2006 release on Regulation NMS:

> "Price protection encourages the display of limit orders by increasing the likelihood that they will receive an execution in a timely manner and helping preserve investors' expectations that their orders will be executed when they represent the best displayed quotation."(p. 36)

This reasoning is in line with the model developed in this section whereby trade-throughs will occur when $\gamma^c < \gamma < 1$. Consider an investor who wishes to place a market order size equal to $q > Y^{I*}(\gamma)$. He should optimally buy part or all of the shares posted at price A_1 in market E. However he will do so only with probability γ. With probability $1-\gamma$, he will instead buy $q - Y^{I*}(\gamma)$ shares at higher prices in market I, a trade-through whose likelihood is therefore $(1 - \gamma)(1 - F(Y^{I*}(\gamma))$. Trade-throughs never happen only if $\gamma = 1$ because in this case all investors optimally execute their market orders against limit orders available in both markets.

Thus, comparing a situation in which $\gamma < 1$ with one in which $\gamma = 1$ is like comparing market structures with and without a trade-through rule. As noted, the number of shares offered in market E is smaller when $\gamma < 1$, because a small γ implies a low likelihood of execution for limit orders placed in market E, and therefore low profitability of limit orders in this market. Hence, a no-trade-through rule effectively encourages investors to submit limit orders in market E by improving their chance of execution.

The model is readily generalized to account for fee differences between trading venues (say, in order submission costs). These fees create asymmetries between the platforms (much like parameter γ does), but do not change the overall logic of the model (see exercise 4).

In this model we have analyzed competition between two platforms, taking their design (both being limit order markets) as given. This is of course incomplete. A full-fledged analysis of intermarket competition would have to allow the design of competing trading platforms to be itself endogenous: the venues choose their fees (which could affect C in the previous model) and design their trading mechanisms. In particular, platforms may want to differentiate their mechanisms to relax competition in fees, in the same way that firms differentiate their products. Such an analysis is needed to see why there is heterogeneity in trading mechanisms and to grasp the logic behind pricing policies such as liquidity rebates. Academic research on these issues is still scanty.

(p.268) 7.5. Regulation

This chapter begins by setting out the risks and costs of fragmentation: violations of price priority across markets, price dispersion, and diminished liquidity externalities. The second part of the chapter, however, shows that fragmentation also benefits investors by fostering competition among trading platforms. Hence, regulators have the problem of capturing the benefits while lowering the costs of intermarket competition. In this section we explain how the U.S. regulator tried to achieve this difficult balance in designing Regulation NMS and the very different way in which the European Union approached the matter with its MiFID legislation.

7.5.1 Regulation NMS

Regulation NMS went into force in 2006 and 2007.[18] It consists of a series of rules to promote intermarket competition in U.S. equities markets while curtailing the harmful effects of fragmentation.

To understand RegNMS, the historical perspective is helpful. In the 1970s, NYSE-listed stocks traded mainly on the NYSE and not elsewhere. But regional U.S. exchanges and the OTC market gradually captured a larger part of this trading. The U.S. Congress was concerned that this evolution could lead to inefficiencies; in 1975, it mandated the SEC to create a National Market System (NMS). Congress envisioned five purposes for the NMS: (i) economically efficient execution of securities transactions; (ii) fair competition among brokers and dealers and between markets; (iii) availability to brokers, dealers, and investors of information about quotes and transaction; (iv) best execution for investors' orders; and (v) the opportunity to execute orders without the participation of a dealer.

The SEC initially proposed to consolidate all limit orders for each stock in a single file where orders would be executed according to price and time priority. However, this proposal met with strong opposition from exchanges and market makers (see Colby and Sirri, 2010). In the end, the SEC opted for a more decentralized approach: the NMS would be composed of multiple trading venues, linked together by technology. The resulting market structure that emerged to achieve this goal was based on two pillars: the intermarket trading system (ITS) and the Consolidated Tape Association (CTA). The ITS was intended to make sure that price priority would be enforced across all markets. Hence, an order routed first to one exchange (say the NYSE) would **(p.269)** then be rerouted to another (a regional exchange perhaps) if the latter posted a better price, unless brokers or market makers on the first exchange decided to improve the price. Implementation of this no-trade-through rule required real-time information on best bids and offers in every trading venue for a security.[19] The CTA was designed to collect and disseminate this information. Importantly, the no-trade-through rule did not apply to stocks traded on the OTC market, in particular stocks listed on Nasdaq.

However, the developments of technology called for revision of this system. The possibility of computerized trading venues to match buy and sell orders with no need for market makers or floor brokers led to the development of electronic communication networks (ECNs), such as POSIT (a crossing network) and Island (a limit order book) in the 1990s. Speed of execution was a strong selling point of theirs against the more traditional markets, such as Nasdaq and NYSE. The no-trade-through rule, however, required orders to be routed to the NYSE when it was quoting a better price, even though it had slower execution (handled manually by the specialist assigned to each stock). For this reason, several ECNs elected to stay out of the NMS. This became problematic as the ECNs' market share grew, in sync with traders' intensifying demand for rapid execution.

The so-called order protection rule (or trade-through rule) of RegNMS is intended to resolve this problem. The rule extends the protection against trade-through to all NMS stocks, but a quote is protected only if it is immediately and automatically accessible. An implication is that manual quotes sourced by floor-based trading systems (such as the NYSE) were no longer protected against trade-throughs. This new regulatory regime lent further impetus to electronic trading and led to an erosion of the NYSE's market share in its listed stocks. As a result, the NYSE had to overhaul its trading system to ensure its quotes were electronic, so that trade execution would be sufficiently immediate to qualify for order protection.

The rise of the ECNs also prompted platforms to devise new pricing strategies. In particular, many ECNs now use the so-called maker-taker pricing model. On a limit order market, each transaction involves a match between a limit order (the “maker”) and a market order (the “taker”). Market orders are viewed as consuming (“taking”) the liquidity supplied (“made”) by limit orders. For a platform, the maker-taker model consists in charging a fee for market orders that fill against limit orders on the platform and rebating a fraction of this fee to the filled limit orders (see table 7.3).

This business model can produce price distortions when combined with the trade-through rule. Liquidity rebates do narrow bid-ask spreads in that they **(p.270)** decrease the cost of providing liquidity.[20] Thus, liquidity rebates are a way for a platform to display the best prices in the market more frequently. It does not follow, however, that routing market orders to this platform is optimal because the total trading cost for takers includes the take fee. Yet the trade-through rule applies to quotes, not quotes cum fees. Thus, by granting very generous liquidity rebates, a platform can capture a large share of the order flow (thanks to the trade-through rule) while still earning significant profits by charging a high take fee.

The access rule of RegNMS addresses this problem by capping the take fee (or access fee) at $0.003 per share. The access rule also prevents platforms from giving preferential treatment (in terms of priority, speed of execution, or fees) to members/subscribers over non-members/subscribers.

Over the year, the CTA’s revenues from the sale of trade and price information became a significant source of income for NMS-members. The allocation of this revenue between members was based on the *number* of trades reported to the consolidated tape, a sharing rule that was creating perverse incentives for platforms. To obtain a larger share, they had an incentive to induce traders to shred their trades into multiple small trades or even engage in wash sales to artificially inflate trading volume.

The market data rules of RegNMS address this issue by changing the revenue-sharing mechanism. The new mechanism is based more on each platform’s contribution to finding the right price. In particular, the new formula rewards the trading venues that frequently set the best bid and ask prices for a given stock.

The last ingredient of RegNMS is the “sub-penny rule,” which imposes a minimum price variation (tick size) of $0.01 for all NMS securities over $1.[21] The intent was to prevent platforms from competing in the coarseness of their grid size. By setting a very fine grid (with a tick smaller than a penny), a platform could attract limit orders that just barely undercut the best bid or offer price in another market. These orders can generate trades for the platform (due to the trade-through rule), but they gain priority by improving prices for insignificant amounts while undermining the incentives of other traders to quickly post good prices.

RegNMS is designed to secure the benefits of competition without incurring the costs of fragmentation, but it stops short of emulating a nationwide consolidated LOB with multiple points of entry. Indeed, while the regulation ensures that small market orders are executed at the best possible price available **(p.271)** on any trading platform, price protection is not given to displayed limit orders that provide market depth at prices outside the best bid and offer. This means that traders who need to fill a large order that cannot be fully executed at the NBBO (after first executing against the protected orders) must hunt around for further market depth, which is often not publicly posted but available in hidden orders on the exchanges or in "dark pools" of liquidity. In this sense, RegNMS does not eliminate all the adverse price disparity and uncertainty effects of fragmentation.

7.5.2 MiFID

Until recently, E.U. security market regulations allowed member countries to impose "concentration rules": under the 1993 Investment Services Directive (ISD), member states could require transactions in equity securities to be carried out on a "regulated market." Therefore some member states–France, Italy, and Spain, among others–maintained rules requiring execution of share trades on their national stock exchange. Others, such as the United Kingdom, left intermediaries free to execute trades off-exchange and also to internalize them, provided they complied with general best execution requirements.

In 2004 the European Union introduced MiFID, a new regulatory regime, which went into effect on November 1, 2007. The main change relating to fragmentation was a ban on the concentration rules, ushering in free competition between trading platforms. Specifically, MiFID allows three types of trading systems: (i) regulated markets (RMs), (ii) systematic internalizers (SIs), and (iii) multilateral trading facilities (MTFs). Regulated markets are the incumbent exchanges (e.g., NYSE-Euronext and Deutsche Börse). Multilateral Trading Facilities are functionally similar but operate under different regulatory requirements (Chi-X or BATS-Europe are examples of MTFs). Systematic internalizers are "investment firms" (i.e., brokers or banks) that opt to match ("internalize") buy and sell orders from their clients in-house (either by acting as market makers or by crossing buy and sell orders from different clients).

The abolition of the concentration rules triggered the entry of a series of MTFs: Chi-X (March 2007), Turquoise (March 2007), BATS Europe (April 2008), Nasdaq OMX (September 2008), NYSEArca (March 2009), and Burgundy (May 2009). On the other hand, anticipating entry by other trading platforms, the existing exchanges have pushed for consolidation: the Paris, Amsterdam, Brussels, and Lisbon stock exchanges merged into Euronext; Stockholm's OMX AB acquired and now operates stock exchanges in Sweden, Finland, Denmark, Iceland, Estonia, Lithuania, and Latvia; the LSE acquired **(p.272)** Borsa Italiana and, more recently, Turquoise. Often, the new trading platforms and regulated markets have also launched "dark pools," which are simply crossing networks that match buy and sell orders at pre-determined points in time at the midquote set in some other market (see Chapter 1). As of April 2010, there were 90 RMs, 135 MTFs, and 12 SIs registered in thirty different European countries.

Thus, with the advent of MiFID, Europe moved closer to the U.S. regulatory framework which encourages competition among platforms. However, there are important differences. The U.S. approach interconnects platforms to enforce best execution at the best price nationwide. In Europe, there is no formal order protection rule requiring the routing of market orders to the platform posting the best price. In fact, the notion of cross-market best bid and offer is not yet defined in Europe. Rather, MiFID mitigates the harmful effects of market fragmentation by best execution rules and order handling rules.

More specifically, Article 21 requires that "investment firms take all reasonable steps to obtain, when executing orders, the best possible result for their clients taking into account price, costs, speed, likelihood of execution and settlement, size, nature or any other consideration relevant to the execution of the order," unless the firm receives a specific instruction from the client. Thus, MiFID does not define best execution only with reference to the price, as RegNMS effectively does with the order protection rule. Consequently, as empirical studies show, trade-throughs now happen relatively frequently in European markets.[22]

In handling orders, investment firms must abide by transparency rules that mimic the U.S. display rule (Rule 11Ac1-1) and quote rule (11Ac1-4). First, when dealing in shares listed on a regulated market, market makers must immediately disclose any unfilled limit orders so as to make them easily accessible to other market participants, unless instructed otherwise by the customers (MiFID, Article 22). This in effect exposes market markers to competition from public limit orders as in the United States. Second, if they deal in shares traded on regulated and liquid markets, investment firms qualifying as "systematic internalizers" must publish firm quotes on a regular and continuous basis during normal trading hours (Article 27).[23] These **(p.273)** must be firm quotes, in the sense that the intermediary must execute at the quoted prices the orders received from their retail clients: the directive prohibits them from offering price improvements to retail customers and limits the scope of price improvements on large orders from professional clients. This measure is designed to encourage firms to display the liquidity that they offer.

Another difference between the European and U.S. equities markets is that clearing and settlement systems are much less unified in Europe. As a consequence, cross-border trading is more costly for investors than domestic trading (see box 7.5). This friction hinders multimarket trading (which in Europe is often cross-border trading) and therefore works against the integration of the various trading platforms. For instance, one French institutional investor may find it optimal to trade on Euronext even if a better price is available on Chi-X, because the extra clearing and settlement costs of the latter exceeds the price gain.

To sum up, RegNMS and MiFID are two different regulatory approaches, aimed at the same objective: securing the benefits of intermarket competition for investors without the adverse effects of market fragmentation. With its trade-through rule, RegNMS enforces strict price priority. That is, "best execution" is essentially defined with respect to prices. MiFID defines best execution more flexibly since there is no rule against trade-throughs in Europe. In making their routing decisions, brokers must consider price, of course, but can also weigh other dimensions, such as speed and likelihood of execution. Another important difference between MiFID and RegNMS lies in the consolidation of market data. In the United States, trading platforms are free to disseminate their trade and quote data, but they must also transmit them to an agency that consolidates them across platforms and provides information in real time on the best bids and offers. Such consolidation of market data does not exist yet in Europe, which makes multimarket trading more complicated and costly.

The jury is still out on how MiFID has affected market quality. For instance, a survey of the CFA institute shows that its members have mixed views:[24] they are practically evenly divided among those who think that MiFID decreased, increased, or had no impact on the illiquidity of European markets. Moreover, 42 percent believe that MiFID has impaired price discovery in European stocks, whereas 26 percent have the opposite view. MiFID is now up for revision with new rules to come into force in the near future. **(p.274)**

Box 7.5 Clearing Houses and Custodians

Trading platforms enable buyers and sellers to find one another and agree on the terms of a trade. After a transaction, however, each party needs post-trading services.

First, the trade has to be cleared. That is, for each match on the platform, the account of the buyer to which the security needs to be delivered and that of the seller to which the payment is due must be identified. This task is performed by a clearing house or central counterparty (CCP). The CCP also serves to minimize counterparty risk (the risk that one party to the transaction fails to honor its obligation). If one party defaults, the CPP completes the trade, so the other party gets the security or the payment due. For these services, the CCP collects a fee on each trade from the buyer and the seller.

Second, the trade needs to be settled. Settlement–the actual delivery of the security to the seller and its payment by the buyer–usually takes place through a central securities depository (CSD). CSDs also act as custodians for securities, as they keep track of ownership and enable the holders to receive the benefits (dividends, issue rights, etc.).

In the United States, there is a single CCP and CSD for all equity trades. In Europe, there are more than twenty CCPs and CSDs for equities alone, and different platforms use the services of different CCPs or CSDs. Moreover, MTFs have often chosen to connect to incumbent CSDs via new CCPs and agent banks. For instance, Chi-X, NASDAQ OMX Europe, and BATS Trading Europe are all using the European Multilateral Clearing Facility (EMCF) as their CCP clearing house. This enables MTFs to reduce clients' trading costs, since the new CCPs often charge much lower fees than the incumbent. For instance, in September 2008 EMCF was charging 5 euro cents to clear a trade while LCH Clearnet (the CCP for Euronext and the LSE) was charging 50 euro cents.

As a consequence, cross-border transactions are more complex and costlier than domestic transactions; they involve more layers of intermediation. Consider a French institutional investor buying a British stock on Chi-X. The investor must become the holder of the security held in the British CSD (of the issuer). This means the custodian of the investor (say, its local CSD) must be linked to the foreign CSD either through another British custodian or a global custodian that is a member of the British CSD.

(p.275) 7.6. Further Reading

Several authors have analyzed thick market externalities in securities markets. Mendelson (1982, 1985, 1987) and Hendershott and Mendelson (2000) show how an increase in the number of participants on a platform (a call market in Mendelson (1982, 1985), a crossing network in Hendershott and Mendelson (2000)) raises the likelihood of finding a match for all traders. Pagano (1989b) considers a different source of thick market externality. In his model, risk-averse traders perceive their demand for the stock as adversely affecting the market price. More market participants implies a lower price sensitivity to each trader's net demand, thus increasing the market's liquidity. The analysis of risk-sharing effects and market fragmentation in section 7.2.2 is based on Pagano (1989b).

The tendency toward agglomeration in a single market also emerges also in models with asymmetric information, such as Admati and Pfleiderer (1988) and Chowdhry and Nanda (1991). Admati and Pfleiderer were the first to consider "discretionary liquidity traders", in a setting in which trading is fragmented across different times of day. Chowdhry and Nanda use a similar model to analyze the fragmentation of trading across different venues; the setting analyzed in sections 7.2.1 and 7.3.1 is a simplified version of their model.

Parlour and Seppi (2003) develop a model of competition between a dealer market and a LOB. Foucault and Menkveld (2008) extend this model to analyze competition between two limit order markets. The model developed in section 7.4.2 is based on Foucault and Menkveld (2008). Foucault and Parlour (2004); Colliard and Foucault (2012); and Foucault, Kadan, and Kandel (2012) endogenize fees charged by trading platforms.

There is also a rich empirical literature on the effects of market fragmentation and intermarket competition on market liquidity and price discovery. Among others see Mayhew (2002); DeFontnouvelle et al. (2003); Barclay, Hendershott, and McCormick (2003); Bessembinder (2003); Boehmer and Boehmer (2003); Battalio, Hatch, and Jennings (2004); Jennings, Boehmer, and Wei (2007); Foucault and Menkveld (2008); Biais, Bisière and Spatt (2010); O'Hara and Ye (2011); Degryse, de Jong, and Van Kervel

(2011); and Gresse (2011).

7.7. Exercises

1. Market consolidation with correlated noise trading.

Consider the model developed in section 7.2.1, but assume that u_A and u_B are correlated. Denote by ρ the correlation between these two variables.

> **(p.276) a.** How does market depth depend on ρ when order flow is consolidated in a single market?
> **c.** Does order flow consolidation increase or decrease market depth?
> **b.** How does the informed investor's expected profit differ from those obtained in the model of section 7.2.1?

2. Optimal order splitting.

Consider an order of total size q that can be split into two orders q_A and q_B to be executed in markets A and B. The security traded in these two markets has expected value μ and variance σ_v^2. Markets A and B are populated by K_A and K_B competitive dealers with no initial inventories and identical risk aversion ρ as in section 7.2.2. Show that equation (7.10) is the optimal split of the total order q between the two markets.

3. Payments for order flow.

Consider the market for a risky security. Its payoff at time 1 is either $v^H =/\mu + \sigma$ or $v =/\mu - \sigma$ with equal probabilities. At time 0, an investor gives to his broker an order to buy or sell one share of the security to a broker. With probability φ, the investor is a retail investor and has no information on the payoff. In this case he buys or sells the security with equal probability. With probability 1 − φ, the investor is an institutional investor. In this case, he is perfectly informed about the payoff with probability α or uninformed with probability $1 - \alpha$. In the latter case, the investor is a buyer or seller with equal probability. Bid and ask quotes for the broker's order are posted by three risk-neutral dealers 1, 2, and 3 before the broker contacts them. The broker cannot split his order among dealers. Dealers have no private information on the payoff of the security. For this exercise, you need also to refer to the material in Chapter 3, section 3.3.2.

> **a.** Assume that there is no payment for order flow between the broker and the three dealers. In this case, the broker randomly selects one dealer among those posting the best price for his order. Compute the bid and ask quotes posted by the dealers.
> **b.** Assume now that dealer 1 has a payment for order flow arrangement under which the broker gives dealer 1 all orders from retail investors and the dealer commits to execute all these orders at the best quotes (i.e., the ask and bid price set by the remaining dealers, 2 and 3). Other orders are sent to dealer 2 or 3 as in question a. What are the quotes posted by dealers 2 and 3? Deduce that the bid-ask spread is higher in this case than where there is no payment for the order flow.

c. Let P be the payment of dealer 1 to the broker. What is the largest possible value of P?
d. Is payment for order flow beneficial or detrimental to investors?

(p.277) 4. Competition between limit order markets with uniformly distributed market orders.

Consider the model of section 7.4.2 and assume that the size of the market order ($\tilde{X}$) has a uniform distribution on $[0, \bar{X}]$. That is, $F(x) = \frac{x}{\overline{X}}$. We denote by $Y^*_{jk}(\gamma)$ the cumulative depth posted at the ask price $A_k = \mu + k\Delta$ in market $j \in \{I, E\}$ when the fraction of investors submitting market orders in both markets I and E is γ, and by c_j be the submission cost in market j.

a. Assume that $2c_I \leq \Delta$ and that $\gamma = 0$. Show that the equilibrium cumulative depth at price A_k is
(7.18)

$$Y^*_{I1}(0) = \overline{X}\left(1 - \frac{2c_I}{\Delta}\right).$$

b. Now suppose that γ is high enough and that the other parameters are such that $Y^*_{I1}(\gamma)\rangle 0$, $Y^*_{E1}(\gamma)\rangle 0$, but $Y^*_{I1}(\gamma) + Y^*_{E1}(\gamma)\left\langle \overline{X}\right.$. Compute $Y^*_{I1}(\gamma)$ and $Y^*_{I1}(\gamma)$ as a function of γ. Deduce further from the result that the conditions $Y^*_{I1}(\gamma)\rangle 0$ and $Y^*_{E1}(\gamma)\rangle 0$ are satisfied if $\frac{4c_I}{\Delta(2-\gamma)+2c_E}\left\langle 1\right.$ and $\frac{4c_E}{\Delta+2c_I}\left\langle \gamma\right.$. Moreover deduce that the condition $Y^*_{I1}(\gamma) + Y^*_{E1}(\gamma)\left\langle \overline{X}\right.$ is satisfied if $4(\gamma c_I + (2-\gamma)c_E)\rangle (2-\gamma)\gamma\Delta$.
c. Deduce from question (b) that the two markets can coexist even if their order submission costs differ and $\gamma = 1$.
d. Why does the cumulative depth at price A_1 in one market decrease with the order submission cost in this market but increase with the cost in the competing market?
e. Consider the case $\gamma = 1$ and suppose that $4(c_I + c_E)\langle \Delta$ and $4c_I \langle \Delta$. Compute $Y^*_{I1}(1)$ and $Y^*_{E1}(1)$.
f. Under the assumptions in question (e), what is the number of shares offered at price $A_k \rangle A_1$? Is the result different when $\gamma = 0$?

Notes:

(1.) Further, the market share of opaque markets–"dark pools" and OTC trading—has increased, but the exact fraction of OTC trading activity is hard to estimate. A study by the Association for Financial Markets in Europe puts it at 16 percent in 2011 (see "The Nature and Scale of OTC Equity Trading in Europe," Association for Financial Markets in Europe, April 2011).

(2.) For instance, Shkilko, Van Ness, and Van Ness (2008) find that ask and bid prices for NYSE-and Nasdaq-listed stocks that are traded on multiple markets are locked or crossed 10 percent and 3.5 percent of the time, respectively.

(3.) For instance, traders had to meet in a physical location, which could involve substantial real estate costs.

(4.) This situation is similar to that of a firm choosing to make its product compatible or not with its competitors' product. For instance, surcharges can be used to deter depositors who are not clients of a bank from using its ATMs.

(5.) A crucial assumption here is that market makers in one market cannot make their quotes contingent on the order flow in the other, and vice versa. If they could, then the equilibrium would simply be that obtaining in a single market. Hence, the speed of information flows between markets is one determinant of the extent of fragmentation.

(6.) To do this, we must compute the expectation of the security's value v conditional on the two prices, p_A and p_B. Recall that the unconditional expectation is $E(v) = \mu$. Denote by Θ the vector of covariances (cov(v, p_A), cov(v, p_B)) and by Ω the variance-covariance matrix of p_A and p_B. From the expressions of the equilibrium prices, we have:

$$\Theta = \frac{\sigma_v^2}{2}\,(1,1)\, and \Omega = \sigma_v^2 \begin{bmatrix} 1/2 & 1/4 \\ 1/4 & 1/2 \end{bmatrix}.$$

As v, p_A, and p_B are normally distributed, $E(v \mid p_A, p_B) = \mu + \Theta\Omega^{-1}\,(p_A - \mu, p_B - \mu)$. Equation (7.8) follows.

(7.) The parameters α_A and α_B can be estimated by a regression of the returns in market A on their own lagged value and the lagged returns in the other market, because equation (7.8) can be rewritten as follows:

$$pl - p_A = -(1-\alpha_A)\,(p_A - \mu) + \alpha_B\,(p_B - \mu),$$

where the differences $pl - p_A$ and $p_A - \mu$ are returns in market A over two consecutive periods. Hence, returns in market A are negatively auto-correlated, *conditional* on returns in market B. In contrast, the return$pl \sum p_A$ in market A is positively correlated with the return $p_B - \mu$ in market B, because a high return in market B is more likely when the informed investor is buying in this market.

(8.) Let Θ and Ω be as defined in Footnote 6. Then $\mathrm{var}(v \mid p_A, p_B) = \mathrm{var}(v) - \Theta\Omega^{-1}\,\Theta'$, which yields equation (7.9).

(9.) In Chapter 10, we see that it also increases the cost of capital to firms, and thereby lowers the level of investment in the economy.

(10.) This can never be optimal: to see why, suppose that the market maker were to bid for a portion of the order routed to the market. Then, he would put competitive pressure on the price and therefore he would lose on the internalized portion of the order. He would, of course, get to trade additional shares in the main market, but he could have traded such shares (without affecting the market price adversely) by internalizing that part of the trade, as well.

(11.) When $\sigma_{Ac}^2 \rangle M\sigma_d^2 + \sigma_{Bc}^2$, the sole equilibrium is such that all trading concentrates in market A.

(12.) Fees will be low only if the elasticity of the demand for trading services is high. The evidence on the elasticity of trading volume with respect to transaction costs is mixed: summarizing, the available evidence, Schwert and Seguin (1993) report estimates ranging from as little as -0.25 to as much as -1.35.

(13.) See BATS website: www.batstrading.com.

(14.) Foucault and Menkveld (2008) place it at 3.5 percent on average in August 2004, with a peak of 6.1 percent for the most actively traded stocks.

(15.) The logic is similar to that of the pro-rata allocation rule in a single limit order market: see Section 6.3.2 in Chapter 6.

(16.) Glosten (1994) considers the case in which the trading platforms use a pro-rata allocation rule and concludes that consolidated depth does not depend on the number of trading platforms. The reason is that the expected profit of limit order traders is zero with the pro rata allocation rule, even when the tick size is strictly positive. See Section 6.3.2 in Chapter 6.

(17.) Note that $\gamma^C \langle 1$, since $2C \langle \Delta$ by assumption.

(18.) It was first released on 9 June 2005. See Securities Exchange Act release n°51808,70 FR 37496.

(19.) The effects of the no-trade-through rule on market liquidity are analyzed in Section 7.4.3.

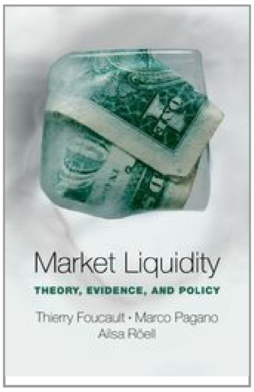
(20.) They can be viewed as a negative order-processing cost (see Chapter 3).

(21.) The sub-penny rule does not prevent trading in increments of less than a penny. For instance, traders can agree to match a trade at a price within the best bid and offer price.

(22.) For instance, Ende and Lutat (2011) estimate the frequency of trade-throughs in the constituent stocks of the Euro Stoxx 50 index traded in eight European markets over twenty trading days in 2007 and 2008 at 12 percent of the trades in their sample.

(23.) Transaction reporting to the competent authory can be made by the investment firm itself, a third party representing the investment firm, a reporting system approved by the competent authority, or the MTF through which the investment firm completes the transaction.

(24.) See “The impact of market fragmentation under the markets in financial instruments directive,” CFA Institute, Centre for Financial Markers Integrity, 2009.



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Thierry Foucault, Marco Pagano, and Ailsa Roell

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Market Transparency

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter discusses the issue of transparency in securities markets. It covers pre-trade transparency, post-trade transparency, and revealing trading motives. It demonstrates that transparency about quotes, orders, and traders' identities generally enhances market liquidity, at least as far as uninformed traders are concerned. The chapter then addresses the question of why markets are so opaque. The final sections provide suggestions for further reading and exercises.

Keywords: securities markets, securities trading, transparency, opacity

Learning Objectives:

- • What market transparency is

- • Its different dimensions
- • How it differs across markets
- • How it affects liquidity and price discovery
- • The implications for regulation

Securities markets are often taken to be the archetype of transparency, whereby all participants are perfectly informed of the terms of past trades and those at which they could trade at every point in time. In practice, of course, this is rarely the case. Consider a retail investor trying to buy a share of IBM. He could check the latest prices available on the internet, but these are delayed, so that he would still be uncertain about the exact current price. For more accurate information, he could subscribe to a real-time data feed such as Bloomberg or Reuters. But these services are costly, and they indicate the price of the most recent past transaction, not the quotes available for the next trade. To get such quotes, our investor must hire a broker or subscribe to Openbook, a service that displays all posted limit orders for NYSE stocks.

And the market for IBM is a relatively transparent one. Consider instead an investor who wants to buy or sell a U.S. corporate bond. This is an OTC market, for which—until recently—no information whatever on transactions was disseminated. After a protracted controversy and intervention by the SEC, **(p.279)** data on past trades are now available —with a delay—but no information on dealers' quotes is published. The only way to learn about the price you might get in the market is—again—to contact a broker or your bank's brokerage office, who will in turn inquire about the prices quoted by the various dealers.

This lack of transparency pervades securities markets worldwide. And by no means does it affect only retail investors: even mutual fund managers or brokers often do not have a full picture of the trading process, especially if the relevant market is highly fragmented. For instance, a broker or dealer in an OTC market, such as the foreign exchange market, cannot possibly know all the opportunities that are available simultaneously. Yet it is important for market professionals to have access to as much information as possible about market conditions, since these professionals quote prices on the market and are crucial to the price discovery process.

The foregoing implies that transparency may refer to different kinds of information, such as past trades and prices—post-trade transparency—or the quotes on future trades—pre-trade transparency. These types of information play different roles. Past trade data help market participants hone their estimates of a security's value and of other participants' strategies. Information on quotes enables them to limit execution cost and risk, and fosters competition between liquidity providers, as we shall see. Transparency can also extend to the identities of market participants—those involved in past transactions or those currently posting a quote. That is, transparency also encompasses the issue of "anonymity" of trading.

Transparency is one of the most hotly debated issues in securities market regulation. In fact, different degrees of transparency mean different distributions of rents across market participants and can even shut some of them out of the market altogether. Moreover, fine-tuning transparency is a crucial choice variable for exchanges, as it affects their relative attractiveness to traders. Each market must disclose some data to attract trading, but in doing so it may enable competitors to piggy-back on price discovery.

8.1. Pre-Trade Transparency

Different market structures impose different constraints on pre-trade transparency. The least transparent are OTC markets, such as those for small company stocks, municipal bonds, corporate bonds, or bespoke derivatives like credit default swaps (CDSs). Typically, these are thinly traded securities; no firm quotes are publicly posted, and prices are only available from a dealer upon request. Such markets, where trading interest is at best sporadic, do not repay the time and effort that dealers would have to expend to monitor price-relevant **(p.280)** information continuously. At the other extreme, there are the electronic LOB markets for blue chips, where any potential trader can purchase real time information on prices and quantities before placing an order. In between, there is a range of intermediate cases. An example is a dealer market with firm quotes publicly displayed on screen for limited trade sizes, where customers can obtain price improvements by contacting individual dealers.

There are three forms of pre-trade transparency: (i) visibility of quotes, (ii) visibility of incoming orders, and (iii) visibility of traders' identities. These three forms of pre-trade transparency have different effects:

> (i) Visibility of quotes reduces dealers' rents and so enhances liquidity (section 8.1.1); moreover it enables customers to fine-tune their orders to the liquidity supply and so reduce execution risk (section 8.1.2).
> (ii) Visibility of incoming orders helps dealers to detect informed investors, leading to narrower bid-ask spreads and better price discovery (section 8.1.3).
> (iii) Visibility of order submitters' identities has an ambiguous effect on liquidity: it reduces trading costs for investors who are identified as uninformed, but it may impair liquidity for the others (section 8.3).

8.1.1 Quote Transparency and Competition between Dealers

This section shows how the lack of pre-trade information on quotes can lessen competition among dealers and thus reduce market liquidity. To see this, consider again the model developed in Chapter 3. There we assumed that dealers' quotes are freely and perfectly observable, so customers costlessly compare quotes and choose the best price. Dealers are accordingly driven to offer zero-profit quotes, and liquidity is maximal. For instance, if there is no asymmetric information, risk aversion, or order-processing cost, the competitive bid and ask quotes are both equal to the expected value of the security, and the spread is zero.

Now suppose instead that dealers' quotes are not visible and clients must contact dealers sequentially to get quotes. On receiving a quote, the investor can either accept it and trade, or reject it and contact another dealer. Getting a new quote is costly: investors pay a search cost c for each request of a quote. Each client can be a buyer or a seller with equal probabilities. Buyers are willing to pay up to $\mu + \tau$ (where μ is the expected value of the security) to buy the security, while sellers want to receive at least $\mu - \tau$ for it. Dealers value the security at μ. These differences in valuation might reflect, for instance, different hedging needs (an investor with a long position in a bond, say, is willing to buy **(p.281)** a CDS at a markup to its fair value). These differences generate a motivation for trading.

In this setup, the presence of a search cost, however small, enables dealers to charge monopoly prices in equilibrium, that is, bid and ask quotes that are respectively equal to buyers' and sellers' reservation values. To see that this is an equilibrium, suppose that dealers quote these prices. Consider a buyer who is matched with one dealer. The dealer offers to sell at $\mu + \tau$. Since the investor expects other dealers to quote the same price, he has no incentive to use resources to shop around. Now consider the dealers. The dealer that the investor contacted has no incentive to offer a better price than $\mu + \tau$, since he expects the investor to accept it. Nor do other dealers have any reason to quote a better price: they cannot advertise their quotes, because the market is opaque. Replicating this argument symmetrically on the bid side, the bid price will be $\mu - \tau$, so the bid-ask spread is 2τ instead of zero.

Interestingly, there is no equilibrium except for the monopoly pricing equilibrium. To see this, suppose that this is not so—that is, that there is an equilibrium in which at least one dealer quotes an ask price less than $\mu + \tau$. Denote him as dealer 1 and suppose the investor is matched with this dealer. Dealer 1 can raise his offer by a small amount (up to c) without losing this customer since the client cannot get a better offer (dealer 1 being the cheapest) and would have to pay the search cost c. As dealer 1 has a profitable deviation, this cannot be an equilibrium. This argument has been standard in the literature on industrial organization since Diamond (1971), and extends beyond our specific assumption regarding the shape of customers' demand and supply curves.

Thus pre-trade opaqueness is conducive to market power for dealers, hence less liquidity. Dispersion in dealers' prices could be a symptom of such market power: if investors have different private value estimates (that is, different τ), the above model suggests that the prices they receive will be highly dispersed, as dealers' market power enables them to price discriminate (assuming that they know investors' private valuations). Moreover, different types of investors have different search costs: sophisticated ones such as hedge funds or mutual fund managers have smaller search costs thanks to their continuous market presence. One therefore expects these investors to get better prices, which should result in a less-dispersed distribution of the prices they get. For instance, in the previous model, if there are investors for which $c = 0$, they all receive a price of μ, regardless of their private valuations.

Evidence from the municipal bond market (SEC, 2004; Harris and Piwowar, 2006; Green, Hollifield, and Schuerhoff, 2007) is consistent with these implications. The trading costs for municipal bonds are substantially higher than for equities, and particularly high for retail-sized rather than institutional trades. This is presumably because retail investors are less sophisticated. Furthermore, **(p.282)** the SEC report (2004) shows that prices for retail transactions (below $10,000) are dispersed: intraday price differences exceeding 3 percent across dealers are quite common (over 9 percent of transactions), even though intraday fluctuations in the fundamental value of municipal bonds are minimal. By contrast, there is less price dispersion for large, institutional transactions: for trades of size around $1 million, a difference exceeding 1 percent is rare (0.4 percent of transactions) in the sample. In 2004, the National Association of Securities Dealers (NASD) sanctioned major dealers that had failed to meet their obligation to buy and sell at fair prices, after finding major discrepancies in pricing: one bond was sold on behalf of a customer for less than half the price that it traded for later in the day; another client received seventy cents on the dollar for a bundle of bonds with a fair value of ninety seven cents.

8.1.2 Quote Transparency and Execution Risk

Investors do not always observe all the limit orders posted in the market in real time. Until 2002, for instance, off-floor investors in the United States could only observe the best bid and ask quotes for NYSE stocks and not the entire LOB for them. This is problematic, since there is considerable time variation in the liquidity available in LOBs. This situation creates uncertainty for investors about the prices they may get: they may end up trading at the wrong time (when price impacts are large) or, the wrong amount (too much when the LOB is thin). In other words, if the market is opaque, it will be harder for investors to adjust to market conditions, so that on average they will gain less from trading.

A simple example can illustrate these points. Assume that quotes are given by the following schedule, as in Chapter 4:

(8.1)

$$p(q) = \mu + \lambda q,$$

where $p(q)$ is the price obtained by a customer who places an order of size q. Unlike the previous chapters, now suppose that the price impact parameter, v, is random. For instance, it could be high ($v, = v^H$) or low ($v, = v^H$) with equal probability. We denote the expected value of v by $\mathrm{E}(v)$.

As in the previous section, consider again a buyer who values the security at $\mu + \tau$ and wishes to place a market order. If the market is opaque, when the investor chooses the size q of this order, he does not know the realization of v, though he correctly anticipates that his order will affect the market price according to price impact function (8.1) and accordingly sets his order size to maximize:

$$\max_q \mathrm{E}\left[(\mu + \tau) q - pq\right] = \tau q - \mathrm{E}(\lambda) q^2 .$$

(p.283) The first order condition of this problem yields the optimal order size, q^O, when the market is opaque:

(8.2)

$$q^{o} = \frac{\tau}{2\mathrm{E}\left(\lambda\right)}.$$

Thus, the investor optimally trades more when the wedge τ between his valuation of the security and that of liquidity suppliers increases, and less if he expects his price impact to be greater.

By contrast, in a transparent market where the investor knows his price impact, v, he sets his order size, q^{T}, to maximize:

$$\max_{q} \tau q - \lambda q^{2}.$$

The first order condition to this problem yields the optimal order size:

(8.3)

$$q^{T}\left(\lambda\right) = \frac{\tau}{2\lambda}.$$

Hence, when the market is transparent, the investor fine-tunes his strategy to the exact amount of liquidity present in the market, v. Thus, in contrast to q^{O}, his order size, $q^{T}\ (v)$, depends on the realized value of v and therefore is itself random. By Jensen's inequality,[1]

(8.4)

$$\mathrm{E}\left(\frac{1}{\lambda}\right)\Bigg\rangle\frac{1}{\mathrm{E}\left(\lambda\right)}.$$

Thus, comparing equations (8.2) and (8.3), the average optimal order size is greater in the transparent than in the opaque market:

$$E\left(q^{T}\left(\lambda\right)\right)\Big\rangle q^{O}.$$

The intuitive reason is as follows. The investor trades more in the transparent market when the price impact of his own trade is small (when $v = v^{L}$) and less when the price impact is large ($v = v^{H}$). The increase in size in the first case more than offsets the restraint in the second. Hence, on average he ends up trading more.

Interestingly, the investor's expected gain from the trade is also greater in the transparent market. To see this, observe that on average his profit in the transparent market is

$$\mathrm{E}\left(\tau q^{T} - \lambda\left(q^{T}\right)^{2}\right) = \frac{\tau^{2}}{4}\mathrm{E}\left(\frac{1}{\lambda}\right),$$

(p.284) whereas in the opaque market, it is:

$$\tau q^O - \mathrm{E}(\lambda)\left(q^T\right)^2 = \frac{\tau^2}{4}\frac{1}{\mathrm{E}(\lambda)}.$$

Using again inequality (8.4), we see that the investor fares better in the transparent market—once again, transparency enables him to make better trading decisions.

These findings carry three implications. First, pre-trade transparency is valuable, so people are willing to pay for real-time quote information or to employ intermediaries who have access to it. Second, pre-trade transparency increases the volume of trade, that is, it encourages participation in the market. Third, if there is some persistence in the illiquidity parameter ν over time, investors have an incentive to make their trades conditional on past measures of market liquidity, since these are informative about current liquidity—a point formalized by Hong and Rady (2002).

8.1.3 Order Flow Transparency

In some markets—foreign exchange or OTC markets, to name two—different orders get filled almost simultaneously by different liquidity providers, who may know very little about trades made simultaneously by their competitors. In this situation, the order flow is not transparent. What effects does this have?

To address this issue, we compare two market structures, using a slightly modified version of the framework developed in Chapter 3: (i) a completely opaque dealer market in which each dealer observes only his own order flow and (ii) a transparent market where liquidity providers see all the orders submitted before setting their prices. The latter situation captures the wealth of information available to speculators on the floor of an open-outcry auction market like the Chicago Board of Trade or the Frankfurt Stock Exchange.

As in Chapter 3, we consider the market for a risky security with payoff ν that can be high (ν^H) or low (ν^L) with equal probability, so that the expected payoff is $\mu = (\nu^H + \nu^L)/2$. Quotes are set by risk-neutral dealers and are valid for one share. Dealers can be contacted either by informed or uninformed investors. Specifically, with probability π two risk-neutral informed traders are present on the market: they both place a market order to buy one share if they know $\nu = \nu^H$ or to sell if $\nu = \nu^L$ With probability $1 - \pi$, there are two liquidity traders, one buyer and one seller. These assumptions represent the simplest possible way to capture the idea that informed trading tends to generate positively correlated orders, while uninformed trading does not (the case in which noise trades are **(p.285)**

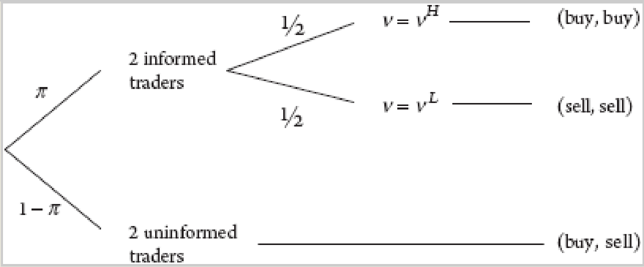


Figure 8.1. Order arrival process in the order flow transparency model

not perfectly negatively correlated yields similar conclusions—see exercise 1). Figure 8.1 describes the order arrival process.

Traders randomly contact one of the dealers quoting the best price and can trade with only one dealer at a time. The crucial difference from the model considered in Chapter 3 is that now two trades (not one) may happen simultaneously in the market. As noted, in the opaque market dealers only observe the order that comes to them, while in the transparent market they see all orders, whether they receive them or not.

At the time he accepts an order in an opaque market, a dealer does not know the direction of the other order that has come to the market. Hence, his ask price, a^O (where the superscript O stands for "opaque") is the expected value of the security, conditional only on the fact that he has received a buy order. As π is the probability that the order comes from an informed trader, equation (3.10) in Chapter 3 yields the dealer's ask price:

(8.5)

$$a^O = \mu + \pi\left(v^H - \mu\right).$$

The bid price symmetrically is

(8.6)

$$b^O = \mu - \pi\left(\mu - v^L\right),$$

so that the bid-ask spread is:
(8.7)

$$s^O = \pi\left(v^H - v^L\right).$$

In the transparent market, dealers have more information when they set their quotes, because they observe the orders that go to their competitors as well. Hence, their quote is the estimate of the security's value, *given all the orders submitted*. Under our assumptions, this information reveals whether the traders are informed and tells the direction of their signal. For instance, if two buy orders are submitted, then dealers infer that informed traders know that $\nu = \nu^H$ **(p.286)** and thereby quote an ask price equal to ν^H. The dealers' valuation and price for each possible configuration is given below:

(i) two buy orders: $E(\nu \mid B,B) = \nu^H$, so that $a^T = \nu^H$.
(ii) one buy order, one sell order: $E(\nu \mid B, S) = \mu$, so that in this case $a^T = b^T = \mu$.
(iii) two sell orders: $E(\nu \mid S, S) = \nu$, so that $b^T = \nu^L$.

The average price paid or received by liquidity traders is μ, which means the bid-ask spread for them is zero. In this example, transparency eliminates their trading cost. Hence, again in this case, opaqueness increases the trading costs for liquidity traders. The reason is that with transparency dealers detect informed traders more easily, so they can charge a lower spread to the uninformed. This inverse relationship between transparency and uninformed investors' trading costs holds more generally, as Pagano and Röell (1996) show. In the transparent market, by contrast, informed investors obtain no profit since they must always pay the maximum bid-ask spread, $\nu^H - \nu^L$. As a result, the average spread in the transparent market is $\pi(\nu^H - \nu^L)$, as in the opaque market. In the transparent market, the allocation of trading costs differs between uninformed and informed investors as transparency-unlike opacity-allows dealers to discriminate between the two groups.

In this model, the lower trading costs for liquidity traders mirror a decrease in trading profits for the informed trader, since trading is a zero-sum game and dealers make zero expected profits. This implies that a change in market transparency can redistribute gains between different types of participants. This explains why transparency is such a controversial issue.

Order flow transparency also has implications for price discovery. To see this, let us use the average squared pricing error as an inverse measure of price discovery. Let p^O denote the price paid to or by a dealer in the opaque market. This will be a^O if a dealer executes a buy market order and b^O if he executes a sell market order. In the opaque market, the average squared pricing error is:

$$\begin{aligned}\mathrm{E}\left[\left(p^O - v\right)^2\right] &= \frac{\pi}{2}\left(a^O - v^H\right)^2 + \frac{\pi}{2}\left(b^O - v^L\right)^2 \\ &+ \frac{1-\pi}{2}\left[\frac{1}{2}\left(a^O - v^H\right)^2 + \frac{1}{2}\left(b^O - v^H\right)^2\right] \\ &+ \frac{1-\pi}{2}\left[\frac{1}{2}\left(a^O - v^L\right)^2 + \frac{1}{2}\left(b^O - v^L\right)^2\right].\end{aligned}$$

The first two terms correspond to the cases in which the dealer trades with an informed investor (receiving good news in the first case and bad news in the second). The third term corresponds to the case in which the dealer receives **(p.287)** an order from an uninformed investor and the true value of the security is high. The last term corresponds to the symmetric situation of an order from an uninformed investor when the true value is low. Substituting out for the ask price a^O from (8.5) and the bid price b^O from (8.6), this expression simplifies to:

$$\mathrm{E}\left[\left(p^O - v\right)^2\right] = \left(1 - \pi^2\right)\left(v^H - \mu\right)^2,$$

which is decreasing in π: as the frequency of informed trading increases, the price discovery process improves.

In the case of transparency, the transaction pricep^T is either equal to the true value if there are informed investors (with probability π), or equal to μ if there are uninformed traders (with probability $1 - \pi$). Hence, the average pricing error is:

$$\mathrm{E}\left[\left(p^T - v\right)^2\right] = (1 - \pi)\left[\frac{1}{2}\left(v^H - \mu\right)^2 + \frac{1}{2}\left(v^L - \mu\right)^2\right] = (1 - \pi)\left(v^H - \mu\right)^2.$$

This pricing error is clearly smaller than the corresponding expression in the opaque market: the difference between the pricing errors is:

$$\mathrm{E}\left[\left(p^O - v\right)^2\right] - \mathrm{E}\left[\left(p^T - v\right)^2\right] = \pi(1 - \pi)\left(v^H - \mu\right)^2 \Big\rangle \mathrm{O}.$$

Thus, transparency also improves price discovery because it helps market participants to learn about the presence of informed traders.

8.2. Post-Trade Transparency

Another issue regarding transparency is the timeliness of disclosure of past trades. In some markets, information is released in real time to all market participants (for a fee); in others, past trades are disclosed with a significant delay, if ever. The speed of publication of information on past trades is one of the most controversial issues in the organization of securities markets. Dealers often oppose prompt disclosure. In this section, we use a variant of the previous section's model to show how the lack of post-trade transparency may enable dealers to capture rents at the expense of other traders. That is, we have another instance of transparency redistributing profits and costs.

Assume that the orders are filled in sequence over two periods, $t = 1$ and $t = 2$, as in figure 8.2. With probability π, two successive identical orders are placed by informed investors and executed at their arrival times, $t = 1$ and $t = 2$. With probability $1 - \pi$, two orders of opposite signs are placed by uninformed traders in random sequence.

Consider first a market with post-trade transparency: dealers have to report their trades as they take place, and the exchange publishes the information **(p.288)**

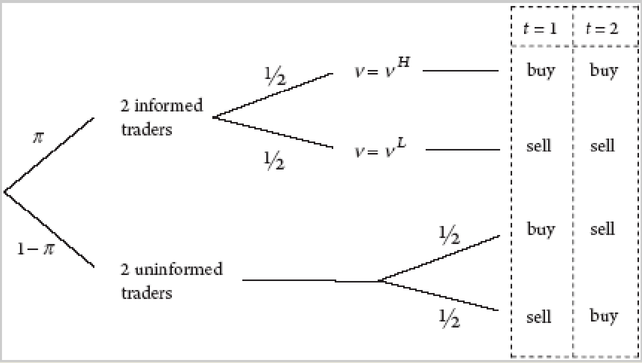


Figure 8.2. Order arrival process in the post-trade transparency model

immediately. In this case, the quotes posted by dealers at time $t = 1$ are the same as those prevailing in the opaque market analyzed in the previous section, since in the first period dealers have no information on forthcoming orders. Hence, in the first period, the bid-ask spread is:

(8.8)

$$s_1^T = \pi \left(v^H - v^L \right).$$

In contrast, at time $t = 2$, the dealer who receives the second order is in the same position as in the transparent market analyzed before: he and all his peers observe his new order, and they have all learned about the previous trade. Consider first the case in which the first-period order was a buy. If another buy order arrives at time $t = 2$, dealers infer that the investor is informed and $v = v^H$. Hence, their ask price is equal to v^H. If instead, a sell order arrives, they infer that the investor is uninformed, so $v = \mu$ and their bid price is μ. Hence, following a buy order in the first period, the bid-ask spread in the second period is $v^H - \mu = (v^H - v^L)/2$. If the first-period order is a sell, the analysis is symmetric. Thus, the bid-ask spread posted in the second period is:

$$s_2^T = \frac{1}{2} \left(v^H - v^L \right).$$

To summarize, with post-trade transparency, the spread will vary over time, and trading costs for uninformed and informed investors will differ. For instance, liquidity traders pay a spread of s_1^T in the first period, and zero in the second. Summed over the two periods, the average trading cost of uninformed investors in the post-trade transparent market is

(8.9)

$$TC^T = \frac{1}{2} \times s_1^T + \frac{1}{2} \times 0 = \frac{\pi}{2} \left(v^H - v^L \right).$$

(p.289) This shows that post-trade transparency is an imperfect substitute for pre-trade transparency: on average, over the two periods, the trading cost for uninformed traders is intermediate between what they would pay with pre-trade transparency and pre-trade opacity.

Let us now compare the outcome with and without post-trade transparency. Where it is absent, first-period transactions are not disclosed before second-period trading. In this case, in the second period the dealer who has already received an order in the first period has an informational advantage: if it was a buy order, he knows that the value of the security is high with a greater probability than other dealers; with a sell order, he knows that it is low. In general, he will exploit this advantage by adjusting his quotes to capture profitable trading opportunities that his peers are not aware of.

To see this as simply as possible, assume that in the second period dealers set their quotes sequentially and that the dealer who received the first-period order goes last.[2] This dealer can always undercut any competitor's quote that could yield a positive expected profit and refrain from undercutting those that are loss-making. This is an extreme instance of the well-known "winner's curse" problem in auction theory: if you win the trade, it can only be because better informed bidders know that the price you are paying is already too high! In response to this problem, the best quotes that competing dealers can offer without losing money are the highest and lowest possible values that the informed dealer may place on the security, i.e. $a = v^H$ and $b = v^L$.

Now consider the best response of the dealer who executed the order arriving at date $t = 1$. Assume first that it was a sell order. If another sell order comes in, the dealer knows that the security has a low value so, like the other dealers, his bid price is v^L. In this case, if he executes, he makes no profit. But if a buy order comes in next, he knows that there is no informed trading and so he estimates the security to be worth μ. He will then slightly undercut the ask price of v^H set by his competitors. Thus, he will get to execute the buy order (as he offers the best price) and will make a profit almost equal to $v^H - \mu$. Therefore, the only scenario in which a dealer makes money in the second period is a sequence of two orders in opposite directions. This only happens with probability $1 - \pi$, since it occurs if and only if the two orders are placed by liquidity traders. That **(p.290)** is, the dealer who manages to capture the first-period trade earns an expected informational rent in the second period of $(1 - \pi)(v^H - \mu)$.

This expected second-period gain comes at the expense of investors, since other dealers are careful to set their quotes so as to make zero profits. Hence, the second-period trading costs are as high as they can possibly be in our setup: the second-period spread is always

$$s_2^O = v^H - v^L.$$

This result shows that lack of post-trade transparency, which increases informational asymmetries among dealers, is a source of informational rents and thereby impairs market liquidity.

At the same time, however, the prospect of such informational rents can sharpen first-period competition between dealers to capture order flow and the attendant information.[3] In our setting, first-period quotes will be driven down to levels such that the total expected profit of each dealer over both periods is nil. The implied first-period ask price is obtained by subtracting the second-period informational rent $(1 - \pi)(v^H - \mu)$ from expression (8.5):

(8.10)

$$a_1^O = \mu + \pi \left(v^H - \mu\right) - (1-\pi)\left(v^H - \mu\right) + (2\pi - 1)\left(v^H - \mu\right).$$

The bid price is symmetric, so that the first-period spread is:

(8.11)

$$s_1^O = (2\pi - 1)\left(v^H - v^L\right),$$

which is clearly smaller than the second-period spread computed above: $s_1^O \langle s_2^O$. Therefore, in a market with post-trade opacity, the time profile of the bid-ask spread is rising: initially dealers accept low spreads and incur losses, which they subsequently recoup by higher spreads. This pattern for the bid-ask spread has been found experimentally by Bloomfield and O'Hara (1999, 2000)[4]. Interestingly, dealers often argue that post-trade opacity enables them to offer better quotes to their clients. This is in fact the case in the first period of the model, but the practice is really just a way to prepare the ground for much less competitive quotes in subsequent periods.

Note that if π is low enough (less than 1/2), the model predicts that with post-trade opacity, the first-period spread can be negative! This may seem paradoxical, although such "crossed quotes" (or "locked quotes") do sometimes **(p.291)** arise briefly in some markets. However, it is a situation that cannot last, since it is a clear opportunity for arbitrage profit-you can buy a security and resell it at the same time for a higher price! If arbitrage prevents a negative bid-ask spread, then dealers will retain strictly positive expected profits over the two periods, which could explain why they generally oppose post-trade transparency. And their hostility to transparency will be further reinforced if first-period competition is too weak to fully dissipate their overall profit.

Who gains and who loses from post-trade opacity? If price competition among dealers in the first period drives their overall expected profits to zero, all we need to do to find out is compute the total trading costs of the liquidity traders in the opaque market. We get:

(8.12)

$$TC^O = \frac{1}{2} s_1^O + \frac{1}{2} s_2^O - \pi\left(v^H - v^L\right),$$

which is twice as much as with post-trade transparency (see equation (8.9)). Thus, post-trade opacity ultimately raises informed traders' expected profits at the expense of liquidity traders.[5] The reason is that in the opaque market the problem of adverse selection persists into the second period, whereas in the transparent market it is eliminated by last-trade publication at the end of the first period. More precisely, in the opaque market adverse selection persists because a dealer gleans from the first-period order flow information that enables him to gain a competitive edge, so as to both outbid other dealers and exploit this edge at the expense of liquidity traders.

This model highlights the fact that dealers may wish to acquire order flow information, either with smaller spreads or through direct monetary inducements such as payment for order flow (i.e., compensation offered to brokers by a dealer in return for channelling orders to him). In the model, these rebates dissipate the informational rents created by opaqueness. In reality, dealers may be able to set them so as to keep some rents. Thus, both dealers and informed traders could benefit from post-trade opacity at the expense of noise traders. This again illustrates how changes in transparency redistribute trading profits and costs among market participants.

As for price discovery, it can be shown that it is less efficient in the opaque market (see exercise 2). Intuitively, in the opaque market, quotes underadjust relative to the information contained in the order flow. In the first period, dealers are willing to execute a buy order at a price below their current value estimate, conditional on receiving a buy order. But in the second period, dealers **(p.292)** set quotes that are independent of the first-period order and that are therefore totally uninformative, generating a discrepancy between transaction prices and fair value. In contrast, in the transparent market, second-period quotes are perfectly informative.

8.3. Revealing Trading Motives

Transparency need not be limited to prices and quantities but may extend to the identity of potential counterparties. This information can be price relevant insofar as it offers insight into the reasons why people want to trade, and specifically whether they have superior information about the value of the security. For example, consider a broker who receives a large sell order from a mutual fund facing substantial customer redemptions. He may get a better price for his client if he can reassure market makers that, given its source, the order is most unlikely to be driven by superior information.

Market designs differ in the scope for conveying such detailed information about trading motives. For example, floor markets allow participants to see one another and interpret subtle cues like impatience and nervousness, as well as to voluntarily share information about their clients. Similarly, dealer markets enable brokers to interact with dealers by phone and explain their clients' reason for trading. This exchange of pre-trade information can be seen as a way of increasing pre-trade transparency for the parties involved.[6] Whether it improves the overall transparency of the market depends on how many people are included in the exchange of information. Knowing their motivations enables liquidity suppliers to offer different prices to different traders, opening up the possibility of price discrimination.

Transmission of information about trading motives—and the attendant price discrimination—is less easy in electronic LOB exchanges, where liquidity providers must post quotes without observing the identities of the brokers who are placing market orders. Of course, there are still ways of getting around the problem. If the exchange reveals the codes of the limit order submitters, which not all exchanges do, then a broker could reveal his identity by placing a marketable limit order at the best existing quotes. But the broker's identity provides at best imperfect guidance about the motives for the order. Alternatively, the broker may call up counterparties to arrange a transaction before clearing it in the main market—but this takes time and effort. **(p.293)**

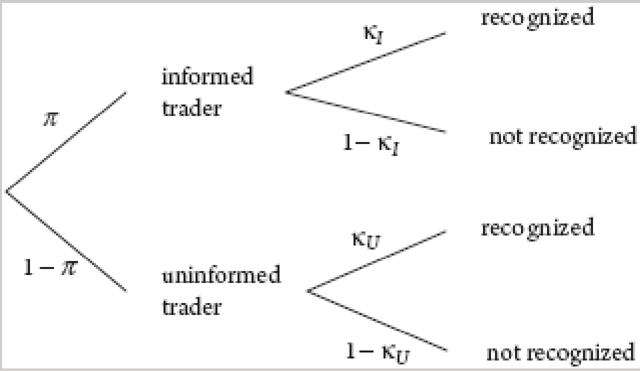


Figure 8.3. Order arrival and recognition of trading motives

In some cases, the need to disclose the trading motives and the identities of the liquidity demanders is so great that non-anonymous trading mechanisms emerge, not necessarily at the exchange authorities' initiative. For instance, an investor in New York who has to carry out a large trade can contact a "block broker" who operates in the so-called upstairs market. The broker elicits the trading interest of large liquidity suppliers after disclosing the identity of the initiator and his motive for trading. Alternatively, the investor could publicly announce the trade and its planned execution date some days in advance. This practice is called "sunshine trading". Finally, in Switzerland since 1997, the exchange has had a "second trading line," where firms can buy their own shares back openly on a separate market segment, to avoid exacerbating adverse selection in the main market (firms being generally considered well informed about their own prospects). See Chung, Išakov and Pérignon (2007).

How does the exchange of information about trading motives affect liquidity? To answer, we resort once more to the one-period model presented in Chapter 3, where π and $1 - \pi$ are the probabilities of an order coming from an informed and a liquidity trader, respectively. But we now assume that before trading, a fraction κ_U of the uninformed traders is recognized as such, perhaps because they manage to reveal their lack of information via a sunshine trading announcement.[7] Symmetrically, a fraction κ_I of the informed traders is also found out, possibly because they have developed a reputation of trading in advance of news. An important assumption, for the moment, is that a trader's identity, if revealed, is observed by all market makers. This assumption will later be relaxed. The order arrival process is summarized in figure 8.3.

Now, a trader who places a buy order will face a different ask price, depending on whether he is recognized or not; if he is recognized, he will pay a different price depending on his type. The easiest cases are those in which traders' **(p.294)** motives are recognized. If the buyer is known to be informed, the market makers' best value estimate is v^H, while if he is known to be uninformed, it remains at its unconditional value μ. Now consider the case in which the trader's type is unknown. We need to compute the probability that this trader is informed, conditional on not having been identified. This probability is

$$\pi' = \frac{\pi(1-\kappa_I)}{\pi(1-\kappa_I) + (1-\pi)(1-\kappa_U)'}$$

that is, the fraction of informed traders within the population of unidentified traders. As this probability plays the same role as π in the model of Chapter 3, we deduce that the ask and bid prices posted for an order of unknown origin are

(8.13)

$$a = \mu + \left[\frac{\pi(1-\kappa_I)}{\pi(1-\kappa_I) + (1-\pi)(1-\kappa_U)}\right]\left(v^H - \mu\right),$$

$$b = \mu - \left[\frac{\pi(1-\kappa_I)}{\pi(1-\kappa_I) + (1-\pi)(1-\kappa_U)}\right]\left(v^H - \mu\right).$$

As expected, the revelation of information on trading motives results in price discrimination. Recognized uninformed traders trade at a zero spread, since they can buy or sell the security at price μ, while informed traders identified as such trade at the fair price and therefore see their trading profits eliminated. Last, all unrecognized traders, regardless of type, trade at the pooled, partially informative prices given in equation (8.13).

The model is open to different interpretations for different values of κ_I and κ_U. The case of $\kappa_U \rangle$ 0 and $\kappa_I = 0$ can be interpreted as the coexistence of two populations of uninformed traders: some are identified as such and obtain a zero spread, because they can access the upstairs market or practice sunshine trading; others are pooled with informed traders. Clearly, uninformed traders in the former group benefit from being recognized, while the others are disadvantaged because they face a larger bid-ask spread than they would under anonymous trading. The bid-ask spread faced by unidentified traders is

$$s = \left[\frac{\pi}{\pi + (1-\pi)(1-\kappa_U)}\right]\left(v^H - v^L\right),$$

which increases in κ_U and is therefore minimal for $\kappa_U = 0$. The reason is that the pool of unidentified traders now has a smaller proportion of uninformed traders than in an anonymous market. However, the average uninformed trader is better off, since only a proportion $1 - \kappa_U$ pay this spread s, and the average spread $(1 - \kappa_U)s$ decreases with

κ_U. Intuitively, since there are fewer uninformed traders in the main market, the expected profits of the informed traders will decrease. So reducing anonymity increases the posted bid-ask spread, even though it reduces average trading costs for uninformed traders.

(p.295) Consider now the opposite case in which some informed traders—but no uninformed traders—can be recognized, that is $\kappa_U = 0$ but $\kappa_I \rangle 0$. This can be thought of as a situation in which regulation obliges some potentially informed traders to disclose their intentions. For example, a potential takeover raider may be forced to disclose that the fraction of the target company's equity (his "toehold") has breached some regulatory threshold (often 5 percent of the company's voting equity): if subsequent large buy orders are attributed by market participants to such an investor, they should have a larger price impact.

In this case, the bid-ask spread for unidentified traders is

$$s' = \left[\frac{\pi(1-\kappa_I)}{\pi(1-\kappa_I) + (1-\pi)}\right]\left(v^H - v^L\right),$$

which is decreasing in κ_I, so identifying informed traders improves market liquidity.

This analysis suggests that disclosing trading motives benefits uninformed traders, at least on average, but the conclusion rests on a crucial assumption: namely, that information on trading motives is disclosed to *all* market participants. If it were revealed only to a subset of price setters, the conclusion might be different. To see this, in the context of the model with $\kappa_U \rangle 0$ and $\kappa_I = 0$, suppose that the customer's type is observed only by the dealer he contacts, possibly owing to a long-standing relationship that confers credibility on the information revealed by the customer—a situation that cannot be replicated with other dealers.

This situation gives the dealer market power, as he can appropriate some or all of the informational rent involved in the transaction. To see this, suppose that a buyer is identified by his dealer as uninformed. The best price that this buyer can then expect from the dealer is μ, which corresponds to zero spread and zero profit for the dealer. At the opposite extreme, the worst possible price that he will accept from the dealer is his outside option $\mu + s/2$, the ask price that he would get in the main market. In this situation of bilateral monopoly, the relative bargaining power of the buyer and the dealer determines which price within the interval $[\mu, \mu + s/2]$ the traders actually settle upon. If the client has some bargaining power, he negotiates a discount from $[\mu + s/2$, his main market price. Such price improvements are in fact quite common in dealer markets, such as Nasdaq and the LSE (Reiss and Werner (2004)).

In the extreme case in which the dealer has all the bargaining power, the client gets the same price as unrecognized traders who buy on the main market, that is $[\mu + s/2$. As a result, all traders will be strictly worse off than under completely anonymous trading, since s is smallest when $\kappa_U = 0$. In fact, it is easy to verify that this remains true whenever the dealer’s bargaining power is great enough, that the discount is low. This shows that revealing trading motives may not **(p.296)** lower the trading costs of uninformed traders (even those whose motives are revealed) if the information remains confidential (disclosed to just one dealer).

Channeling the orders of uninformed clients to specific dealers or markets is known as cream skimming, the practice for which brokers often obtain a payment for order flow. If brokers also have a market-making capability (i.e., they are broker-dealers), they may skim off the uninformed orders for in-house execution, a practice known as internalization.

There is evidence that cream skimming does take place: Easley and O’Hara (1996) find that orders diverted from the NYSE are less informative than those that remain on the NYSE. Grammig and Theissen(2012) obtain the same result for orders internalized on Deutsche Börse’s Xetra trading system. They also find that investors whose orders are internalized do not capture the entire benefit of lower adverse selection costs. Since skimming deprives the main market of a proportion κ_U of the uninformed order flow, it should reduce market liquidity according to this model. However, the evidence is mixed (Battalio, 1997; and Battalio, Greene, and Jennings, 1998).

We deal with internalization, payment for order flow, and cream skimming when discussing market fragmentation in Chapter 7. Questions about these practices resurface here, because opacity and fragmentation are intertwined. For instance, order flow transparency (discussed in section 8.1.3) is harder to achieve when traders can split market orders among multiple trading platforms, unless there are extremely efficient linkages among them.

8.4. Why Are Markets So Opaque?

The previous sections lead to the conclusion that transparency about quotes, orders, and traders’ identities generally enhances market liquidity, at least as far as uninformed traders are concerned. The natural question, then, is why we see so little of it: why are so many real-world securities markets so opaque along one or more of the dimensions we have analyzed. A simple explanation may be that these market structures are not designed to benefit uninformed traders but market-making intermediaries or informed traders (see section 8.4.1). Indeed there is good reason to think that these market professionals have far greater influence in the design of trading rules than uninformed and occasional traders.

Even if market making is competitive, so that it yields no rents to intermediaries, the incentive to offer opacity to large trades still exists, even though this may decrease the liquidity available to retail traders. This issue is examined in section 8.4.2.

But the opacity of securities markets may also reflect concerns about economic efficiency, at least in some cases. For instance, transparency may be **(p.297)** problematic—it may even reduce liquidity—if it exposes limit-order placers to a high risk of being picked off by more informed traders. Moreover, if markets are opaque, market makers may find it more difficult to sustain collusive agreements because it will be harder to detect and punish violations of the cartel's rules. In this sense, opacity may foster competition between market makers. Section 8.4.3 discusses these brighter sides of market opacity.

8.4.1 Rent Extraction and Lobbying

As section 8.1.1 shows, market makers can extract rents in a market with little pre-trade transparency on quotes, and informed traders can do so in one with little post-trade transparency. In some cases these two categories actually coincide: intermediaries may have superior information (partly due to their advance knowledge of the order flow), and so they may resist making their quotes visible, in part to retain informational rents.

This is consistent with the empirical evidence that transaction costs fall when markets become more transparent. For example, the bid-ask spread on the NYSE declined when outside limit orders were allowed to compete with the specialist, and execution costs in the muni bond market fell by half when the TRACE trade reporting system was introduced. And even in such inherently opaque markets as real estate or travel services, the introduction of web-based search engines has reduced the profits of incumbent firms.

Trading platforms themselves can also extract rents from opacity by restricting access to data on prices and trades, thus increasing the data's value for sale at a premium, even to the point of provoking complaints from some participants. For instance, the NYSE's decision to charge a fee for real time information on quotes and trades in Archipelago (a trading platform acquired in 2006) stirred up strong opposition from investors. Similarly, the fee charged by Nasdaq for the dissemination of corporate bond prices has been very controversial. In 2003, the sale of market data generated a revenue of $386 million for U.S. equity markets, at a dissemination cost estimated at $38 million.

A way in which trading platforms can squeeze even more profits from the data that they generate is by selling "low-latency" (super-fast) access to data feeds, even selling to some traders the opportunity to colocate their computers in physical proximity to the platform's own computer. Such preferential highspeed access is keenly sought by algorithmic traders, who can make money by beating their competitors even by nanoseconds. This again creates tiered access to market data.

Besides enabling intermediaries to retain rents from market power and informed traders to extract profits from superior information, opacity may **(p.298)** allow brokers to cheat their customers by misreporting trade prices, or to simply conceal the fact that they exerted little effort or care to make sure their customer got the best price available on the market at the time of execution. This may occur even where best-execution rules are in place, because enforcing such rules is harder in opaque markets: investors may more easily fail to see that they were not well served by their brokers, and be more unlikely to assemble the evidence necessary to demonstrate dishonesty or negligence.

Therefore, market professionals should be expected to be generally interested in limiting the transparency of the markets where they operate. And it is no surprise that their interests influence market regulation more than those of uninformed market users. Not only do they have a more regular market presence, but nowadays they largely own, control, and manage stock exchanges and multilateral trading facilities, as a result of the trend towards demutualization (described in Chapter 2).

8.4.2 Opacity can Withstand Competition

The previous section may suggest that there is an intrinsic connection between opacity and rent extraction (or collusion) by trading intermediaries. If so, one might expect that competition from new entrants, unaffiliated with incumbents, would spark a race towards transparency. But in fact opaque markets are resilient to competition.

To see why, consider again the model of post-trade transparency presented in section 8.2. Suppose that dealers are allowed to choose whether to publicize their trades. In this case the only equilibrium is such that none chooses to publicize trades. The reason is that the market maker who does not publicize can undercut those who do, because he can use the information gleaned from the first-period trades to make a profit in the second period. To see this, recall that the competitive bid-ask spread at time $t = 1$ in the opaque market (equation (8.11)) is lower than the spread that a transparent market maker could charge (equation (8.8)). Thus, in that model all first-period trading will be captured by opaque market makers. Note that the choice about the degree of opacity can be made not only by individual dealers but also at the level of trading platforms on which they operate. Under this interpretation, what the model predicts is that competing platforms will inevitably opt for opacity in the design of their trading system, and such platforms will capture all trading.

In reality, opaque market makers or platforms will compete aggressively only for large orders: small trades, which are unlikely to be informative, will be of little or no interest, since they can be filled just as well by transparent market **(p.299)** makers. This leads to a two-tier market, where large trades execute opaquely and small ones transparently.

The foregoing implies that market forces alone are unlikely to produce ex-post transparency, at least for large trades. Regulation is required to mandate a minimum level of transparency. For instance, the U.S. corporate bond market was opaque until 2002, when the SEC mandated a system (called TRACE) to disseminate price and trade information, albeit with a lag.

Even mandatory disclosure, however, encounters considerable problems of enforcement. First, there may be regulatory arbitrage: if trading can migrate to venues that do not impose less post-trade transparency, it will. Many observers of the migration of the wholesale markets in European blue-chip equities to SEAQ International in London in the early 1990s argued that a primary determinant was the lack of trade publication there. Europe-wide negotiations on market regulation and transparency typically pitted the looser, more freewheeling regime of the London wholesale markets against the more centralized, transparent regime prevailing in the home markets. Nowadays, the increasing role of dark pools and of the OTC market in the United States and post-MiFID Europe can be explained using the same logic.

Second, it may be possible to evade ex-post transparency requirements: Franks and Schaefer (1995) point out that U.K. trade reporting deadlines can be circumvented by using “protected trades”: market makers informally offer a price to a customer for a large deal (with the understanding that they will, in practice, honor the commitment), but the deal is not officially finalized and confirmed until they have had time to unwind the resulting inventory. This delays the trade report—and its publication—beyond the regulators’ intention. Moreover, it reduces immediacy and imposes some execution risk on large traders. Porter and Weaver (1998) find evidence that Nasdaq market makers do use late trade reporting to manage the flow of information: the trades that are reported out of sequence are disproportionately large and much too common to be due to legitimate causes (such as computer glitches).

8.4.3 The Bright Side of Opacity

Is the self-interest of market professionals the only explanation for the prevalence of opaque trading systems? The fact that even the most transparent marketplaces allow for some opaque forms of order placement suggests otherwise. For instance, most electronic order book systems—otherwise known for their high degree of pre-trade transparency—allow traders to enter hidden limit orders or iceberg orders; In this case, only a small portion of the order is visible to other traders, and the rest gradually becomes visible as the order executes **(p.300)** against incoming orders. Of course, other investors will eventually realize when a hidden order is present on the LOB, and adapt their trading strategies accordingly (De Winne and D’hondt, 2004). This takes time, however, and even then the total size of the order remains unknown.

One reason why an investor may want to place such an order is to avoid giving away his private information to the market by placing a visible large order.[8] This motivation would be in line with the idea that opacity benefits informed traders. But the opposite may be true as well: a trader may consider placing a hidden order to protect himself against the risk of being picked off by better informed traders, when if he is afraid, say, that his information may be (or may become) stale and his order accordingly mispriced. Indeed, as Copeland and Galai (1983) observe, placing a buy limit order is tantamount to offering a free put option to other market participants; likewise, a sell limit order is a free call option. Investors who receive new information can exploit such an option by hitting existing limit orders before they can be cancelled. Therefore, traders who submit limit orders bear an adverse selection cost similar to that facing market makers: the difference is that with limit orders, the possible losses to informed trading are more limited, orders vanish once hit (and they are typically hit as soon as they go into the money), while dealer quotes remain available for trading until the dealer manages to update them.[9]

Therefore, if uninformed investors could not place hidden orders, they would most likely place market orders, possibly trickling them into the market to minimize price impact. As a result, the LOB would be less deep than with hidden orders, which in practice do provide a considerable amount of liquidity. For example, in an active LOB such as that for the CAC 40 blue-chip French stocks on average, the displayed depth at the best five quotes accounts for less than 55 percent of the total depth at these prices, that is, more than 45 percent are provided by hidden orders (De Winne and D'hondt, 2004). Here, then, we have an instance in which imposing a greater degree of transparency (banning hidden orders) would be likely to reduce liquidity, by discouraging the use of limit orders.

A similar argument holds for anonymity, again insofar as it applies to limit-order traders, who provide liquidity, rather than to market-order traders, who consume it. Recall again the analogy between limit orders and options: as the value of an option depends on the volatility of the underlying security, the value **(p.301)** of a limit order to the market—and hence the cost to its placer—increases with volatility. Hence in anticipation of increased volatility, traders should be less willing to place limit orders, or at least bid less aggressively to reduce the risk of being picked off. As a result, the bid-ask spread should widen. So a wide bid–ask spread signals that limit-order traders expect high volatility. Now suppose that some traders know future volatility and others do not: the latter can infer volatility from the limit order book, which also contains offers posted by the informed traders. But if trading is anonymous, uninformed traders can no longer pick out the limit orders from informed participants. So, when they see a wide spread, they don't know precisely whether or not it reflects high future volatility.

In this setting, Foucault, Moinas, and Theissen (2007) show that if the fraction of informed traders is high, the uninformed will tend to see a large spread as a hint that future volatility is high. They will cave in, thus contributing to a large bid-ask spread. But if the fraction is low, the uninformed traders will consider a large spread as relatively uninformative, and so will trade aggressively, thereby tending to narrow the spread. So in the first scenario anonymity increases the spread, while in the second it reduces it. They show empirically that the latter result is consistent with the data from Euronext (the French Stock Exchange), where identifiers for the brokers placing limit orders ceased to be disclosed after April, 23 2001. After this date, the average quoted spread became significantly smaller; in this case, anonymity increased liquidity. Notice that this result does not contradict those of section 8.3, which refer to the anonymity of market-order, not limit-order placers.

The anonymity of quotes may also deter collusion between quote setters, or at least make it more complicated. When identities of quote setters are unknown, it may be difficult or impossible to "punish" quote setters who violate previous accords, so quote setting is likely to become more competitive. This is another channel through which pre-trade opacity may increase liquidity: Simaan, Weaver, and Whitcomb (2003) find that market makers are more likely to quote on odd ticks and actively narrow the spread when they can do so anonymously by posting limit orders on ECNs. Thus, from a public policy perspective, decreasing the level of pre-trade transparency by allowing anonymous quotes could improve price competition and narrow spreads.

8.5. Further Reading

In this chapter, we have mainly analyzed transparency in models of trading with asymmetric information. However, the literature on pre-trade quote transparency also covers situations where market-maker pricing is determined by **(p.302)** inventory holding costs. Biais (1993) posits that market makers differ in inventories (and thus in their inventory holding costs) but are otherwise approximately risk neutral and in agreement on the fundamental value. He compares two market structures: a transparent market in which dealer inventories (or equivalently quotes) are observed by all market participants and an opaque market in which dealers have no information on the inventories (quotes) of their competitors. Effectively, the former is an English auction and the latter is a sealed-bid auction, and the two produce different market maker bidding strategies. The revenue equivalence theorem of auction theory applies to this setting (risk neutral bidders with independent, identically distributed private values) so that, as Biais concludes, the expected spread will be the same in the two markets.

Beyond this stylized setting, auction theory states that revenue equivalence breaks down if the bidders (the market makers) are risk averse, if their private valuations are correlated or asymmetrically distributed, or if there is a an imperfectly known common value generating a winner's curse problem (see, for example, Bolton and Dewatripont, 2004). Within Biais's framework, de Frutos and Manzano (2002) relax the assumption of low market-maker risk aversion and conclude that the opaque market is more liquid. Yin (2005) extends Biais's analysis by incorporating the idea that quote transparency exerts competitive pressure by eliminating customers' search costs. He considers the case in which customers must pay a search cost in the opaque market and finds that the average spread is smaller in the transparent market.

A few empirical studies have examined how a change in pre-trade transparency affects market quality. Hendershott and Jones (2005) consider the impact of an SEC regulatory action that prompted the Island ECN to stop displaying limit orders in three of the most actively traded ETFs for about a year. They find that this was associated with a drop in liquidity for these ETFs. Boehmer, Saar, and Yu (2005) find that liquidity increased when the NYSE started releasing information on the LOBs of its listed stocks in 2002, as Baruch (2005) had predicted. However, Madhavan, Porter, and Weaver (2005) consider a similar change on the Toronto Stock Exchange and obtain an opposite conclusion.

The empirical evidence on post trade transparency is also mixed. Gemmill (1996) considers three different reporting regimes (no publication, ninety-minutes delay, and twenty-four hours delay) for dealers on the LSE, and finds no significant impact on liquidity. In contrast, more recent empirical work (Bessembinder, Maxwell, and Venkataraman, 2006; Edwards, Harris, and Piwowar, 2007; and Goldstein, Hotchkiss, and Sirri, 2007) suggests that the implementation of post-trade transparency in the U.S corporate bond market led to a significant fall in trading costs. Röell (1988, 1995) models post-trade **(p.303)** transparency and finds that post-trade transparency benefits retail uninformed raders in particular.

Foucault, Pagano, and Röell (2010) offer a brief survey of the theoretical and empirical literature on transparency.

8.6. Exercises

1. Uncorrelated noise traders' orders.

Modify the setting of section 8.1.3 by assuming that if noise traders are present, each independently chooses to place either a buy or a sell order. In other words, we can have the following orders by noise traders: (buy, buy), (buy, sell), (sell, buy), (sell, sell), each occurring with probability $(1 - \pi)/4$. Compute the bid and ask prices under pre-trade transparency and opaqueness. How do the predictions of the model change?

2. Price discovery and transparency.

Consider the model of post-trade transparency described in section 8.2. Consider the time-averaged expected squared deviation between the transaction price and the true value of the security, that is

$$\frac{\mathrm{E}\left[\left(p_1^k - v\right)^2\right]}{2} + \frac{\mathrm{E}\left[\left(p_2^k - v\right)^2\right]}{2},$$

where p_t^k is the transaction price in period $t = 1,2$ in regime $k = T,O$ (transparent, opaque). Show that price discovery is more efficient in the transparent market. You may limit your analysis to the case $\pi > ½$ in which the equilibrium first-period spread is positive.

3. Asymmetric information and dealers.

(Hard question!) Analyze the opaque regime of the model described in section 8.2 under the assumption that in the second period, the market maker who has inherited order flow information from the first period posts his quotes simultaneously with the other market makers—that is, that there is no Stackelberg leader in the second period. First provide an intuitive argument that there is no pure-strategy equilibrium of the quote-setting game among market makers in the second period. Then, characterize the ensuing mixed-strategy equilibrium. **(p.304)**

Notes:

(1.) This inequality states that, for any random variable x, $\mathrm{E}[f(x)] > f[\mathrm{E}(x)]$ if $f(\cdot)$ is a convex function. Here we apply this principle to the function $1/x$, which is convex.

(2.) This dealer behaves as a "Stackelberg follower." Though unrealistic, this assumption ensures the existence of a pure-strategy equilibrium in the price-setting game between asymmetrically informed dealers. If instead dealers were assumed to set their quotes simultaneously, the equilibrium is necessarily a mixed-strategy one. This has been shown for first-price common value auctions with differentially informed bidders, to which our game is equivalent (see Engelbrecht-Wiggans, Milgrom, and Weber, 1983). For a treatment of the problem in our setting, see Röell (1988). See exercise 3 as well.

(3.) In this respect, dealers have been compared to the bookmaker who said he was happy to lose money to a successful bettor: "He is my most valuable client. I always shorten the odds when he bets and he saves me a fortune." (*The Financial Times*, December 6, 1987).

(4.) Most experimental work regarding pre-trade transparency has found that markets with greater pre-trade transparency feature lower bid-ask spreads. See for instance Flood et al., 1999.

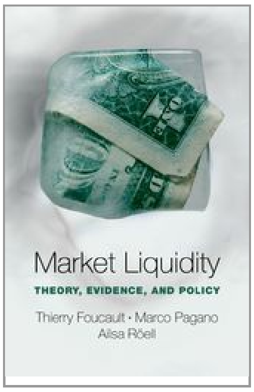
(5.) At first glance, this is surprising since competition among dealers in the first period dissipates their rents. Hence one would expect uninformed investors to entirely recoup their second period losses with lower trading costs in the first period. But this is not the case, because informed investors capture part of the benefits of the lower spread in the first period.

(6.) In principle, the exchange of information need not be only from traders to liquidty suppliers, as is assumed in this section. For instance, market makers may also reveal their identities to traders who place orders with them, or they may not—an issue touched on in the next section.

(7.) For models that study how investors can signal their trading motives, see Seppi (1990) or Admati and Pfleiderer (1991) for instance.

(8.) A cost however is that hidden orders have a lower execution probability and longer time to completion. See Bessembinder, Panayides, and Venkataraman (2009).

(9.) Equivalently, they can be seen as auction participants exposed to the winner’s curse problem, because their buy limit orders are more likely to be filled when they overestimate the true value, and their sell orders are more likely to be filled when they underestimate it (see Chapter 6).



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Thierry Foucault, Marco Pagano, and Ailsa Roell

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Liquidity and Asset Prices

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter first explains how liquidity affects the returns required by investors, and hence asset prices. It then examines how prices in illiquid markets may diverge from underlying long-run values, especially in the context of market freezes and financial crises. It also highlights that noise trading itself can play an important role in pushing securities prices away from fundamental values. This occurs especially when noise trading is correlated among investors and over time. The final sections provide suggestions for further reading and exercises.

Keywords: illiquidity, investment returns, asset prices, noise trading, stock prices

Learning Objectives:

- • Why illiquidity and liquidity risk affects security prices and returns
- • Sources of illiquidity premia in security returns
- • Illiquidity and deviations from the law of one price
- • How funding liquidity impacts on market liquidity

9.1. Introduction

A recurrent theme of this book is that the various costs of liquidity provision (attributable to adverse selection, inventory holding, and order processing) create a wedge between transaction prices and the fundamental values of assets. This wedge is a measure of market illiquidity. But we have not yet considered that illiquidity may affect the asset's value.

There are two main reasons for this. First, transaction costs reduce the return to investors, as a tax on capital gains does. Hence, other things being equal, investors will pay less for less liquid assets, so asset returns contain an illiquidity premium in addition to a risk premium, as section 9.2 explains. After analyzing the asset pricing implications of illiquidity in a simple model where bid-ask spreads are exogenous, section 9.2 considers situations where illiquidity and prices are jointly determined, positing that illiquidity is due either to informational asymmetry (section 9.2.4) or to search costs (9.2.5).

(p.308) Second, liquidity varies over time, and the fluctuations may add "liquidity risk" to the basic risks characterizing financial assets, such as default risk and unknown future dividend streams. As usual in asset pricing theory, if investors are sensitive to risk, they will require compensation for liquidity risk as well, unless it can be diversified. Liquidity risk and how it affects illiquidity premia is discussed in section 9.3.

Why can't arbitrageurs play on the differences in prices and returns generated by differences in liquidity, and so eliminate the illiquidity premium? As section 9.4 explains, one reason is that arbitrageurs need money to finance their positions and accordingly risk forced liquidation before they can realize their profit. This implicit limit to arbitrage has been put forward to resolve several asset pricing puzzles that apparently refute the law of one price, such as persistent pricing discrepancies for dually traded stocks and closed-end mutual funds.

It is often observed that in crashes and panics liquidity dries up.[1] Such evaporation of liquidity exacerbates securities price drops, dramatically lowering the value of the collateral that arbitrageurs can pledge to build their positions. As a consequence, the fall in market liquidity worsens their terms for financing; that is, it lowers "funding liquidity." In turn, this tightens the limits to arbitrage, and further reduces market liquidity. This interaction between market and funding liquidity is a feature of many financial crises, a relationship discussed in section 9.4.3.

The chapter concludes by highlighting that noise trading itself can play an important role in pushing securities prices away from fundamental values. As explained in section 9.5, this occurs especially when noise trading is correlated among investors and over time.

9.2. Illiquidity and Asset Prices

This section presents a simple framework drawn from Amihud and Mendelson (1986), which illustrates the effect of liquidity on asset prices and required returns and shows how this relationship is affected by the investors' time horizon, with empirical evidence on the relationship.

9.2.1 The Illiquidity Premium

To illustrate how the level of liquidity affects the prices of asset, consider a simple example. U.S. Treasury notes and bills are bonds of different initial **(p.309)** maturity. They have exactly the same default risk but are traded on distinct markets. At issue, notes have longer maturities (two to ten years) than bills (twelve months or less). But at any given point in time there will coexist bills and notes with identical residual maturity of less than six months, with a single payout remaining at maturity. As those securities are identical in risk and payout timing, standard arbitrage arguments would imply that in a frictionless world they should have the same price per dollar of payout. However, Amihud and Mendelson (1991) find that notes typically trade at a discount relative to identical bills; on average, the annualized yield on notes exceeds that on bills by fourty-three basis points, in a sample with an average maturity of about ninety-five days.

A natural explanation for this significant premium is that the markets for bills and notes are not equally liquid. Amihud and Mendelson (1991) report that the bid-ask spread on notes averages four times as great as that on bills (3 and 0.7 basis points respectively). Brokerage fees also differ (0.75 and 0.12 basis points, respectively, for transactions of one-million-dollar face value). Hence notes must offer extra compensation to attract investors.

To see this, consider the following simple model. An investor buys a security that he plans to sell after h periods; during this time the security does not pay dividends or interest. The market is illiquid, and the proportional bid-ask spread at date t is s_t, so that the ask and bid prices at that time are given by:

(9.1)

$$a_t = m_t\left(1 + \frac{s_t}{2}\right),$$

(9.2)

$$b_t = m_t\left(1 - \frac{s_t}{2}\right).$$

Henceforth, we assume that the midquote is equal to the fundamental value of the security, that is, $m_t = \mu_t$. For brevity, we call s_t a bid-ask spread, but it should be interpreted more broadly as a measure of trading cost, that includes, for instance, the price impact of large transactions and brokers' commissions. Suppose that investors require a return of*r* per period on the security, given its risk characteristics. For example, for a riskless security like a Treasury bill, the required return is simply the risk-free rate. If the market were perfectly liquid (i.e., s_t = 0 at any time) then μ_t would grow at rate *r* on average.

Under these assumptions, the maximum price that the investor is willing to pay is given by the standard discounted cash flow model:

(9.3)

$$a_t = \frac{b_{t+h}}{(1+r)^h},$$

(p.310) that is, using (9.1) and (9.2):

$$\underbrace{\mu\left(1+\frac{s_t}{2}\right)}_{a_t} = \underbrace{\mu_{t+h}\left(1-\frac{s_{t+h}}{2}\right)}_{b_{t+h}}\frac{1}{(1+r)^h}.$$

This can be used to express the current value of the asset as its discounted future value adjusted for current and future transaction costs:

(9.4)

$$\mu_t = \mu_{t+h} \times \frac{1}{(1+r)^h} \times \frac{1-\frac{s_{t+h}}{2}}{1+\frac{s_t}{2}}.$$

The last term is a measure of illiquidity since it decreases in both s_t and s_{t+h}. The fundamental value at date *t* is correlated with illiquidity: the greater the current or future spread, the higher the transaction costs for investors and the lower the value of the asset to them. This is consistent with our example, in which the more liquid Treasury bills traded at a premium over notes, notwithstanding their identical payoffs at maturity.

Empirical researchers often find it easier to test asset pricing models by investigating the properties of returns rather than prices. Therefore, we use equation (9.4) to derive a relationship between gross returns and bid-ask spreads:

$$\frac{\mu_{t+h}}{\mu_t} = (1+r)^h \times \frac{1+\frac{s_t}{2}}{1-\frac{s_{t+h}}{2}},$$

and thus *R*, the gross return per period, solves:
(9.5)

$$(1+R)^h = (1+r)^h \times \frac{1+\frac{s_t}{2}}{1-\frac{s_{t+h}}{2}}.$$

The gross return R is the average per-period percentage change in the security's fundamental value required to induce the investor to hold the security. As the last term on the right-hand side of the equation exceeds 1, R is greater than the net return r: the difference compensates investors for transaction costs. This explains why the yield on notes can be greater than the yield on bills.

The gross return in (9.5) also depends on the investors' holding period h. It is easiest to see the effect of this factor if the percentage spread s_t is assumed constant, so that $s_t = s_{t+j} = s$. Using this assumption in equation (9.5), the gross return R is given by the following approximation for r and s:[2]

(9.6)

$$R \simeq r + s/h.$$

(p.311) This expression shows that R is increasing in the spread s at a rate that is inversely proportional to the holding period h. The reason is that an investor who holds the security incurs the transaction cost only once every h periods, and thus the per-period cost is reduced.

The foregoing analysis may seem restrictive, in that it assumes the asset to be safe. But this assumption is not indispensable: if μ_t (the fundamental value) is uncertain, the discount rate should be the rate of return required by investors, given the risk of the security. According to the capital asset pricing model (CAPM), this risk adjustment is determined by adding to the risk-free rate r a premium that is proportional to its "beta," that is, the covariance between the return on the stock and that on the market portfolio, divided by the variance of the latter. Specifically, the required rate of return on asset j should be

(9.7)

$$\mathrm{E}\left(r_j\right) = r + \beta_j \left[\mathrm{E}\left(r_M\right) - r\right],$$

where $\mathrm{E}(\cdot)$ is the expectation operator, r_{M} is the uncertain return of the market portfolio, and $\beta_j \equiv \mathrm{cov}(r_j, r_M)/\mathrm{var}(r_j)$. This expression indicates that the risk premium is the product of the contribution of asset j to the risk of a well-diversified portfolio (β_j) multiplied by the risk premium on the market portfolio ($\mathrm{E}(r_M) - r$). Hence, the β of a security captures its specific risk, which cannot be suppressed by holding it in combination with many others. For this reason, this parameter is often called the security's systematic risk.

For asset j, equation (9.5) becomes

(9.8)

$$\left[1 + \mathrm{E}\left(R_j\right)\right]^h = \left[1 + \mathrm{E}\left(r_j\right)\right]^h \times \frac{1 + \frac{s_{jt}}{2}}{1 - \frac{s_{jt+h}}{2}},$$

where R_j is the gross return, and s_{jt} the bid-ask spread at date t. Substituting $E(r_j)$ from equation (9.7) into (9.8) and making the same approximation as before, we obtain (9.9)

$$E(R_j) = r + s_j/h + \beta_j [E(r_M) - r].$$

This expression shows that the required gross rate of return on asset j is equal to the risk-free rate plus the illiquidity premium s_j/h plus the risk premium β_j $[E(r_M) \mp r]$. This shows that the CAPM relationship (9.7), which applies to net returns (because that is what investors care about), must be suitably modified if it is estimated using gross returns (i.e., in our setting, returns based on mid-quotes instead of transaction prices). Figure 9.1 depicts the relationship between the expected gross return on asset j and the spread, where the intercept is the expected net return, $E(r_j) = r + \beta_j [E(r_M) - r]$, and the slope is $\frac{1}{h}$ the inverse of the holding period.

Thus gross expected returns have two determinants: (i) the classical determinants of required asset returns, such as the beta in the CAPM, and (ii) an **(p.312)**

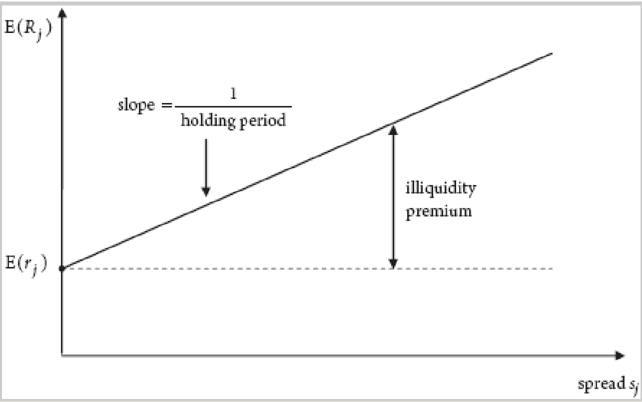


Figure 9.1. Expected return and bid-ask spread

additional premium that compensates for transaction cost. We refer to this premium s_j/h as the illiquidity premium. If all investors have the same holding period h, then the illiquidity premia are approximately linearly related to spreads, with slope $1/h$, as depicted in figure 9.1. If equation (9.9) holds in reality, the inverse of the coefficient of the spread in a regression for the gross rate of return should provide an estimate of the holding period of the representative investor.

Another assumption in the foregoing analysis is that the future spread is known with certainty at the initial date. But in reality the spread may change unpredictably so that the riskiness of the liquidation price will be determined both by fluctuations of the fundamental value μ_{t+h} (due to news about future cash flows, for example) and by fluctuations in the spread s_{t+h}. The latter generate what is known as "liquidity risk." If the liquidity risk of a security is idiosyncratic—that is, uncorrelated with market returns—then the analysis is unchanged, except that the actual spread $st+h$ must be replaced by its expected value $E(s_{t+h})$. Investors will not require any compensation for liquidity risk, since they can diversify it away[3]

In reality, however, future liquidity tends to worsen precisely at times of low market returns (i.e., when μ_{t+h} is low) and high fundamental volatility. This means that liquidity risk amplifies the systematic fundamental risk of securities-it increases their beta. In equilibrium, this prompts investors to **(p.313)** demand an additional compensation for liquidity risk. Section 9.3 addresses this point.

Finally, the analysis can be extended to the case in which the security pays cash flows (coupons or dividends) before the liquidation date $t+h$. These cash flows do not affect the basic insight developed so far, but they do reduce the magnitude of the liquidity premium: intuitively, this is because investors receive part of the return directly via these cash flows, without having to liquidate the security (see exercise 1).

9.2.2 Clientele Effects

So far, we have implicitly assumed that all investors have the same holding period. Actually, though, different investors typically have different holding periods. For instance, pension funds tend to hold securities for longer periods than mutual funds. The cost of trading weighs more heavily on investors who trade frequently, so one would expect them to prefer highly liquid stocks, while low-turnover investors should be more willing to invest in illiquid securities. Amihud and Mendelson (1986) show that this “clientele effect” implies that equilibrium returns should be a concave rather than a linear function of the spread.

To see this, consider a simple example with just two securities, a liquid zero-coupon bond with spread $s1$ and a relatively illiquid one with spread $s_2 \rangle s_1$, and two types of investors, one with short holding period h_1 and another with long holding period $h_2 \rangle h_1$. Consider a situation in which the first group buys only the liquid bond at the gross rate R_1 and the second buys the illiquid bond at the rate R_2.

This situation is an equilibrium if each of the two clienteles has no incentive to switch out of its candidate “preferred habitat” (in the wording of Modigliani and Sutch, 1966). This is the case for the first group of investors, if they earn at least as large a net return r_1 by investing in the liquid asset as by investing in the illiquid one:

$$R_1 - s_1/h_1 \geq R_2 - s_2/h_1.$$

Conversely, the second group of investors will not want to switch to the liquid security if their net return on the illiquid security, r_2, is at least equal to the return per period on the liquid security:

$$R_2 - s_2/h_2 \geq R_1 - s_1/h_2.$$

These two conditions, taken together, imply the inequality:

(9.10)

$$\frac{1}{h_2} \leq \frac{R_2 - R_1}{s_2 - s_1} \leq \frac{1}{h_1}.$$

(p.314)

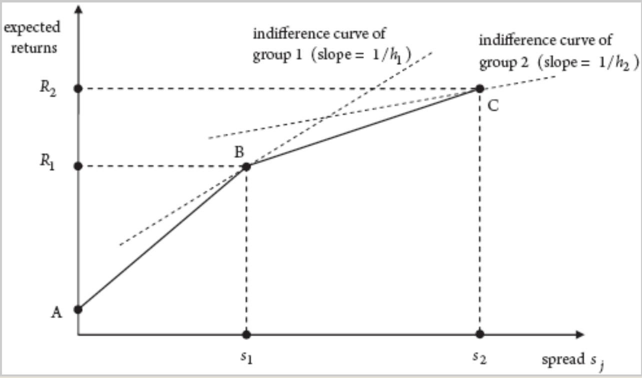


Figure 9.2. Expected return and spread with two clienteles

This condition implies that investor 1 is in his preferred habitat purchasing security 1, while investor 2 prefers security 2. Graphically, the middle expression in inequality (9.10) is the slope of the segment $\overline{BC}$ that connects the equilibrium return-spread pairs for the the two assets in figure 9.2. The condition requires it to lie between the indifference curve of clienteles 1 and 2: the indifference curve of clientele i depicts the gross return-spread combinations that yield a given net return r_i to clientele i. Thus, the indifference curve of clientele i has slope $1/h_i$, for $i = 1,2$. In the figure, the short-horizon clientele 1 prefers point B to point C (i.e., prefers security 1 to security 2). It also prefers point B to point A, which corresponds to a zero-spread asset. Conversely, clientele 2 is best off at point C, and therefore will hold security 2. Group 2 cares less about liquidity because it has a longer holding period, and so finds the extra return associated with a high spread s_2 sufficiently attractive to prefer C over B, while group 1 cares more about liquidity and therefore prefers B over C.

Extending the logic to three or more securities ranked by liquidity, we would find that a “preferred habitat” equilibrium requires gross returns to form a (weakly) concave locus such as that drawn in figure 9.3. Clienteles with different holding periods congregate at different points on the curve. Therefore, the gross return increases with the spread s_h, but at a decreasing rate.

9.2.3 Evidence

The cross-sectional prediction that one can draw from equation (9.9) is that illiquid securities must provide investors with a higher expected return to compensate them for their higher transaction costs, controlling for the determinants **(p.315)**

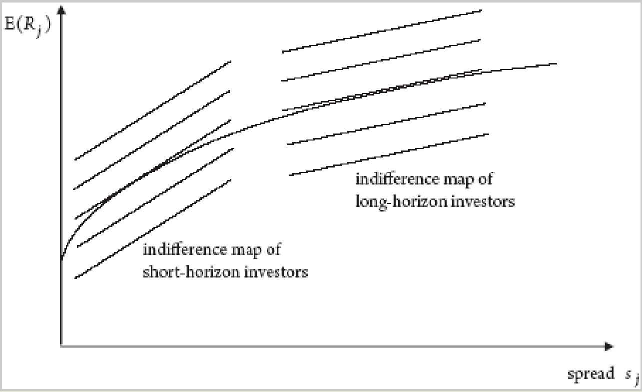


Figure 9.3. Expected return and spread with many clienteles

of the required net rate of return: the security's market risk, as measured by its beta. Observe that the realized gross return R is the expected return $E(R_j)$ plus an expectation error. If investors have rational expectations, this error will have zero mean and be independent of any other observed variable. Thus, according to equation (9.9), the observed gross return for security j should satisfy

(9.11)

$$R_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 s_i + \varepsilon_i,$$

where γ_0 is an estimate of the risk-free return, γ_1 is an estimate of the risk premium on the market portfolio and γ_2 is an inverse measure of the average holding period.

Equation (9.11) can be tested and estimated by running a cross-sectional regression of stock returns on betas and relative bid-ask spreads. Such an estimation was first applied to monthly returns by Amihud and Mendelson (1986) using NYSE and AMEX stock data for 1961–80.[4] In their basic specification, which also includes year dummies, they obtain the following estimates for equation (9.11):

$$R_i = 0.0036 + \underset{(6.18)}{0.00672}\,\beta_i + \underset{(6.83)}{0.211} s_i,$$

(t-statistics in parenthesis). These estimates imply that a 1-percentage-point increase in the bid-ask spread for a stock is associated with a 0.211 percent increase in its monthly expected return-that is, more than 2.5 percent per year. **(p.316)** This coefficient also suggests that the marginal investor trades once every five months

$$\left(\text{holding period} = \frac{1}{0.211} \simeq 5\right)$$

If—as is argued in section 9.2.2—the relationship between expected return and spread is concave, it can be estimated as a piecewise linear regression. Amihud and Mendelson (1986) do this and find that the slope of the relationship does tend to decline as the spread increases. They also estimate other specifications, controlling for additional firm-specific variables that may affect required stock returns, such as firm size, tax treatment and market-to-book ratio; they find that the effect of the spread is robust to such changes in specification and that it accounts for a good portion of the so-called small firm effect (small firms offering higher returns than larger firms with comparable risk).

Amihud and Mendelson point out that the increases in required returns called for by larger spreads translate into substantial reductions in asset valuations:

> "Consider an asset which yields $1 per month and has a bid-ask spread of 3.2% (as in our high-spread portfolio group) and its proper opportunity cost of capital is 2% per month, yielding a value of $50. If the spread is reduced to 0.486% (as in our low-spread portfolio group), our estimates imply that the value of the asset would increase to $75.8. about a 50% increase, suggesting a strong incentive for the firm to invest in increasing the liquidity of the claims it issues." (p. 246).

This estimate makes it clear that corporate financial policies that enhance liquidity have a considerable payoff in terms of asset valuation. For instance, a company may reduce its cost of capital by listing on a more liquid stock exchange. Foerster and Karolyi (2000) provide evidence of a fourty-four basis points decline in intraday effective spreads for fifty-two Canadian companies listing in the United States, and Miller (1999) finds that the market reaction to the issuance of advanced depository receipts is highest for firms that list on more liquid and better-known markets such as the NYSE and Nasdaq. We will return to the impact of liquidity on corporate policies in Chapter 10.

Amihud and Mendelson's study has spawned a vast empirical literature. Many subsequent studies confirmed the significant positive cross-sectional association between various illiquidity measures and asset returns, controlling for risk (e.g., Brennan and Subrahmanyam 1996 and Amihud 2002). Brennan and Subrahmanyam use the coefficient of price impact regressions (see Chapter 5) to develop a measure of illiquidity. If λ_i is the price impact coefficient for stock i, then its illiquidity in a given month is measured by $\lambda_i q_i / P_i$, where q_i is the average trade size in that month for stock i and Pi is its average price. Using NYSE stock returns for 1984–1991, they also find a positive cross-sectional **(p.317)** relationship between monthly average stock returns and this measure of illiquidity, after controlling for other factors that are known to affect stock returns (e.g., the so-called Fama-French factors).

One problem with this approach is that illiquidity may simply be proxying for some risk factor that has not yet been identified. One way to cope with this problem is to compare returns for securities that have similar risks but different illiquidity. Bonds issued by highly rated sovereign issuers are ideal candidates, since they present no risk of default. They should therefore offer the same yields if they have the same residual maturity. Amihud and Mendelson (1991) were first to use this approach. As the beginning of section 9.2.1 explains, they show that the yield to maturity of U.S. Treasury notes with six months or less to maturity exceeds that on Treasury bills with similar maturity. This supports the theory since Treasury bills are more liquid. Their findings are also confirmed by subsequent studies (e.g., Kamara 1994; or Longstaff 2004).

Differential liquidity also provides a possible explanation for the "on/off-the-run" yield differential: "off-the-run" issues, like T-bonds and T-notes issued before the last auction for the same instruments, are typically less liquid than “on-the-run” issues for the same maturity. Thus, on-the-run bonds should offer lower yields. And this is indeed the case, as found empirically in various studies (Krishnamurthy, 2002, or Goldreich, Hanke, and Nath, 2005, among others).[5] Goldreich et al. show that the yield differential between on-the-run and off-the-run securities narrows as the date at which securities go off-the-run nears. This finding is interesting: it shows that investors are forward looking and care about future rather than current liquidity in pricing on-the-run securities (this is consistent with the fact the gross required return on a security depends both on current liquidity, s_t, and future liquidity, s_{t+h}).

Another way to assess the impact of liquidity on asset prices and the cost of capital is to consider changes in trading rules known to affect liquidity (see exercise 2). For instance, Muscarella and Piwowar (2001) study a sample of 134 stocks listed on the Paris Bourse that switched from call trading to continuous trading (86 stocks) or vice versa (48 stocks). Those switching from call to continuous trading improved their liquidity, while those going over to call trading suffered a deterioration. Accordingly, a switch from periodic call auctions to continuous trading should reduce the cost of capital and produce a jump in stock’s price between the date at which the switch is announced and the date at which it is implemented. A switch from continuous trading to a call **(p.318)** market should have the opposite effect. This is precisely what Muscarella and Piwowar (2001) find: stocks that switch to continuous trading register a price rise of about 5.5 percent while those switching to call trading undergo a decline of 4.9 percent.

9.2.4 Asymmetric Information, Illiquidity and Asset Returns

In the previous section, the bid-ask spread was taken to be exogenous. In reality, we have seen it depends on factors such as asymmetric information, order-processing costs, inventory holding costs, and search costs. Hence, ultimately, the determinants of illiquidity studied in this book should affect asset prices. And by the same token, they should affect asset returns, via the illiquidity premium component of the required rate of return (section 9.2.1). Here, we illustrate this point for the case in which illiquidity arises from asymmetric information and show that the illiquidity premium is increasing in the likelihood of informed trading. The next section considers the similar role of search costs.

To see this in the simplest possible way, consider the one-period version of the model presented in section 3.3.1 of Chapter 3, where informed and liquidity traders place orders with uninformed, risk-neutral dealers. There, the opening bid-ask spread is $S = \pi(v^H - v^L)$, and the midquote m is equal to the unconditional estimate μ of the fundamental value. Expressing the final value of the security as $v^H = \mu(1 + \sigma)$ when high and $v^L = \mu(1 - \sigma)$ when low, where σ measures the advantage of informed traders, the relative spread can be written as

$$s = \frac{S}{m} = \frac{\pi\left(v^H - v^L\right)}{\mu} = 2\pi\sigma.$$

As in Chapter 3, the relative spread is increasing in the likelihood π of informed trading and in the informational advantage σ. If uninformed investors hold the asset for one period only ($h = 1$), by equation (9.5) they will require a gross return of:

$$1+R = (1+r)\times\frac{1+s/2}{1-s/2} = (1+r)\times\frac{1+\pi\sigma}{1-\pi\sigma}.$$

For small values of π and σ, the gross return can be approximated by:

(9.12)

$$R \simeq r + 2\pi\sigma.$$

Hence, the illiquidity premium, $R - r$, should be greater for securities where informed traders are more numerous and have more valuable information.

This economic intuition would also hold in a more general model of asymmetric information with multiple securities. For instance, Easley and O'Hara **(p.319)** (2004) consider a model with multiple securities and risk-averse investors. All investors observe public signals about payoffs and a fraction of them also get private signals. In equilibrium, the expected return on a security is higher when the ratio of private to public signals is larger. In the presence of informed investors, the uninformed end up holding more securities with low payoffs and fewer securities with high payoffs than they would in a market without asymmetric information. Thus, they require a higher return on their holdings.[6] Easley and O'Hara (2004) note that holding several securities does not alleviate this problem, since uninformed investors always end up on the wrong side of the trade.[7]

Easley, Hvidkjaer, and O'Hara (2002) test the prediction that stocks more exposed to informed trading command a higher risk premium, using the probability of information-based trading (PIN) in a stock as a proxy for the level of informed trading. In Chapter 5, section 5.4, we explain how to estimate this probability using intraday data on buy and sell orders. More specifically, Easley et al. estimate the following regression in each month from 1984–1998 for a large sample of stocks listed on the NYSE:

(9.13)

$$R_{it,l} = \gamma_{0t} + \gamma_{1t}\widehat{\beta_i} + \gamma_{2t}PIN_{it-1} + \gamma_{3t}SIZE_{it-1} + \gamma_{4t}BM_{it-1} + \eta_{it},$$

where $R_{it,l}$ is the excess return of stock i in month l of year t; PIN_{it-1} is an estimate of the PIN measure for stock i using order data in year $t-1$; $SIZE_{it-1}$ is the logarithm of the market capitalization of stock i at the end of year $t-1$; BM_{it-1} is the logarithm of the book-to-market value of stock i at the end of year $t-1$; and $\widehat{\beta}_i$ is an estimate of the β for stock i over the entire estimation period. The authors control for market capitalization and book-to-market ratios, as these variables are known to be priced characteristics (i.e., to explain the cross-section of stock returns).

Over all the monthly regressions, the average effect of PIN is positive and significant, in various specifications of equation (9.13). Thus, stocks with a higher likelihood of informed trading have higher returns on average, as predicted by equation (9.12).

(p.320) Another possibility is that the PIN measure captures other characteristics that affect returns. In particular, high-PIN stocks may also be illiquid, but for reasons other than just asymmetric information. However, the effect of PIN on returns does not vanish when the bid-ask spread is included as a control variable in equation (9.13). In a more recent study, Duarte and Young (2009) estimate equation (9.13) over a longer sample period (1984–2004) using the Amihud ratio (defined in Chapter 2) to control for illiquidity. They find that once this variable is controlled for, the PIN measure loses its significance.

9.2.5 Illiquidity Premia in OTC Markets

As Chapter 8 explains, it takes time in OTC markets to locate the liquidity suppliers who post good prices, and this friction enables dealers to charge non-competitive spreads. Thus, illiquidity premia in OTC assets (such as swaps, CDSs, corporate bonds, etc.) should, at least in part, reflect search costs (e.g., opportunity costs due to delays in finding a counterparty) and dealers' bargaining power. This point has been highlighted by a recent strand of research on asset pricing in OTC markets, pioneered by Duffie, Gârleanu, and Pedersen (2005, 2007).

In this section, we briefly outline the approach of these authors. This analysis is interesting for at least two reasons. First, it explicitly relates illiquidity premia to search costs and dealers' bargaining power, thus paving the way to better understanding of required rates of return for securities traded over the counter. Moreover, it shows that the asset pricing model developed in section 9.2.1 may be grounded on the existence of search costs rather than asymmetric information, as we have just seen in section 9.2.4.

Consider a continuum of risk-neutral investors who can invest in two different assets: a riskless security paying one dollar per period forever (a "consol bond") and a bank account offering a rate of interest r. Investors can either hold one share of the security or none, and short-sales are not allowed. Hence, all trades are for a single share. Some investors have a holding cost of c per period: when they hold the asset, their cash flow is only $\$(1-c)$ per period. We call them "low-valuation" investors; those who bear no holding cost are "high-valuation" investors. The holding cost of low-valuation investors may reflect, for instance, a less favorable tax rate than that faced by high-valuation investors.

Thus, in each period t, there are four groups of investors: (i) high-valuation investors who own the security, (ii) high-valuation investors who do not own the security, (iii) low-valuation investors who own the security, and (iv) low-valuation investors who do not own the security. The per-capita supply of the consol bond is denoted by q.

(p.321) In each period, after receiving their payoff, investors switch valuation with probability ψ: this is the frequency with which high-valuation investors turn into low-valuation investors or vice-versa. Upon making their new valuation, investors can trade the security. High-valuation non-owners want to buy and low-valuation owners want to sell. They can do so by a trading with dealers at bid and ask prices b and a. Finding a dealer takes time. Specifically, in each period, the probability of an investor who needs to trade finding a market maker is φ, so that on average he will have to search for $1/\varphi$ periods before finding a market maker. This delay generates a search cost for low-valuation investors: since in each period they bear the holding cost c, their expected search cost is c/φ.[8]

To sum up, *in each period,* the time line of the model is as follows:

(i) Investors holding the asset receive their payoff, \$1 or \$$(1-c)$.
(ii) Each investor changes his valuation for the security with probability ψ.
(iii) High-valuation non-owners and low-valuation owners search for a dealer and find one with probability φ.

The fractions of investors' types and the prices quoted by dealers in each period are endogenous (the appendix of this chapter explains how they are determined). When $q\langle\frac{1}{2}$, the price paid by investors for the security turns out to be (see the appendix)

(9.14)

$$a=\frac{1}{r}-\underbrace{\frac{2\psi}{r(1+z)}\left(1-\phi\frac{1-z}{2}\right)s}_{\text{illiquidity premium}},$$

where S is dealers' bid-ask spread and z is an index of dealers' market power: when $z = 1$, this is maximal and investors obtain no surplus from trading. To grasp the intuition for the ask price in expression (9.14), consider first the case in which investors do not expect their valuation to change over time, that is, $\psi = 0$. Then buyers are willing to pay a price $\frac{1}{r}$ for the asset: since high-valuation non-owners expect to earn \$1 forever once they buy, they are willing to pay at most the discounted value of a payment of \$1 per period forever (their holding period h being infinite, in terms of the notation of section 9.2.1). In equilibrium, they pay exactly this price when per-capita supply $q\langle\frac{1}{2}$, because under our assumptions the fraction of high-valuation investors is always $\frac{1}{2}$ (as **(p.322)** shown in the appendix). Hence, when $q\langle\frac{1}{2}$ and $\psi = 0$, there would be excess demand for the asset at any ask price below $\frac{1}{r}$.

This argument explains why when $\psi = 0$, the second, negative term in expression (9.14) vanishes and the ask price simplifies to $\frac{1}{r}$. By contrast, if $\psi\rangle 0$, the second term does not vanish: the value of the security is less than $\frac{1}{r}$, because buyers anticipate that they will seek to resell it as soon as their holding cost becomes positive. As their holding horizon is finite, they value the security less than $\frac{1}{r}$, by a discount directly proportional to the bid-ask spread S charged by dealers, precisely as in the model analyzed in section 9.2.1.

This discount—the second term in expression (9.14)—can be viewed as the illiquidity premium for the asset. The premium is large when dealers have strong bargaining power z, so that they can extract more surplus from investors when they resell. This lowers the security's resale value and hence the price that buyers are initially willing to pay. Instead, the illiquidity premium is small if there is a high probability φ of meeting with a dealer so that an investor who wants to sell the security because his valuation has been revised downward can do so more readily. Thus, he sustains his holding cost for a shorter time, his expected search cost c/φ is small, and he is willing to pay more for the asset initially. This shows how search costs in OTC markets can impact on illiquidity premia.

These parameters also affect the bid-ask spread, whose expression can be shown to be:

(9.15)

$$S = a - b = \frac{(1+z)\,c}{2\,(r+2\psi) + \;(1-2\psi)\,\phi\,(1-z)}.$$

Thus the model predicts that in OTC markets, the spread is increasing in the holding cost c and in dealers' bargaining power z: when their holding costs are high, sellers' valuation of the asset is smaller and if dealers have substantial bargaining power, sellers will be forced to accept a low price. Hence, on both accounts the bid price quoted by dealers will be low. The effect on the spread of the probability φ of meeting a dealer is ambiguous, however: finding the rationale for this result is left as an exercise (see exercise 6).

9.3. Liquidity Risk and Asset Prices

In the previous sections, we have seen why the level of future expected liquidity should affect required rates of return. However, intuitively, stock returns should depend not only on future liquidity but also on the uncertainty over its level. Indeed, as section 9.2.1 points out, the liquidity of any security fluctuates over time, contributing to the volatility of returns. This source of risk is called **(p.323)** liquidity risk. Empirical studies, such as Hasbrouck and Seppi (2001); Chordia, Roll, and Subrahmanyam (2000); and Huberman and Halka (2001), show that there is "commonality" in liquidity risk. That is, liquidity measures for various securities (e.g., bid-ask spreads or measures of price impact) are usually positively correlated. This co-movement in liquidity implies that the liquidity risk on a security cannot be easily diversified away and so contributes to its systematic risk. In this situation, the logic of asset pricing models (such as the CAPM) implies that investors will require compensation for liquidity risk, thereby creating a second link between the price and the liquidity of financial assets.

Acharya and Pedersen (2005) present an extension of the CAPM that accounts for the effect of liquidity risk. The essence of their model is given by normalizing the holding period to $h = 1$ in equation (9.6) for asset j:

(9.16)

$$R_j \simeq r_j + s_j,$$

and assuming that both the net return r_j and the spread s_j are random. For the sake of exposition, we assume that this approximation holds exactly, so that the net return on asset j is $r_j = R_j - s_j$. Since the investor's concern is the net return, the CAPM relationship $E(r_j) = r + \beta_j [E(r_M) - r]$ still applies, so that:

(9.17)

$$E\left(R_j - s_j\right) = r + \beta_j \left[E\left(R_M - s_M\right) - r\right],$$

where R_M is the gross return on the market and s_M is a measure of market illiquidity, weighting the illiquidity of each stock by its fraction of the value of the market portfolio. Recalling the definition of β_j, we can re-express this as:

$$\begin{aligned}\beta_j &\equiv \frac{\operatorname{cov}(r_j, r_M)}{\operatorname{var}(r_M)} = \frac{\operatorname{cov}(R_j - s_j, R_M - s_M)}{\operatorname{var}(r_m)} \\ &= \underbrace{\frac{\operatorname{cov}(R_j, R_M)}{\operatorname{var}(r_M)}}_{\beta_{1j}} + \underbrace{\frac{\operatorname{cov}(s_j, s_M)}{\operatorname{var}(r_M)}}_{\beta_{2j}} - \underbrace{\frac{\operatorname{cov}(R_j, s_M)}{\operatorname{var}(r_M)}}_{\beta_{3j}} - \underbrace{\frac{\operatorname{cov}(s_j, R_M)}{\operatorname{var}(r_M)}}_{\beta_{4j}}.\end{aligned}$$

This expression implies that the required risk premium on stock j can be expressed as

(9.18)

$$E\left(R_j\right) - r = \beta_{1j}\lambda_M + \underbrace{E\left(s_j\right) + \beta_{2j}\lambda_M - \beta_{3j}\lambda_M - \beta_{4j}\lambda_M}_{\text{illiquidity premium}},$$

where we use the shorthand $\lambda_M \equiv E(R_M - s_M) - r$ for the risk premium on the market portfolio. This decomposition yields a "liquidity-adjusted CAPM," which is relevant for any asset pricing test of the CAPM that relies on gross returns—that is, for tests relying on returns measured as percentage changes **(p.324)** of midquote prices, rather than net returns (percentage changes of transaction prices).

Equation (9.18) shows that the illiquidity premium (calculated using gross returns) should depend both on the expected level of trading costs at the end of the holding period, $E(s_j)$, and on various sources of illiquidity risk, which are measured by the three terms β_{2j}, β_{3j}, and β_{4j}. Each coefficient β captures one source of illiquidity risk.

(i) Parameter β_{2j} measures the co-movement between the illiquidity of stock j and market-wide illiquidity (s_m). Holding stocks that remain liquid when others go illiquid is a way to hedge a drop in asset values due to market-wide illiquidity. Thus, other things being equal, these stocks are attractive and should have a higher price (lower risk premium). Conversely, stocks with a high β_2 must have greater returns on average, because investors require compensation for holding a security that becomes illiquid when the overall market does.

(ii) Parameter β_{3j} measures the co-movement between the gross return on stock j and market-wide illiquidity (s_M). A high value of β_{3j} means that the stock does

well when market-wide illiquidity increases. Thus, stocks with high β_{3j} offer a hedge against a drop in market-wide liquidity, so investors require a smaller risk premium to hold these stocks.
(iii) Parameter β_{4j} measures the co-movement between the illiquidity of stock j and the gross return on the market portfolio. Stocks with a high β_4 require a lower expected return, because they tend to remain liquid when the market is down: investors value such stocks more highly, as they allow them to sell at low transaction costs in an adverse market phase.

Acharya and Pedersen (2005) estimate equation (9.18) on CRSP data for all common stocks listed on NYSE and AMEX from 1962–1999. They measure illiquidity by the Amihud ratio, i.e., the ratio between the absolute value of daily returns and the daily volume. They obtain three main results. First, illiquid stocks (those with high $E(s_j)$) tend to be more exposed to illiquidity risks; they also have higher β_{2j} and more negative β_{3j} and β_{4j}. Second, the dispersion of stock returns is explained better by the liquidity-adjusted CAPM (equation (9.18)) than by the traditional CAPM. Last, liquidity risk is priced. Overall, the various sources of liquidity risk contribute to a 1.1 percent difference in average annual returns between a portfolio of illiquid stocks and a portfolio of liquid stocks. This yield spread is largely due to the third source of illiquidity risk (captured by β_4). Thus, investors seem to value securities that remain liquid when the market is down especially highly.

(p.325) In this approach, the level of liquidity at a given point in time and the sources of liquidity risk are exogenous (i.e., the covariation between market illiquidity and security j illiquidity is exogenous). Thus, it does not explain why there is covariation in illiquidity across stocks or between illiquidity and stock returns. Section 9.4 describes how stochastic changes in illiquidity may occur and how commonality in liquidity can therefore emerge, especially in the context of financial crises.

9.4. Liquidity and Limits to Arbitrage

The absence of arbitrage is a central tenet of asset pricing: two assets with identical cash flows should command the same price. Yet there are well-known cases in which this principle appears to be violated. For instance, take the opening example in section 9.2.1: in the United States, Treasury notes typically trade at a discount relative to otherwise identical bills. We argued that this difference in gross rates of return can be accounted for by the greater liquidity of bills. But why don't arbitrageurs eliminate this difference by buying notes and selling bills? This looks like a textbook example of a profitable arbitrage opportunity—the kind that according to asset pricing theory cannot last.

A reason for its persistence is that arbitrageurs may not have enough wealth to exploit the opportunity. Of course they could borrow so as to carry out the arbitrage. But unless they can secure long-term funding, borrowing exposes them to the risk of having to liquidate their position before the prices of notes and bills eventually converge at maturity. This risk is particularly great if the mispricing that they try to correct widens: as a result, they may have to meet larger margin requirements or may face a decrease in funding due to a loss of confidence in their strategy. So, paradoxically, they may be forced to liquidate their position just when its future profitability is largest.

To understand this point, it is important to notice that in practice arbitrage requires capital. In our simple example, it looks as if no capital is required to build the arbitrage portfolio: if the notes are worth 95 percent of the bills with the same maturity, the arbitrageur can use the proceeds from selling the bills short (say, $100,000) to buy the notes ($95,000), and immediately cash in the difference ($5,000), which is an instantaneous arbitrage profit (a “free lunch”), provided he can hold the position to maturity. In reality, though, this kind of standard arbitrage portfolio requires capital. In the United States, to sell bills short an arbitrageur must borrow them from a broker, who will require a margin of 150 percent of the value of the short sale as collateral (in our example, $150,000); buying the notes will cost another $95,000. Therefore, the total capital, net of the proceeds of the short sale, committed to **(p.326)** the arbitrage portfolio is $145,000, a huge multiple of the arbitrage profits ($5,000).

Limited wealth generally forces arbitrageurs to raise external funds. If these funds come in the form of short-maturity debt or can be withdrawn at will, arbitrageurs risk being unable to refinance their positions before their arbitrage portfolio pays off. For instance, in our example, the lender of the bills may recall them at any point in time, forcing the arbitrageur to either borrow from another broker or to close out his position, possibly at a loss. Arbitrage may also be disrupted if, before it pays off, the arbitrageur must roll over the loan taken to purchase the notes and cannot do so.

One way out of these problems is to obtain long-term financing, as some arbitrageurs do by imposing lockup covenants, the condition that investors cannot withdraw their funds for some minimum period. For instance, hedge funds typically raise money in the form of shares that can be redeemed at their market value only after a lock-up period. However, if the claims are short-term, financiers can better control the use of their funds by arbitrageurs: if the latter do not perform well, external investors can simply pull the plug. This threat helps to discipline arbitrageurs and prevent excessive risk taking. Alternatively, short-term financing could be an optimal response to uncertainty: it allows the financier to progressively increase his stake as he becomes more confident about the real ability of arbitrageurs. As noted above, it exposes arbitrageurs to the danger of early liquidation. The demise of LTCM in 1998 (see box 9.1) offers a vivid illustration.

This reasoning has been used to explain the empirical evidence of persistent arbitrage opportunities. For instance, Rosenthal and Young (1990); Froot and Dabora (1999); and de Jong, Rosenthal, and Van Dijk (2009) show that the prices of dually listed stocks (so-called twin stocks) often differ even though they are claims on the same cash flows.[9] de Jong, Rosenthal, and Van Dijk (2009) show that arbitrageurs who try to exploit these price differences must hold their positions for long periods of time with all the attendant risk. They write: “Since there is no identifiable date at which dual listed companies prices will converge, arbitrageurs with limited horizons who are unable to close the price gap on their own face considerable uncertainty. In some cases, arbitrageurs would have to wait for almost nine years before prices have converged and the position is closed. In the short run, the mispricing might deepen. In these situations, arbitrageurs receive margin calls, after which they would most likely **(p.327)** be forced to liquidate part of the position at a highly unfavorable moment and suffer a loss” (p. 497).

Box 9.1 The Ltcmcrisis

Long-Term Capital Management (LTCM) was a hedge fund founded in 1994 with equity of $1.3 billion. Investors had to put up a minimum of $1 million and could not withdraw it for three years. The first four years of its life were extremely profitable. Its main investment was of “market-neutral arbitrage,” i.e., arbitrage positions that involve no systematic risk, mainly buying high-yield illiquid bonds (e.g., emerging market bonds and non-investment-grade corporate bonds) and shorting low-yield liquid bonds (e.g., U.S. Treasuries). Many these arbitrage positions were taken indirectly via derivative contracts such as rate swaps. Thus LTCM was betting that the yield spread between low- and high-risk bonds would narrow. Even by the standards of hedge funds, LTCM’s leverage was enormous (over $20 of borrowing per $1 of equity), so that even a slight narrowing of yield spreads would translate into huge profits.

In the spring of 1998, however, spreads widened in the wake of the Southeast Asian financial collapse and Russia’s troubles. The situation deteriorated further in August, when Russia devalued the ruble and declared a moratorium on public debt payments. By September, the yield spread on the JP Morgan index of emerging market bonds had increased to over seventeen percentage points, five times more than in October 1997. Yield spreads on U.S. non investment-grade bonds also rose fivefold over the period. As a result, LTCM started making huge losses, cutting its equity value to $600 million by September (from $4.8 billion at the start of 1998). At this point, both LTCM and other funds with similar strategies came under pressure to meet margin calls and post additional collateral with creditors and swap counterparties, resulting in pressure to liquidate their positions at a further loss.

However, infear that ageneralized flight to quality might trigger a downward price spiral and a cascade of bankruptcies of financial institutions, the Federal Reserve persuaded a consortium of LTCM’s major creditors to inject over $3.6 billion into the fund in return for a 90 percent equity stake. After the rescue, LTCM was run by these creditors and, helped by the recovery of financial markets, it gained 13 percent by December 1998. In the following months its portfolio was unwound and LTCM was liquidated.

The sequence of events in the LTCM crisis offers a good illustration of the forces at work in the model of section 9.4.1: LTCM was arbitraging away yield differences that it considered excessive, but in doing so it was exposed to the risk of early liquidation by its creditors. Its positions were fundamentally sound, in that its portfolio bounced back once its creditors were persuaded to extend long-term financing, but the differences were not eliminated as quickly as LTCM had bet they would be. Other arbitrageurs did not intervene on a sufficient scale, because the Russian and Asian crises had raised volatility and tightened the funding available for arbitrage activity.

(p.328) 9.4.1 Risk of Early Liquidation as a Limit to Arbitrage

To illustrate these points, consider the following simple model, which is inspired by Shleifer and Vishny (1997). There are three dates (0, 1 and 2) and two zero-coupon bonds (A and B) with the same certain payoff V at date 2. The risk-free interest rate is set to zero, so that absent arbitrage opportunities, the price of the two bonds should equal V at both dates 0 and 1.

Suppose that an arbitrage opportunity arises at date 0: the price of bond A falls below that of bond B by an amount $M_0 > 0$ ($P_{0A} = P_{0B} - M_0$), while bond B is priced correctly ($P_{0B} = V$). That is, bond A is undervalued, and the size of this mispricing at date 0 is M_0. At date 1, the mispricing increases to $M_1 > M_0$ with probability κ or disappears with probability $1 - \kappa$. In the first case, the price of bond A is $P_{1A} = P_{1B} - M_1$; in the second case it is simply $P_{1A} = P_{1B} = V$. In the next section, we explain why such mispricing may arise and persist. At date 2, the assets pay off so that any mispricing is eliminated: $P_{2A} = P_{2B} = V$ with certainty.

As the prices of bonds A and B will converge at date $t = 2$, the arbitrage portfolio is a short position in asset B and an offsetting long position in asset A.[10] We assume that arbitrageurs must choose whether to build their arbitrage portfolio at date 0 or at date 1, denoting the decision to intervene at date t by the indicator function $I_t \in \{0,1\}$: if $I_t = 1$, the arbitrageur sells one share of B and buys one share of A at date t. If $I_0 = 1$, the portfolio is built at date 1; if $I_1 = 1$, at date 1. Otherwise the arbitrageur is inactive at both dates ($I_0 = I_1 = 0$). There are two restrictions: the arbitrageur cannot be active in both periods, that is, he cannot choose $I_0 = I_1 = 1$, and cannot take a position of more than one **(p.329)**

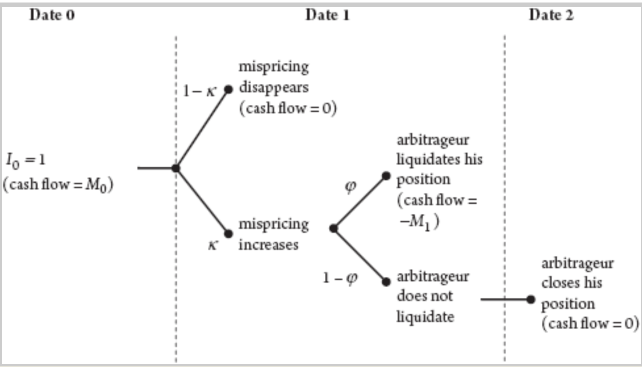


Figure 9.4. Actions and cash flows if the arbitrageur intervenes at date 0

unit in the two bonds. These are intended to capture the funding constraints discussed above.

Suppose that the arbitrageur chooses to intervene at date 0. In this case, figure 9.4 shows the cash flows of his portfolio at each date and in each contingency. The arbitrageur pockets the mispricing M_0 at date 0. With probability $1 - \kappa$ the mispricing disappears at date 1, so he closes his position. But with probability κ the mispricing increases, so that the marked-to-market value of the portfolio declines from 0 to $M_0 - M_1 < 0$.

When the mispricing persists at date 1, outside investors may choose to cut their funding of the arbitrageur's activities because they cannot tell whether the increase in mispricing is transient or reflects a true loss in value of the asset (in which case the arbitrageur was wrong to bet on a price recovery). Then the arbitrageur must close his position at a loss: so the model captures what Shleifer and Vishny (1997) call "performance-based arbitrage." We assume that such forced liquidation occurs with probability ϕ. When forced to liquidate, the arbitrageur has a negative cash flow of $-M_1$, because he resells security A at price $V - M_1$ and covers his short position by buying security B at price V. If he is not forced to liquidate at date 1, he has zero cash flow at $t = 1$ and $t = 2$, as at this date the net payoff of his arbitrage portfolio is zero. Hence, intervention at date 0 yields an expected profit equal to $\Pi_0(\phi) = M_0 - \kappa\phi M_1$.

If instead the arbitrageur has not intervened at date 0, he will intervene at date 1 if the mispricing persists. The cash flows of his arbitrage portfolio in this case are illustrated in figure 9.5. By definition he has zero cash flows at date 0. At date 1, he has the opportunity to set up an arbitrage portfolio only if the mispricing of bond A increases: hence he gets a cash flow M_1 with probability **(p.330)**

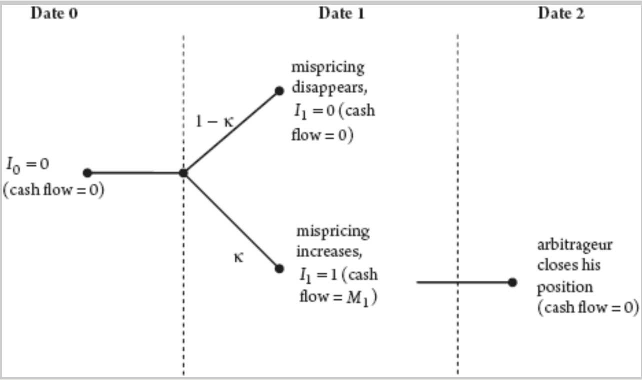


Figure 9.5. Actions and cash flows if arbitrageur intervenes at date 1

κ. If instead the mispricing disappears, the arbitrageur can no longer intervene profitably, and will get no cash flow. Thus, the arbitrageur's expected profit is $\Pi_1 = \kappa M_1$.[11]

The choice hinges on the following trade-off. If the arbitrageur chooses to intervene at date 1, he expects a gain M_1 if the mispricing persists (which happens with probability κ), and avoids the risk of early liquidation. But he forgoes the gain M_0 from exploiting the mispricing at date 0. Therefore, if $\kappa M_1 \geq M_0$, all arbitrageurs prefer to defer their intervention, as on average the mispricing will be greater at date 1. Hence, from now on, we focus on the more interesting case in which $\kappa M_1 \langle M_0$, that is, mispricing is expected to decrease. In this case, the optimal strategy depends on the sensitivity of funding to interim performance, which is to say, on the probability ϕ of early liquidation. Waiting is optimal if and only if $\Pi_1 \rangle \Pi_0$, that is, if

$$\kappa M_1 \rangle M_0 - \kappa\varphi M_1.$$

Hence, waiting is preferable if the risk of forced liquidation is high enough:

(9.19)

$$\varphi \Big\rangle \widehat{\varphi} = \frac{M_0 - \kappa M_1}{\kappa M_1}.$$

Even when mispricing is expected to decline ($\kappa M_1 \langle M_0$), the arbitrageur will prefer to postpone intervention to date 1 if the sensitivity ϕ of the risk of early liquidation is great enough. If this risk is low instead $\left(0 \Big\langle \varphi \Big\langle \widehat{\varphi}\right)$, he intervenes **(p.331)**

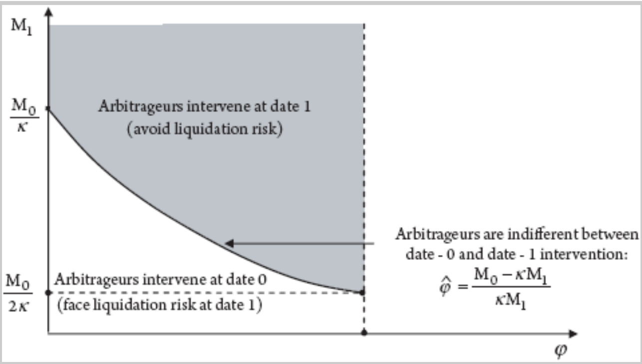


Figure 9.6. Allocation of arbitrage capital between dates 0 and 1

at date 0. But liquidation risk is not the only relevant factor: the choice will also depend on the profits M_1 from delayed intervention. Even if the liquidation risk ϕ is low, investors may prefer to postpone intervention if the profits M_1 from waiting are large enough. Hence, the liquidation risk $\widehat{\varphi}$, which by equation (9.19) makes arbitrageurs perfectly indifferent between intervening at date 0 or at date 1, is decreasing in the future mispricing M_1, as shown in figure 9.6. In the shaded region above the indifference curve $\widehat{\varphi}$, liquidation risk and/or future mispricing are large, so that arbitrageurs prefer to defer intervention. Below it, liquidation risk and/or future mispricing are low enough that they intervene immediately.

9.4.2 Limited Speculative Capital as a Barrier to Arbitrage

So far, we have considered mispricing (M_0 and M_1) at either date as exogenous. The literature on limits to arbitrage often assumes that mispricing arises from supply shocks that drive prices away from fundamental values, which may occur for various reasons. For instance, some investors ("noise traders") may become irrationally over-pessimistic or over-optimistic (investors' sentiment) about the prospects of a security.[12]

Alternatively, some investors (e.g., mutual funds or insurance companies) may suddenly be forced to liquidate positions in financial assets to deal with an unexpected funding need (e.g., to pay back creditors) or **(p.332)** a prudential regulation requiring them to scale back their positions in certain asset classes. Such forced asset liquidations are often called "fire sales."[13]

To capture this in the simplest possible way, suppose that the mispricing M_1 that occurs in the market for bond A at date 1 is due to a shock to noise traders' supply—"a fire sale"—that occurs with probability κ and pushes the bond's price further below its fundamental value. Otherwise, the price of bond A reverts to its fundamental value V. When this shock occurs, noise traders' aggregate supply of asset A at date 1, $y(P_{A1})$, is assumed to be:

(9.20)

$$y\left(P_{A1}\right) = 1 + \delta\left(P_{A1} - V\right) = 1 - \delta M_1,$$

where $\delta > 0$. That is, with probability κ, noise traders sell the bond at any price above V-$\frac{1}{\delta}$: they are overly pessimistic, as they value the bond at V-$\frac{1}{\delta}$, even though it will pay V for sure at date 2. Their sell orders exert a downward price pressure as long as the price is higher than V-$\frac{1}{\delta}$, that is, provided its undervaluation M_1 does not exceed $\frac{1}{\delta}$.

Hence, noise traders' orders are a source of mispricing, which can be corrected only if arbitrageurs lean strongly enough against them. However, the arbitrageurs' ability to do this is limited, since they must allocate capital between different strategies (in our model, interventions at date 0 and at date 1). Suppose that mispricing at date 0 is very pronounced. In this case, arbitrage capital will be massively invested to harness the date 0 mispricing, leaving little available to bet against mispricing at date 1. Mispricing at date 1 will tend to persist and increase in the mispricing at date 0.

The exact relationship between the mispricing at dates 1 and 2 depends on the distribution, among arbitrageurs, of the exposure to early liquidation risk. Suppose that there is a continuum of arbitrageurs of mass 1 differing in their probability $\phi(i)$ of liquidation in case of poor performance at date 1. We assume that $\phi(i) = i$, where i is uniformly distributed over [0,1]. Recall that the arbitrageurs for whom $\widehat{\varphi}$ are perfectly indifferent between intervening at date 0 and date 1. Hence, those with a smaller likelihood of liquidation at date 1 will choose to intervene at date 0. As the distribution of arbitrageurs is uniform, $\widehat{\varphi}$ and $1-\widehat{\varphi}$ are the fractions of arbitrageurs who intervene at date 0 and date 1, respectively.

The fraction $\widehat{\varphi}$ of arbitrageurs who intervene at date 0 is determined by M_1 through the indifference condition (9.19) plotted in figure 9.6. This condition implies that the greater the mispricing at date 1, the smaller the fraction of arbitrageurs intervening at date 0. That is, the effect of M_1 on $\widehat{\varphi}$ is negative.

(p.333) But the fraction of arbitrageurs, $\widehat{\varphi}$, intervening at date 0 in turn affects the degree of mispricing M_1 through the market-clearing condition at date 1: intuitively, a larger fraction $\widehat{\varphi}$ of arbitrage capital deployed at date 0 implies that less will be available at date 1 to counter the negative pressure of noise traders, resulting in a greater mispricing at date 1. This induces a positive effect of $\widehat{\varphi}$ on M_1.

To see this, consider the determination of the equilibrium price of bond A at date $t = 1$ when a supply shock occurs. This price must be such that the supply is equal to the demand. There are two types of sellers: (i) noise traders who collectively sell an amount $y(P_{A1})$, which is positive for $\mathrm{M}_1 \left\langle \frac{1}{\delta} \right.$, and (ii) arbitrageurs who are forced to liquidate their position prematurely. As each arbitrageur i who intervened at date 0 must liquidate his position with probability $\phi(i)$, the aggregate supply from arbitrageurs who liquidate at date $t = 1$ is $\int_0^{\widehat{\varphi}} \varphi(i)\, di$. Sell orders from these two categories of investors must be absorbed by those arbitrageurs who still have capital to invest at date 1 because they did not intervene at date 0. Hence, the equilibrium price of asset A at date $t = 1$ is given by the condition:

(9.21)

$$\underbrace{y(P_{A1})}_{\text{noise traders' sell orders}} + \underbrace{\int_0^{\widehat{\varphi}} \varphi(i)\, di}_{\text{arbitrageurs' sell orders}} = \underbrace{1-\widehat{\varphi}}_{\text{arbitrageurs' buy orders}} .$$

Recalling that $\phi(i) = i$ and replacing $x\,(P_{A1})$ with its expression in (9.20), this clearing condition can be rewritten as

$$1-\delta V+\delta P_{A1}+\frac{1}{2}\widehat{\varphi}^2 = 1-\widehat{\varphi},$$

which yields the equilibrium price of bond A at date $t = 1$
(9.22)

$$P_{A1} = V-\frac{1}{\delta}\left(\frac{1}{2}\widehat{\varphi}^2+\widehat{\varphi}\right).$$

Hence, as announced earlier, the undervaluation of bond A at date $t = 1$ when a supply shock occurs is increasing in $\widehat{\varphi}$.

To sum up, the expected level of mispricing at date $t = 1$ determines the fraction of arbitrageurs who intervene at date $t = 0$ since, according to the indifference condition:

(9.23)

$$\widehat{\varphi} = \frac{M_0-\kappa M_1}{\kappa M_1}.$$

(p.334)

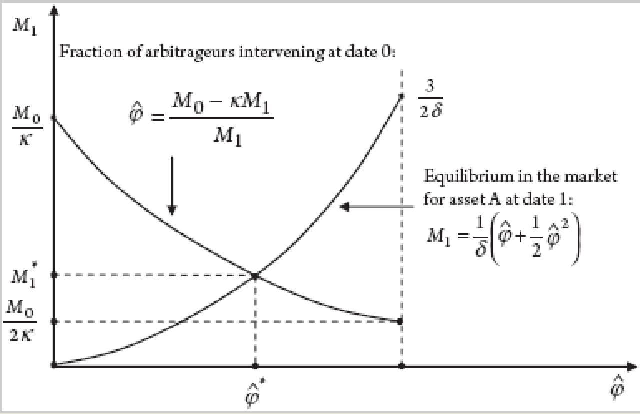


Figure 9.7. Mispricing and allocation of arbitrage capital between dates 0 and 1

The fraction who intervenes at date 1 determines the mispricing at this date, since according to equation (9.22):

(9.24)

$$M_1 = \frac{1}{\delta}\left(\frac{1}{2}\widehat{\varphi}^2 + \widehat{\varphi}\right).$$

Thus, an equilibrium is given by the fraction $\widehat{\varphi}^*$ of arbitrageurs intervening at date 0 and the date-1 mispricing M_1^*, that jointly solves equations (9.23) and (9.24).[14] Figure 9.7 illustrates the determination of this equilibrium. The downward-sloping curve represents the indifference equation (9.23): as the expected mispricing κM_1 decreases, intervening at date $t = 0$ becomes relatively more attractive, so that the fraction of arbitrage capital $\widehat{\varphi}$ attracted to speculation at date 0 increases. The upward-sloping curve coincides with the indifference condition already plotted in figure 9.6, and shows the equilibrium mispricing at date 1 (equation (9.24)) given the fraction $\widehat{\varphi}$ of arbitrage capital deployed at date 0: as this fraction increases, the arbitrage capital still available to correct mispricing at date 1 decreases, so that M_1 rises.

Figure 9.7 helps understanding the relationship between mispricing at date 0 and at date 1. If there is no mispricing at date 0 so that $M_0 = 0$, the downward-sloping curve (which gives the fraction of arbitrageurs intervening at date 0 as a function of mispricing at date 1) flattens on the horizontal axis and the equilibrium point shifts to the origin, where $\widehat{\varphi}^*$ and M_1^*. Intuitively, as there is no mispricing at date 0, all arbitrageurs intervene at date 1 $\left(1 - \widehat{\varphi}^* = 1\right)$, so no mispricing appears at this date $\left(M_1^* = 0\right)$, because enough arbitrage capital is available to counter the negative effect of noise traders' pessimism on prices.

(p.335) As M_0 increases, the downward-sloping curve shifts upward and steepens, so that both M_1^* and $\widehat{\varphi}^*$ increase: intuitively, this is because as mispricing at date 0 increases, it attracts scarce arbitrage capital to early intervention, causing mispricing at date 1 to increase. Hence, when arbitrage capital is limited, mispricing persists and is self-reinforcing. Interestingly, it has been observed that in derivatives markets arbitrage opportunities can be relatively persistent, taking a long time to disappear (see, for instance, Mitchell, Pedersen, and Pulvino, 2007; or Gabaix, Krishnamurthy, and Vigneron, 2007). This generally is ascribed to the slow entry of arbitrageurs, but it could also be due to strategic allocation of arbitrage capital over time (or more generally over various assets), as in the model developed here.

Figure 9.7 can also be used to clarify the role of noise traders in generating this persistence. As the probability κ of their presence increases, the downward-sloping curve shifts downward, so that $\widehat{\varphi}^*$ decreases: intuitively, more noise trading at date 1 attracts more arbitrage capital to date-1 speculation, and so reduces that available at date 0. Thus the magnitude of the mispricing M_1^* also decreases, but its expected value κM_1^* increases (the reader is invited to prove this result), so that the expected profits from delayed intervention actually increase. In the limiting opposite case of noise trading vanishing ($\delta \to 0$), the downward-sloping curve shifts upward without bounds, so that the only equilibrium is such that $\widehat{\varphi}^* = 1$: all the arbitrage capital is concentrated at date 0, since at date 1 it has no use.

In this section we have taken date-0 mispricing M_0 as given. But this could be generated by exactly the same mechanism illustrated so far for date-1 mispricing M_1: also at date 0 the arrival of pessimistic investors can push the price below fundamental value. The exact level of date-0 mispricing will then depend on how much arbitrage capital is allocated to intervention at date 0 (we leave this analysis as exercise 5). In general, mispricing at date 0 will not disappear because in our model the arbitrage capital available at this date is limited: the prospect that the mispricing of the bond may be greater at date 1 induces some arbitrageurs to wait rather than exploit the opportunity at date 0.

In reality, arbitraging activity may be curtailed not only by the risk of forced liquidation, but also by two other frictions. First, the cash flows generated by arbitrage portfolios are uncertain when there is a risk of early liquidation, as in our model here. If arbitrageurs are specialized (say, because their activity requires expertise), they will not be able to diversify this risk and will accordingly limit their positions. Second, as in the example at the beginning of section 9.4, arbitrageurs are required to put up cash as collateral on their short positions, funds that usually earn less than the riskless interest rate (zero in our model), which lowers the overall return. When this cost is large relative to the benefit of **(p.336)** an arbitrage opportunity, arbitrageurs will simply refrain from taking advantage, and the opportunity will persist.

The extent to which all these frictions limit arbitrage activity is amplified by the length of time for which mispricing can persist, which itself depends on market characteristics (as well as on the amount of arbitrage capital available). Duffie, Gârleanu, and Pedersen (2007) show that the speed with which transaction prices recover after “fire sales” depends to a large extent on search costs and the liquidity of the market: in the case of illiquid environments, price recovery may take considerable time as market participants have to wait for the arrival of counterparties. That is, limits to arbitrage are themselves a function of market liquidity.

9.4.3 Implications for Market Making and Liquidity Crises

The arbitrageurs modelled in the previous sections can be viewed as providing liquidity to noise traders. For instance, arbitrageurs with capital at date 1 are "leaning against the wind" by absorbing the sell orders for asset A of noise traders and the fire sales of arbitrageurs who liquidate at that date. In doing this and selling asset B short, they act exactly as market makers, who fill sell orders and simultaneously hedge their positions.[15] Thus, one can interpret the magnitude of the mispricing M_1 as the price impact of sell orders due to the lack of liquidity. Indeed, if all arbitrage capital is deployed at date 1, the market at that date is fully liquid, in the sense that it absorbs noise traders' sell orders with no discount.

This interpretation highlights some aspects of market-making activity that we have yet to discuss. First, market participants such as hedge funds, which take and hold positions for long periods, play a similar role conceptually to that of the market makers described earlier (e.g., NYSE specialists or Nasdaq dealers). The main difference is that they do not operate at the same frequency. Hedge funds that follow "contrarian" strategies for extended periods of time, such as a month, are good examples. They build portfolios with long positions in underperforming stocks and short positions in overperforming stocks (see Khandani and Lo, 2011). Thus, they seek to exploit the price reversals that should occur if the price movements in each group are due to transient liquidity shocks. Like traditional market makers, they sustain inventory risks because the extent and the speed of price reversals are uncertain; the price reversals that they exploit simply take place at a lower frequency than those on which traditional market makers thrive. The analogy between the role of arbitrageurs and market **(p.337)** makers is highlighted by Grossman and Miller (1988) and Huang and Wang (2009, 2010).[16]

Second, this interpretation underscores the importance of financing for market makers. To provide liquidity, they must be able to fund their positions. A shortage of external funding can force them to scale down their activity or even withdraw completely from some markets, leading to wider bid-ask spreads and greater price impact of orders.[17] This explains why a credit crunch affecting market makers often leads to reduced liquidity in security markets: in the words of Brunnermeier and Pedersen (2009), a sudden drying up of "funding liquidity" reduces market liquidity as well. Not only is this mechanism often at work in large-scale financial crises such as that of 2007–2008, but it can also operate in normal times, helping to explain the "commonality" in market liquidity-that is, the fact that bid-ask spreads often widen simultaneously on many security markets, as funding to market makers is curtailed across the board, such as after a monetary policy restriction.[18]

The funding requirements of market makers can lead to a drying up of liquidity just when it is most needed, and hence to a very sharp fall in asset prices. To capture this point in the model of the previous section, suppose that with very low probability the arbitrageurs who saved their capital for intervention at date 1 are hit by an unexpected cut in financing at that date, possibly due to a credit crunch or to a capital loss in unrelated business. We call this a "crisis" state. As the probability of the crisis is low, it does not affect the choice of a strategy at date 0, so that $\widehat{\varphi}$ is determined by equation (9.19), as before.[19] Hence, if the economy is not in crisis, the equilibrium mispricing M_1^* is determined by equation (9.24), as in figure 9.7.

If a crisis does occur however, the clearing condition (9.21) is altered since the arbitrageurs who did not intervene at date 0 are no longer available to absorb **(p.338)** the sell orders placed by other market participants: formally, the new clearing condition is

(9.25)

$$y\left(P_{A1}\right) + \int_0^{\widehat{\varphi}} \varphi\left(i\right) di = 0,$$

so that the magnitude of the mispricing in crisis state is
(9.26)

$$M_1^{crisis} = \frac{1}{\delta}\left(\frac{1}{2}\widehat{\varphi}^{*2} + 1\right).$$

Compared to its equilibrium value in "normal times" (given in equation (9.24)), in the crisis the mispricing of asset A increases by

$$M_1^{crisis} - M_1^* = \frac{1 - \widehat{\varphi}^*}{\delta},$$

implying that the price of asset A is lower than in the absence of crisis at date 1. This is illustrated by figure 9.8, where the downward-sloping locus representing the allocation of arbitrage capital between the two dates does not shift (as it was determined giving a negligible weight to the possibility of crisis), while the vertical intercept of the upward-sloping curve representing equilibrium mispricing shifts up from 0 to $1/\delta$. The equilibrium level of mispricing in the crisis is found on the new market-clearing curve in correspondence with the original value of $\widehat{\varphi}^*$. As figure 9.8 indicates, the increase in mispricing triggered by the crisis—the market crash—will be more dramatic, the smaller $\boldsymbol{\delta}$ is. The reason is that the noise traders' valuation of the asset is $V - \frac{1}{\delta}$: hence, lower δ means noise traders value the asset less. Now, since in the crisis the arbitrageurs' ability to absorb the asset is crippled by lack of funding, noise traders are left alone to hold the asset and in equilibrium must become net buyers. Hence, the lower their valuation of the asset, the larger the price drop. That is, the

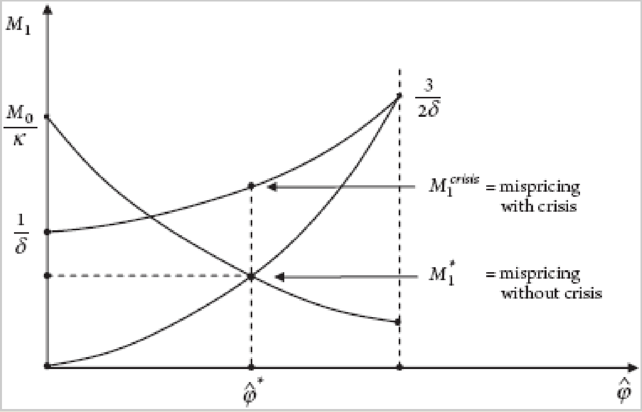


Figure 9.8. Mispricing and allocation of arbitrage capital with and without crisis

(p.339)

Table 9.1. Mispricing with and without Crisis

	M_1^*	M_1^{crisis}	$M_1^{\text{crisis}} - M_1^*$
$\delta = 0.9$	1.29	1.55	0.26
$\delta = 0.1$	2.8	10.27	7.47

disappearance of fresh arbitrage capital at date 1 leads to a sharper fall in prices when δ becomes small.

This is illustrated by table 9.1, which compares mispricing at date 1 in normal and in crisis times. Wecan see that the crisis is associated with greater mispricing but that its magnitude is much greater if $\delta = 0.1$ than if $\delta = 0.9$. When δ is small, the drying-up of the liquidity normally supplied by arbitrageurs has a more dramatic effect. In fact, the arbitrageurs who are forced to close their positions at date 1 help to push the price even below the valuation of the noise traders, who in the crisis end up absorbing the arbitrageurs' sell orders.

9.5. Correlated Order Flow and Noise Trader Risk

In the previous section, noise traders were seen to be an important element in financial crises, because the price pressure they generate may be so great that arbitrageurs are unable to absorb their orders and keep market prices from diverging further from fundamentals. One reason is that in some circumstances their orders tend to be highly correlated, so that they sell or buy in waves. Dorn, Huberman, and Sengmueller (2008), using data on a sample of retail investors with accounts at a large online broker in Germany, find that their trades are positively correlated.

This can either reflect correlation of their liquidity needs or a tendency to follow the same trading rules (reacting in the same way to the same information). Liquidity needs may be correlated because of macroeconomic shocks to many households at once. In recession, for instance, many workers may be laid off or put on shorter hours, and so they may all have to liquidate their asset holdings at the same time. Moreover, investors may display “herd” behavior, in the sense that they may imitate one another’s trading strategies, stampeding into or out of specific investments. Alternatively, they may follow common mechanical trading “momentum strategies,” which lead them to sell at the same time when securities prices are falling (and buy when they are rising).

This type of behavior is modelled by De Long et al.(1990) via the assumption of “positive feedback traders,” who buy when prices rise and sell when they **(p.340)** fall. In a similar spirit, Gennotte and Leland (1990) assume that in certain circumstances the aggregate demand curve for stocks takes a reverse S-shape, negatively sloped at relatively high and low prices and positively sloped at intermediate ones. The positive-sloped portion of the aggregate demand curve results from the presence of “dynamic hedgers” (such as investors committed to “portfolio insurance” or “stop-loss” trading strategies), who sell whenever the stock price falls below a certain threshold. Both models show that such behavior tends to amplify the fluctuations of asset prices. In the model by Gennotte and Leland it may also trigger sharp price adjustments, in other words, market crashes.

A simple way to grasp this point is to consider the inventory model of Chapter 3, where the dealers’ quotes are centered on the price

$$m_t = \mu_t - \rho\sigma^2 z_t,$$

which decreases with the dealers’ risk aversion ρ, the riskiness of the security σ^2, and the dealers’ inventory z_t. In that setting, larger orders (q_t) increase the size of dealers’ inventory (the absolute value of z_t), and thereby their risk. If the orders of noise traders tend to be correlated, the swings in their trade sizes, and hence in z_t and m_t, are larger. Seen ex ante through the eyes of investors, this phenomenon tends to increase risk: I will consider the stock to be more risky if I expect many other investors to sell when I want to sell. Hence noise traders’ behavior is in itself a source of risk, distinct from the fundamental risk due to new information on the asset payoff (i.e., changes in μ_t).[20]

Interestingly, rational speculators-or arbitrageurs-may not offset the price swings induced by feedback noise traders or by dynamic hedgers, but rather exacerbate them. In De Long et al. (1990), this is because speculators may gain by jumping on the bandwagon and trading ahead of the noise traders; in Gennotte and Leland (1990) it is because they may incorrectly interpret selling by dynamic hedgers as driven by bad news, inducing them to join the latter in a wave of selling.

This risk induced by noise trading is greater in thin markets, which cannot absorb large bulges of buy or sell orders without wide price fluctuations. Pagano (1989a) and Allen and Gale (1994) show that in certain circumstances noise trader risk can trap the market in a self-perpetuating high-volatility, low-volume equilibrium, where noise trader risk deters new investors and low volume makes for acute sensitivity to noise traders’ orders.

(p.341) 9.6. Further Reading

Amihud, Mendelson and Pedersen (2005) and Vayanos and Wang (2009) offer in-depth examinations of the effects of illiquidity on asset prices (see also Cochrane, 2005). The seminal study by Amihud and Mendelson (1986) was followed by a spate of works confirming the positive cross-sectional relationship between liquidity and returns, controlling for risk: see Brennan and Subrahmanyam (1996); Chordia, Roll, and Subrahmanyam (2000); and Datar, Naik, and Radcliffe (1998). Similarly, the liquidity effects found by Amihud and Mendelson (1991) for fixed-income securities were confirmed by other studies, such as Warga (1992), Daves and Ehrhardt (1993), Kamara (1994) and Krishnamurthy (2002). Some researchers have also considered the effect of exogenous transaction costs on asset prices in dynamic portfolio choices models (e.g., Constantinides 1986, and Vayanos, 1998).

It is worth pointing out that asymmetric information poses several important issues for asset pricing beyond those discussed in section 9.2.4. One question is whether a CAPM-like relationship still obtains. Even under the usual CAPM assumptions (i.e., CARA utility functions and normal distributions of securities' payoffs), investors with heterogeneous information have different expectations for returns and their covariance structure. Hence, they do not have the same mean-variance frontier, implying that under asymmetric information the usual two-funds separation theorem of the CAPM does not hold. Biais, Bossaerts, and Spatt (2010) show however that a CAPM-like relationship can be obtained by considering the weighted-average beliefs of informed and uninformed investors (see also Admati, 1985).

The model of asset pricing in OTC markets in section 9.2.5 is based on Duffie, Gârleanu, and Pedersen (2005), from which our presentation differs in two respects. First, their model is cast in continuous time, while we consider a discrete time. Second, in their model investors can trade directly with other investors, as well as with dealers. This model has been extended in various directions. For instance, Duffie, Gârleanu, and Pedersen (2007) consider the case of a risky security and risk-averse investors, and analyze the dynamics of prices after an aggregate liquidity shock (an exogenous drop in the fraction of investors with a high valuation). Weill (2007) allows market makers to carry inventories over time and thus to counter the negative price effects of liquidity shocks. Vayanos and Wang (2007) posit multiple securities and derive cross-sectional implications for asset returns.

Pastor and Stambaugh (2003)were among the first to study whether liquidity risk is priced. They build a measure of market-wide illiquidity based on the observation that illiquid stocks should feature larger reversals in returns (see Chapters 3 and 5). They then measure a stock's exposure to illiquidity by the **(p.342)** co-movement between its return and their market-wide measure of illiquidity. They find that this source of liquidity risk is priced, that is, that stocks with higher exposure to illiquidity risk have higher returns on average.

In contrast to the cross-sectional evidence, the longitudinal evidence on the relationship between bid-ask spread and returns is somewhat inconclusive. Using a very long series of monthly data, Jones (2002) finds that at times when spreads are large and so expected to be high in the near future, then expected returns are also high. However, Hasbrouck (2005) gets mixed results using daily data. These conflicting findings may be partly produced by the difficulty of disentangling the impact of time-varying liquidity from that of time-varying volatility, induced perhaps by bouts of noise traders' buy or sell orders.

A few papers highlight the interactions between illiquidity and fundamental risk. For instance, in Vayanos (2004), fund managers are subject to withdrawals when their performance falls below a given threshold. Hence, they are more likely to liquidate when the market is volatile, which increases the illiquidity premium at times of high volatility. In contrast, Favero, Pagano, and von Thadden (2010) predict a negative relationship between the illiquidity premium and volatility, as in their model investors have less use for liquidity when volatility is high since outside investment opportunities also deteriorate.

The theoretical literature on limits to arbitrage can be traced back to Dow and Gorton (1994) and Shleifer and Vishny (1997). Kondor (2009) develops a continuous-time model of convergence trading in which arbitrageurs do not immediately allocate all their capital to an opportunity, anticipating that it may become even more profitable in the future. He shows that, in equilibrium, the size of the opportunity must necessarily increase over time. Gromb and Vayanos (2002) explicitly model the link between arbitrageurs' past performance and the amount of arbitrage capital available at a given date, in the presence of margin requirements. If performance is bad, arbitrageurs' wealth declines. Thus, margin requirements limit the amount of capital that can be devoted to arbitrage activities more stringently, further depressing prices. Brunnermeier and Pedersen (2008) extend their model to the case of multiple arbitrage opportunities, showing how financial constraints (such as margin requirements) can generate co-movement in liquidity in various securities. In his AFA presidential address, Duffie (2010) surveyed recent research on these price effects across multiple markets associated with slow-moving capital. Duffie and Strulovici (2009) model the equilibrium movement of capital between asset markets with different levels of invested capital, and show that the greater the difference in capital levels, the greater the intermediaries' effort to re-balance the distribution of capital across markets, and the faster the convergence of the mean rates of return on different assets toward a common level. Xiong (2001) and Kyle and Xiong (2001) consider a dynamic setting in which arbitrageurs have logarithmic **(p.343)** utility functions, so that their demand for risky securities is inversely related to their wealth. As a consequence, losses on arbitrageurs' positions induce a decrease in their demand for risky securities and further price decline. Gromb and Vayanos (2010) survey the theoretical literature on the limits to arbitrage.

Many of the empirical studies on limits to arbitrage analyze the temporary underpricing and subsequent price reversal induced by "fire sales." Coval and Stafford (2007) investigate stock market transactions induced by the redemptions of open-end mutual funds, and find that funds undergoing large outflows tend to decrease the size of their positions, which puts pressure on prices of the securities held in common by distressed funds. Mitchell, Pedersen, and Pulvino (2007) study the price reactions of convertible bonds to forced redemptions of hedge funds. Pulvino (1998) shows that financially constrained airlines get lower prices than their unconstrained rivals when they sell used narrow-body aircraft. Campbell, Giglio, and Pathak (2011) show that forced sales of real estate due to foreclosures are associated with substantial price discounts followed by mean-reversion, while unforced sales prices are close to a random walk. Ellul, Jotikasthira, and Lundblad (2011) examine fire sales of speculative-grade corporate bonds by insurance companies arising from regulatory constraints and/or capital requirements. They find that more severely constrained companies are, on average, more likely to sell downgraded bonds, and that bonds whose probability of regulatory-induced selling is higher exhibit significant price declines and subsequent reversals, especially when insurance companies as a group are more distressed and other potential buyers have scarce capital.

9.7. Appendix. The Derivation of the Search Model

This appendix shows how to derive the expressions for the ask price and the bid-ask spread in section 9.2.5. In each period *t,* there are four groups of investors in the market: (i) high-valuation investors who own the security, (ii) high-valuation investors who do not own the security, (iii) low-valuation investors who own the security, and (iv) low-valuation investors who do not own the security. Let π_h^o, π_h^{no}, π_l^o, and π_l^{no} be the fractions of investors in each group, with superscript *h* and *l* referring respectively to a high-valuation and a low-valuation individual, and subscripts *o* and *no* to an owner and a non-owner.

In each period, the fraction of investors willing to buy the security is $\pi_b = \Psi\pi_l^{no} + (1-\Psi)\pi_h^{no}$, and the fraction willing to sell is $\pi_s = \Psi\pi_h^o + (1-\Psi)\pi_l^o$. We first show that when $q\langle \frac{1}{2}$, we have $\pi_b \rangle\ \pi_s$, that is, there is excess demand for the security (so that its price is determined by the maximum value that buyers place on it).

(p.344) To see this, let π_h be the steady-state fraction of high-valuation and π_l be the steady-state fraction of low-valuation investors. It must be that $\pi_h + \pi_l = 1$. Next, posit that in each period a fraction ψ of high-valuation investors become low-valuation investors, and vice-versa. Hence:

(9.27)

$$\pi_h = (1-\psi)\pi_h + \psi\pi_l = (1-\psi)\pi_h + \psi(1-\pi_h).$$

Solving this equation for π_h yields $\pi h = \frac{1}{2}$. By definition, $\pi h = \pi_h^o + \pi_h^{no}$ and $\pi_l = \pi_l^o + \pi_l^{no}$. Moreover, all shares are necessarily owned either by high-valuation investors or by low-valuation ones. Hence $q = \pi_l^o + \pi_h^o$. Thus, we have:

$$\pi_h^o + \pi_h^{no} = \frac{1}{2},$$

$$\pi_l^o + \pi_l^{no} = \frac{1}{2},$$

$$\pi_l^o + \pi_h^o = q.$$

Using these equations, after some straightforward steps one obtains:

$$\pi_b = \pi_s + \left(\frac{1}{2} - q\right),$$

so that $\pi_b > \pi_s$ if and only if $q < \frac{1}{2}$.

Let $\bar{a}$ be the maximum price that an investor is willing to pay and $\bar{b}$ be the minimum price that a seller is willing to accept from a dealer. Following Duffie et al. (2005), we assume that dealers cannot hold inventories: their aggregate inventory at the end of each period must be zero, and dealers with long positions sell the asset to those with short positions. We assume that these inter-dealer transactions take place at price:

(9.28)

$$\mu = \frac{\bar{a} + \bar{b}}{2}.$$

At any ask price a less than $\bar{a}$, there is excess demand, since $\pi_b > \pi_s$. At $a = \bar{a}$, buyers are indifferent. We assume that they choose to buy with probability $\rho^b = \phi \cdot \frac{\pi_s}{\pi_b}$. In this way, the number of buy orders received by dealers in the aggregate is just equal to the number of sell orders, so that their aggregate inventory is zero as required. Hence, the ask price a set by dealers equals buyers' reservation price $\bar{a}$.

Dealers' bid price cannot be determined by the same reasoning, because at any bid price below μ there is no excess supply of the security. Hence, following Duffie et al. (2005), we assume that dealers and sellers bargain over the bid price, producing a bid price that is the average of the dealers' and the sellers' **(p.345)** valuations, weighted by their respective bargaining power:

$$b = z\bar{b} + (1 - z)\,\mu,$$

with $0 \leq z \leq 1$. As dealers' bargaining power z increases, they extract a larger surplus from sellers. At the limit, for $z = 1$, the surplus left to sellers is zero.

To obtain the equilibrium ask and bid prices, we must compute $\bar{a}$ and $\bar{b}$. We first compute the discounted value of the future stream of cash flows that each type of investor expects to receive *just after trading in a given period* (i.e., just after stage 3). Let V_j^k be this discounted value for a trader with valuation $j \in \{h, l\}$ and type $k \in \{o, no\}$.

Now, consider a high-valuation non-owner contacting a dealer: he buys the security if and only if

$$V_h^o - a \geq V_h^{no},$$

since otherwise he is better off staying a non-owner. Similarly, a low-valuation owner will sell the security to a market maker if and only if

$$V_l^{no} + b \geq V_l^o.$$

Thus, $\bar{a} = V_h^o - V_j^{no} = \Delta V_h$ and $\bar{b} = V_l^o - V_l^{no} = \Delta V_l$. To determine ΔV_h and ΔV_l, we first calculate V_j^k for $j \in \{h, l\}$ and $\kappa \in \{o, no\}$. The value placed by a high-valuation owner on the security is

(9.29)

$$V_h^o = \frac{1}{1+r} + \frac{(1-\psi)V_h^o}{1+r} + \frac{\psi(1-\phi)V_l^o}{1+r} + \frac{\psi\phi\left(V_l^{no}+b\right)}{1+r}.$$

To understand this expression, observe that a high-valuation owner always receives \$1 with certainty at the beginning of the next period, which explains the first term here. The last three terms are simply the weighted average of the discounted cash flow for the investor in each of his possible states at the end of the next period, the weights being the respective probabilities. With probability $1 - \psi$, the investor remains a high-valuation owner and therefore values the discounted cash flow of the asset at V_h^o. This explains the second term in the equation. With probability $\psi(1 - \varphi)$, he turns into a low-valuation owner who does not manage to sell to a dealer, and thus the subsequent period ends up valuing it at V_l^o; this explains the third term. Finally, with probability $\psi\varphi$, the investor becomes a low-valuation investor who does manage to resell the security at price b in the next period. In this state, the investor receives b but also keeps the option of buying at some point in the future. The value of this option is V_l^{no}. This state is captured by the last term.

(p.346) Proceeding in the same way, we obtain the discounted value of future cash flows for a high-valuation investor who does not own the asset yet:

(9.30)

$$V_h^{no} = \frac{\psi V_l^{no}}{1+r} + \frac{(1-\psi)\left(1-\rho^b\right)V_h^{no}}{1+r} + \frac{(1-\psi)\rho^b\left(V_h^o - \bar{a}\right)}{1+r}.$$

The first term here corresponds to the state in which the investor's valuation drops, so that in the next period he wants to buy the asset, the second to the state in which his valuation stays high but he does not manage to buy the security from a dealer, and the last term to the situation in which the investor does buy the asset from a dealer (at price $\bar{a}$ with probability ρ^b) and therefore owns the asset at the end of the next period. Following the same reasoning, we get:

(9.31)

$$V_l^o = \frac{1-c}{1+r} + \frac{\psi V_h^o}{1+r} + \frac{(1-\psi)(1-\phi)V_l^o}{1+r} + \frac{(1-\psi)\phi\left(V_l^{no}+b\right)}{1+r},$$

(9.32)

$$V_l^{no} = \frac{(1-\psi)\, V_l^{no}}{1+r} + \frac{\psi \left(1-\rho^b\right) V_h^{no}}{1+r} + \frac{\psi \rho^b \left(V_h^o - \bar{a}\right)}{1+r}.$$

From equations (9.29), (9.30), (9.31), and (9.32), we obtain:

$$\Delta V_h = \frac{1+\psi(1-\phi)\,\Delta V_l + \left(\psi b + (1-\psi)\,\bar{a}\right)\phi - (1-\psi)\left(\phi-\rho^b\right)\left(\Delta V_h - \bar{a}\right)}{(1+r) - (1-\psi)\,(1-\phi)}.$$

Recalling that $\bar{a} = \Delta V_h$, this expression can be rewritten as:

(9.33)

$$\Delta V_h = \frac{1+\psi(1-\phi)\,\Delta V_l + \left(\psi b + (1-\psi)\,\bar{a}\right)\phi}{(1+r) - (1-\psi)\,(1-\phi)}.$$

Proceeding in the same way, we obtain

(9.34)

$$\Delta V_l = \frac{(1-c) + \psi(1-\phi)\,\Delta V_h + (1-\psi)\,\phi b + \psi\phi\bar{a}}{(1+r) - (1-\psi)\,(1-\phi)}.$$

Hence

(9.35)

$$\Delta V_h - \Delta V_l = \frac{(1-c) + S\,(1-2\psi)\,\phi}{(1+r) - (1-\psi)\,(1-\phi)},$$

where $S = a - b$ is the bid-ask spread charged by dealers.

Now, recalling that

(9.36)

$$\bar{a} = \Delta V_h,$$

(9.37)

$$b = z\bar{b} + (1-z)\,\mu.$$

and that $\mu = \frac{\bar{a}+\bar{b}}{2} = \frac{\Delta V_h = \Delta V_l}{2}$, we deduce from equations (9.36) and (9.37) that:
(9.38)

$$\Delta V_h - \Delta V_l = \frac{2\,(\bar{a}-b)}{1+z} = \frac{2S}{1+z}.$$

(p.347) Substituting $\Delta V_h - \Delta V_l$ from this expression into equation (9.35) and solving for

S, we get:

(9.39)

$$S = \frac{(1+z)\,c}{2\,(r+2\psi) + \,(1-2\psi)\,\phi\,(1-z))},$$

which is expression (9.15) in section 9.2.5.

Next, using the fact that $\Delta V_l = \Delta V_h$ from equation (9.38) and $\bar{a} = \Delta V_h$, we can solve equation (9.33) for ΔV_h and to obtain the ask price:

$$\bar{a} = \Delta V_h = a = \frac{1}{r} - \frac{2\psi}{r\,(1+z)}\left(1 - \phi\,\frac{1-z}{2}\right)s,$$

which is expression (9.14) in section 9.2.5.

9.8. Exercises

1. Liquidity premium in the presence of dividend income.

Consider a stock with a dividend yield d per period and with fundamental value μ_t at date t(equal to its midprice m_t at that date). Investors hold the stock for one period and can trade it at a constant percentage bid-ask spread s in each period. Their required rate of return on the stock is given, equal to r.

a. Define the gross-of-transaction-cost return $1 + R$ in terms of μ_t, μ_{t+1}, and d.
b. Determine the equilibrium gross return $1 + R$ as a function of r, s, and d alone.
c. How does the liquidity premium respond to an increase in the dividend yield d? What is the intuitive reason for this result?

2. Effect of changes in market structure on expected returns.

Consider an investor who plans to buy stock X at date t and resell it, after one year, at date $t + 1$. The required annual rate of return on this stock is r and each year the stock pays a dividend equal to $\bar{D}$ on *average*. Let b_{t+1} be the expected resale price at $t + 1$. As the market for stock X is illiquid, the resale price is lower than the expected fundamental value μ_{t+1}. Specifically, $b_{t+1} = \mu_{t+1}\ (1 - s/2)$ where s is the bid-ask spread. Let $a_t = \mu_t(1 + s/2)$ be the ask price that the investor is willing to buy at at date t.

a. Show that

$$\mu_t = \frac{\overline{D}}{\left[(1+r)\frac{1+s/2}{1-s/2} - 1\right]\left(1 - \frac{s}{2}\right)}$$

where r is the net expected return on the stock.

(p.348) b. Let $r_t = \frac{D_t + b_t + 1}{a_t} - 1$ be the actual return of the stock over the period $[t, t + 1]$. Note that this differs from the net expected return simply because the actual dividend may differ from the expected dividend in every period. Of course, $E(D_t) = \bar{D}$. Let $r_t - r$ be the "abnormal" return of the stock over the period $[t, t + 1]$. What is the average abnormal net return?

c. Now suppose that at some date τ, the stock exchange on which stock X is listed introduces a new trading system. Following this change, the bid-ask spread on stock X becomes s^*. What is the effect of this change in trading organization on the price of the stock at date τ? If this change is unexpected until date τ, what is the effect on the expected abnormal return of the stock from date $\tau - 1$ to date τ.

d. Suppose you measure the average gross and net annual returns of stock X over ten years before the change at date τ, excluding the year preceding date τ; then you perform the same calculation with ten years of data after the change in the trading system, and finally compare the two figures. If the change in the trading system makes the market more liquid (i.e., $s^* < s$), what should be the outcome of this comparison?

3. Liquidity risk and asset returns.

Assume that the holding period of the representative investor, h, is not equal to 1, as is assumed in section 9.3. How would you have to modify equation (9.18) in this more general case?

4. Limits to arbitrage and mispricing.

Consider the model of arbitrage in section 9.4.2. Suppose that the continuum of arbitrageurs has a mass K and that arbitrageurs differ in their probability $\phi(i)$ of liquidation at date 0 in case of bad performance. Assume that $\varphi(i) = \frac{i}{k}$ where i is uniformly distributed over $[0,K]$. Analyze the effect of K on equilibrium mispricing at date 1.

5. Endogenous mispricing at date 0 and 1.

Consider the model of arbitrage in section 9.4.2. Assume that, at date 0, the total demand from noise traders is

(9.40)

$$y(P_{A0}) \equiv 1 - \delta_0 (V - P_{A0}),$$

where P_{A0} is the price of asset A at date 0. Otherwise the model is unchanged.

a. Write the system of equations that must be satisfied by M_0, M_1 and $\widehat{\varphi}$ in equilibrium.

b. Solve numerically for $\widehat{\varphi}$ and the equilibrium levels of mispricings at dates 0 and 1 when (i) $\lambda = 0.1$, $\delta_0 = 0.4$, $\delta = 0.1$, and (ii) $\kappa = 0.3$, $\delta_0 = 0.4$, $\delta = 0.1$.

c. Explain the differences in findings in both cases.

(p.349) 6. Effect on the bid-ask spread of the probability of finding a dealer.

Consider the equilibrium bid-ask S in expression (9.15) in the search model of section

9.2.5.

a. Assuming $z \langle 1$, how does the bid-ask spread respond to changes in the probability φ of finding a dealer? How does the answer depend on the value of the probability ψ that the investor’s valuation will change in the future?
b. What is the intuitive explanation for this result?

Notes:

(1.) The Flash Crash of May 6, 2010 is a vivid example: inthe early afternoon, the liquidity available in the LOBs of hundreds of securities (stocks and ETFs) on U.S. exchanges evaporated in a few minutes.

(2.) A way to derive this approximation is to take logarithms of both sides in equation (9.5) and recall that $\ln(1 + x) \simeq x$, for x small.

(3.) According to the CAPM, an asset risk premium is determined by the co-variation of its net return with the net return of the market portfolio. If trading costs are uncorrelated across securities, they do not contribute to the cross-sectional covariations in net returns and therefore do not affect the risk premium required to hold a security.

(4.) They use forty-nine portfolios sorted according to the previous-year average bid-ask spread for the stocks in the portfolios and the previously estimated betas of the portfolios. They regress the monthly returns of these portfolios on their average bid-ask spreads and their betas in the previous year.

(5.) On-the-run bonds are often borrowed by traders making short sales (to play on the yield difference between these bond and the off-the-run bonds). Holders of on-the-run bonds can therefore earn lending fees from short sellers. These fees also help to widen the yield spread between on and off the run bonds (see Duffie et al., 2002 and Krishnamurthy, 2002).

(6.) Another reason is that uninformed investors face greater uncertainty when the number of public signals decreases relative to private signals. As they are risk averse, they require a higher premium.

(7.) Wang (1993) considers a dynamic model of trading with asymmetric information and a single security. He finds that the effect of asymmetric information on expected returns conditional on a liquidity shock is ambiguous. An increase in the fraction of informed traders in his set up has two opposite effects. It increases adverse selection risk for uninformed investors, which works to increase expected returns. But it also increases the amount of information available to investors, which reduces the uncertainty over the final payoff and with it the risk premium required by investors to absorb liquidity shock.

(8.) As is pointed out by Duffie, Gârleanu, and Pedersen (2005), in OTC markets, this delay can be due to the fact that it might take time to check the credit standing of an investor, to obtain trade authorization, etc.

(9.) A “dually listed company” is a corporate structure that involves two listed companies with two distinct stocks that are claims on the same assets. A “cross-listed” company is a single firm with a single stock traded in multiple markets; researchers have found that arbitrage opportunities on these stocks are much smaller and less common than for the dually listed.

(10.) This type of arbitrage which bets on the convergence in the prices of two related securities, is also called “convergence trading.”

(11.) One can imagine a more complex setting in which an arbitrageur is allowed to invest only part of his resources at date 0 and “hoard” the rest to intervene at date 1. This case is more realistic but does not deliver additional insights.

(12.) Hence, the literature on limits to arbitrage is also related to that on behavioral finance, which studies how investors’ psychological biases affect securities prices and can explain market anomalies (such as the existence of arbitrage opportunities).

(13.) See Shleifer and Vishny (2011) for examples of fire sales and a review of the literature on this topic.

(14.) Observe, using equation (9.24), that the equilibrium mispricing at date 1 will be greater than at date 0, $\langle M_1^*$, as assumed so far if δ is small enough since $\widehat{\varphi}^* \langle 1$.

(15.) For instance, a market maker taking a long position in a stock can hedge the systematic risk by selling futures on a stock index.

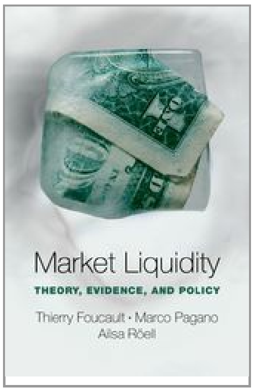
(16.) They present a model of market making with inventory risk. If a large number of investors decide to sell a security for liquidity reasons, its price will drop to attract buyers even though its cash flow is unchanged. This temporarily low price makes it attractive for investors to step in and buy, providing liquidity to the sellers-effectively playing the role of market makers. If there are many such investors large sell orders have little or no impact on price. But if these liquidity suppliers have limited capital, their ability to absorb orders may be impaired, in which case the price will be sensitive to sell orders.

(17.) Huang and Wang (2009) present a theory of market crashes based on the idea that the demand for liquidity may become suddenly too great relative to the supply.

(18.) Coughenour and Saad (2004) consider the comovements in liquidity for stocks handled by the same specialist on the NYSE, finding that they are explained both by movements in the liquidity of the specialist portfolio and market-wide movements in liquidity. This suggests that increases in the cost of capital or increased risk exposure for a specialist leads to a simultaneous drop in liquidity for the stocks assigned to that specialist.

(19.) We proceed as if arbitrageurs who decide to defer their interventions ignore the possibility of not being able to intervene at all at date 1, because the probability of this is extremely low.

(20.) Building on this observation, several recent papers attempt to quantify the contribution of noise traders to the volatility of stock returns (see, for instance, Foucault, Thesmar, and Sraer (2011) or Hendershott et al. (2010)).



Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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Liquidity, Price Discovery, and Corporate Policies

Thierry Foucault
Marco Pagano
Ailsa Röell

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Abstract and Keywords

This chapter focuses on how liquidity affects real investment decisions and corporate policies. Section 10.2 examines the mechanisms through which liquidity affects firms' investment decisions. Section 10.3 considers how high liquidity affects the governance of firms, and specifically whether it discourages the formation of large shareholding stakes and therefore the probability that management will be monitored by a large shareholder. But liquidity is not the only aspect of market microstructure that can affect corporate policies: the market's ability to keep prices in line with fundamentals may also be important for firms. Section 10.4 investigates the link between the accuracy of price discovery, the quality of firms' investment decisions, and their ability to incentivize managers by indexing executives' compensation on stock prices. Section 10.5 considers the reverse causal link, which is to say, actions that firms themselves can take to fine-tune

the liquidity of their securities. The final sections provide suggestions for further reading and exercises.

Keywords: market liquidity, corporate investment, corporate governance, investment decisions, price discovery, executive compensation

Learning Objectives:

- • Effects of the financial markets' liquidity on firms' investment and corporate governance
- • Effects of price discovery on corporate investment and executive compensation
- • How corporate policies affect market liquidity

10.1. Introduction

A key function of securities markets is to channel funds from savers to firms, to finance their investment. What role do market liquidity and price discovery play in this process? For instance, is a more liquid market conducive to more investment? Does it lead to a better allocation of funds among alternative investment projects? These are old questions to which the great economists of the past have responded in very different ways.

John Hicks, for instance, argued that the liquidity of securities markets—and not technological progress per se—is what made the industrial revolution possible (Hicks, 1969): "According to Hicks, the products manufactured during the first decades of the industrial revolution had been invented much earlier. Thus, technological innovation did not spark sustained growth. Many of the existing innovations, however, required large injections and long-run commitments of **(p.351)** capital. The critical new ingredient that ignited growth in eighteenth century England was capital market liquidity" (Levine, 1997, p. 692).

Keynes, by contrast, argued that liquidity can lead to inefficient investment decisions, by encouraging short-term speculation at the expense of sound investment decision, based on firms' long-term prospects: "If I am allowed to appropriate the term *speculation* for the activity of forecasting the psychology of the market, and the term *enterprise* for the activity of forecasting the prospective yield of assets over their whole life, it is by no means always the case that speculation predominates over enterprise. As the organization of investment markets improves, the risk of the predominance of speculation does however increase…These tendencies are a scarcely avoidable outcome of our having successfully organized 'liquid' investment markets. It is usually agreed that casinos should, in the public interest, be inaccessible and expensive. And perhaps the same is true of Stock Exchanges." (Keynes, 1936, p. 158–9).

While these views conflict, they nevertheless both rest on the idea that liquidity is of paramount importance to firms' investment, and therefore ultimately to their performance. In section 10.2, we go deeper into the mechanisms through which liquidity affects firms' investment decisions and discuss the evidence that has built up. Section 10.3 considers how high liquidity affects the governance of firms, and specifically whether it discourages the formation of large shareholding stakes and therefore the probability that management will be monitored by a large shareholder. But liquidity is not the only aspect of market microstructure that can affect corporate policies: the market's ability to keep prices in line with fundamentals may also be important for firms. So section 10.4 investigates the link between the accuracy of price discovery, the quality of firms' investment decisions, and their ability to incentivize managers by indexing executives' compensation on stock prices. Finally, section 10.5 considers the reverse causal link, which is to say, actions that firms themselves can take to fine-tune the liquidity of their securities.

10.2. Market Liquidity and Corporate Investment

Chapter 9 provides an important reason why liquidity should enhance investment: investors will pay a higher price for more liquid securities or else, to the same effect, require a lower expected rate of return to hold them. Hence, a more liquid securities market lowers the cost of capital to firms, which should boost investment, as Hicks argued with reference to the industrial revolution: cheaper capital will increase the number and size of the investment projects with positive net present value, so more liquid markets should be associated with more investment and higher firm valuation.

(p.352) There is evidence to support these predictions. Levine and Zervos (1998a), with data for forty nine countries from 1976 to 1993, find that investment and economic growth are positively correlated with measures of stock market turnover, taken to be a proxy for liquidity. They estimate cross-sectional regressions where the dependent variable for a country is alternatively the investment rate, the growth rate of real per–capita GDP, or the growth rate of productivity, averaged over the sample period. The explanatory variables include measures of stock market turnover in 1976, scaled by stock market capitalization or by GDP, plus several control variables capturing other factors associated with capital accumulation and growth (initial income per capita; education; political stability; indicators of exchange rate and trade, fiscal, and monetary policy). The estimated coefficient of stock market turnover is always positive and significant, implying that stock market liquidity is positively and significantly correlated with subsequent investment.

Related evidence is drawn from the analysis of stock market liberalizations via policies relaxing restrictions on foreign investors' share purchases. These policies are associated with an increase in liquidity, a jump in stock prices, and a drop in the cost of equity capital, as well as an increase in private investment. For instance, in a sample of eleven developing countries that liberalized their stock markets, Henry (2000b) finds that the growth rate of private investment rose above the median pre-liberalization investment rate one year later in nine countries, two years later in ten, and in three years later in eight. The average growth rate of private investment in the three years after liberalization exceeds the mean of Henry's sample by 22 percentage points (see section 10.6 for related studies).

There is also firm-level evidence that liquidity is correlated with company value. Fang, Noe, and Tice (2009) report that firms with more liquid stocks have higher market-to-book value ratios. To identify the causal effect of liquidity on firm performance, they focus on an exogenous shock to liquidity—the decrease of the tick size for U.S. securities in 2000 (see Chapter 6)—and show that the resulting increase in liquidity around that year raises company values. One mechanism that may explain this finding, as explained above, is the decline in firms' cost of capital due to better liquidity. An alternative mechanism is that a more liquid market may induce managers to select better investment opportunities, as the next section explains. Fang, Noe, and Tice (2009) try to discriminate empirically between these alternative mechanisms, and conclude that the cost-of-capital channel plays a minor role compared with the managerial decision channel.

Stock market liquidity is particularly important for the firms that first approach public markets via an IPO: their value is very uncertain, so the stocks of recent IPOs are particularly exposed to problems of asymmetric information. **(p.353)**

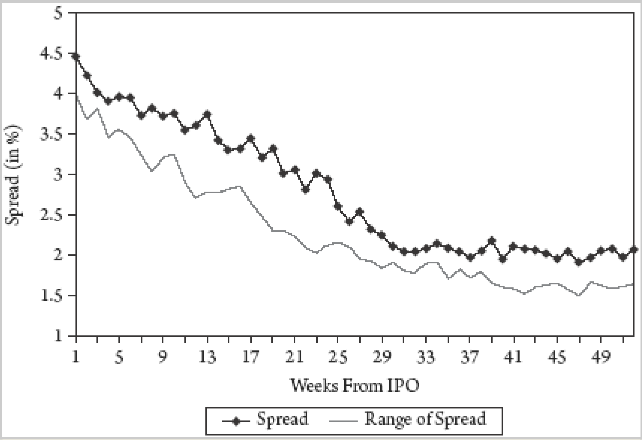


Figure 10.1. Bid-ask spread after an IPO: average value and range of variation (based on data from Ellul and Pagano, 2006)

As information about the company progressively emerges in secondary market trading, the problem becomes less severe and market liquidity improves. Ellul and Pagano (2006) show that in a sample of 337 British IPOs between 1998 and 2000, the average bid-ask spread declined from 4.5 percent in the immediate aftermarket to about 2 percent several weeks after, as shown by figure 10.1—a decline that largely reflects the decrease in the adverse-selection component. The figure also shows that the range of variation of the spread is greatest immediately after the IPO and declines steadily thereafter. This observation indicates not only that illiquidity is most severe in concomitance with the IPO, but that illiquidity risk is too. Accordingly, the illiquidity premium that investors require to buy new stocks should be particularly large, especially if these stocks are purchased by "flippers" who want to buy and quickly resell.

Consistent with this argument, IPOs sell at a discount relative to the immediate post-IPO market price, known as "IPO underpricing": relative to the first day of trading in the sample, this underpricing is 47.7 percent, which implies that firm owners leave a lot of money on the table when their firm goes public. In fact, the underpricing is more pronounced for firms that turn out to be more illiquid (figure 10.2). In other words, the more illiquid the primary market, the higher the cost of equity capital for the companies that tap it. Ellul and Pagano (2006) show that this relationship persists even after controlling for other determinants of IPO underpricing, such as the reputation of the underwriter.

Another benefit of a liquid market is that it provides an "exit option" for venture capitalists —a class of intermediaries who invest in startup firms, acting not only as financiers but also as advisors and monitors in the early stages. As **(p.354)**

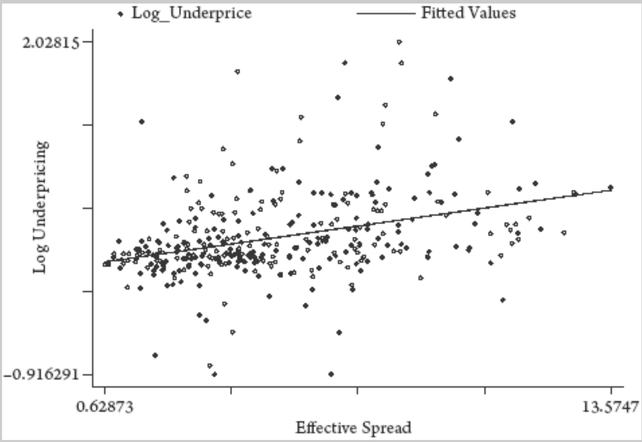


Figure 10.2. IPO underpricing and effective bid-ask spread after an IPO (based on data from Ellul and Pagano, 2006)

these advisory and monitoring services require highly specialized human capital, venture capitalists typically must limit their operations to just a few companies at a time. An increase in market liquidity enables them to exit more rapidly from their investments, because better prices are available to them. Michelacci and Suarez (2004) show that this more quickly frees their scarce human capital for redeployment in a new round of financing, and thereby increases the total number of firms they are able to serve.

10.3. Market Liquidity and Corporate Governance

We now turn to a more indirect channel through which liquidity affects firms' investment policies, namely via their corporate governance. In particular, the question is whether greater liquidity encourages investors to monitor companies in order to improve managerial decisions. As section 10.1 indicates, Keynes would have probably answered in the negative. He viewed liquidity as discouraging investors' from emphasizing the long-term prospects of their investments, as they can liquidate their holdings quickly and cheaply if dissatisfied with their performance. In line with this view, some have argued that the U.S. market-friendly regulatory framework, designed to enhance liquidity, ultimately discourages the activism of large shareholders.

Coffee (1991) states this view as follows: "*Liquidity and control are antithetical*. American law has said clearly and consistently since at least the 1930s **(p.355)** that those who exercise control should not enjoy liquidity and vice versa.…the separation of liquidity and control is not only a cause of institutional passivity, but to some degree should be. In short, those institutions that most desire liquidity would make poor monitors." He adds that other countries have chosen a different point along the trade-off between liquidity and control: “Advanced industrial economies can be classified along a continuum ranging from those, such as Japan and Germany that permit financial institutions to control corporate managements, but effectively deny them liquidity, to those that inhibit institutional control, but maximize their liquidity.” This is represented by the United States, where disgruntled investors sell their shares rather than challenge management—the well-known “Wall Street Rule.” In this vein, Bhide (1993) argues that in the U.S. “public policy has favored stock market liquidity over active investing,” at the cost of impaired corporate governance.

The trade-off between liquidity and control described by Coffee (1991) and Bhide (1993) is a special case of A. O. Hirschman’s idea that members of a dysfunctional organization choose between exit and voice. If it is easy and cheap to exit by selling, shareholders will have little interest in exercising a costly voice to intervene actively in the affairs of the company, replacing unsatisfactory managers, and so on. But if exit is blocked because the market is illiquid, then exercising one’s “voice” becomes more attractive.

To capture these ideas, consider the time line of events illustrated by figure 10.3: at date 0, a potentially activist shareholder amasses a block of shares amounting to a fraction φ of the firm’s equity, whose current value is V; at date 1 the blockholder decides whether to use his control rights to intervene in the company’s operation (voice) or to sell his stake (exit). We first analyze the date-1 problem facing the investor after he has already purchased a sizable block of shares. Then we consider whether, given what he expects to happen at date 1, he wants to buy a block of shares at date 0. **(p.356)**

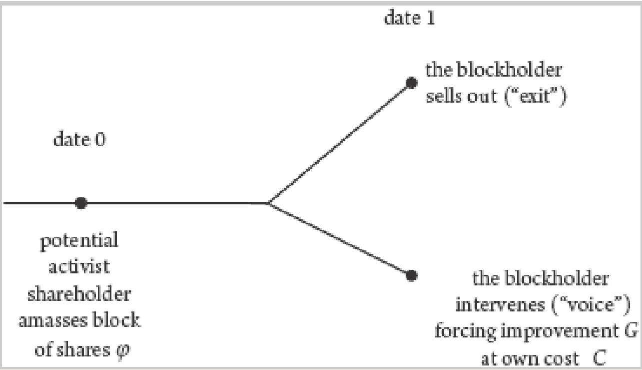


Figure 10.3. Stock market trading and shareholder activism: timeline

If the blockholder intervenes at date 1, say by replacing lazy or incompetent management, he brings about a gain G in the firm’s value. But, to do so he must exert effort at a cost C. His decision whether to intervene or to walk away depends on the liquidity of the stock at date 1. If he stays for the long haul and intervenes, the value of his stake, net of the intervention cost, will be:

$$\varphi(V+G)-C.$$

To make the problem interesting, we assume that the blockholder’s intervention produces an increase in the value of his stake that exceeds the cost of intervention: $\phi G > C$. Otherwise, there is no chance of activism.

If at date 1 the blockholder sells his stake, he gets:

$$\varphi\left(\mu_1 - S_1\right),$$

where μ_1 is the market's perception of the stock's value at date 1, including any downward revision induced by the blockholder's decision to sell, and the half-spread S_1 measures any market illiquidity over and above such price impact. For example, if trading is anonymous and the market believes the blockholder will intervene with probability π, then $\mu_1 = V + \pi G$. If instead regulation forces him to disclose his intention to exit before trading, then $\mu_1 = V$, because the market anticipates no improvement.

Thus, for voice to prevail over exit—i.e., for the shareholder to intervene rather than sell—we must have

(10.1)

$$\varphi\left(V+G\right) - C \geq \varphi\left(\mu_1 - S_1\right).$$

Let us consider the case most favorable to exit, namely that the blockholder can sell anonymously at a price $\mu_1 = V + \pi G$, where π is a rational estimate of the probability of intervention. Then the previous condition (10.1) becomes

(10.2)

$$\varphi G - C \geq \varphi\left(\pi G - S_1\right).$$

Clearly, this condition always holds if it is true for $\pi = 1$, so that there will always be intervention by the blockholder. Simple algebra shows that this occurs if $S_1 \geq C/\varphi$, that is, if the market is so illiquid that exit is excessively costly. If $S_1 < C/\varphi$ instead, there is a mixed-strategy equilibrium with the probability of intervention π such that the blockholder is just indifferent between voice and exit. In that case, condition (10.2) holds with equality:

(10.3)

$$\pi = \min\left\{1 - \frac{1}{G}\left(\frac{c}{\varphi} - S_1\right), 1\right\}.$$

This expression shows that market illiquidity encourages voice by making exit more costly: the probability π of blockholder intervention increases with the **(p.357)** illiquidity parameter S_1, rising to 1 when $S_1 \geq C/\varphi$, as shown above.[1] This illustrates the "lock-in effect" described by Coffee: the more illiquid the market, the more advantageous is active monitoring rather than exit. Note also that blockholder activism is more likely, the greater the potential gain G, the lower the intervention cost C, and the larger the blockholder's stake, as one would expect.

The lock-in effect just described is heightened if the blockholder is forced to disclose that he is walking away from his stake, so that the market prices the company at non-intervention value $\mu_1 = V$. In this case, exit becomes prohibitively expensive because there is no capital gain from selling, but the trading cost S_1 must still be paid. We leave it to the reader to verify that in this case activism is guaranteed, that is, $\pi = 1$ (see exercise 1)0.

Up to this point, we have taken the presence of a blockholder for granted. But we must consider whether a potential activist would find it worthwhile to build up a block of shares in the first place. As Bolton and von Thadden (1998) and Maug (1998) point out, the decision to purchase a block is affected by the liquidity of the market at the date of purchase: here liquidity favors activism by reducing the cost of building up the block. Formally, at date 0 the per-share price paid by the potential activist is $\mu_0 + S_0$, where μ_0 denotes the market's valuation of the company and S_0 is the transaction cost. The market valuation μ_0 will range from V when the market does not expect a blockholder to emerge, to $V + \pi G$ when it does expect it.

As before, consider the case where a blockholder's presence is not foreseen, so that $\mu_0 = V$. Recall that, from the left-hand side of expression (10.1), the net value of a block after intervention at date 1 is $\varphi(V + G) - C$. Since the cost of amassing the block at date 0 is $\varphi(V + S_0)$, the ex-ante net gain from doing so is

$$\varphi(V+G) - C - \varphi(V+S_0)$$
$$= \varphi(G - S_0) - C,$$

which is decreasing in the market illiquidity parameter S_0. Hence illiquidity is a double-edged sword: it discourages the original formation of a block, but once the block is in place, it locks the blockholder in and encourages activism.

However, if at date 0 the market fully anticipates the presence of a block-holder, then any improvements that he may bring to the company will already be reflected in the date-0 share valuation $\mu_0 = V + \pi G$. In this case, he is certain to lose money if he goes ahead and acquires the block. To see this, note **(p.358)** that his net gain is then

$$\varphi(V+G) - C - \varphi(V+\pi G+S_0)$$
$$= \varphi((1-\pi)G - S_0) - C,$$

which, inserting the probability of his date-1 intervention from equation (10.3), becomes

$$-\varphi S_0 - \min(C, \varphi S_1)\langle 0.$$

Intuitively, the blockholder incurs a transaction cost of S_0 per share when acquiring his stake, and subsequently incurs either the monitoring cost C or the exit transaction cost of S_1 per share, whichever is lower. Any improvements he makes to the company's value are already incorporated into the share price at purchase. Not surprisingly, he loses money on the round-trip transaction and so prefers to avoid it. This idea was first articulated by Grossman and Hart (1980), who argue that other shareholders free-ride on the value improvements they expect the activist investor to make, by refusing to sell at any price that does not fully reflect them. Thus, the potential activist cannot recover his costs, unless he can secretly build up a toehold stake at a lower price. This will largely depend on his ability to camouflage his trades behind noise trading, as Kyle and Vila (1991) note.

The model presented above indicates that the assertion that "liquidity and control are antithetical" (Coffee 1991) is an oversimplification: greater liquidity may actually encourage active intervention in corporate control by blockholders.[2] The relationship between liquidity and activism is further complicated by the fact that, from the standpoint of a large shareholder, liquidity will also depend on the price impact of his trades, in addition to S_0 and S_1, which may discourage the accumulation of a large stake and penalize its decumulation. As noted, the magnitude of this adverse price impact depends on the visibility of the blockholder's trades. If regulators want to promote the formation of activist blockholders, they will have to facilitate accumulation of significant blocks of shares without disclosure to other market participants. But once a **(p.359)** blockholder does have a controlling stake, sale of this stake should be highly visible to make his exit as costly as possible. In other words, the formation of a block requires relative opacity, while its retention requires great transparency, a delicate balancing act for regulators.

In practice, regulators have addressed this dilemma by allowing the buildup of an initial stake to go undisclosed below some threshold (in the United States, 5 percent of firm's equity); over it, all further transactions by the blockholder must be disclosed immediately. Thus, once the stake is taken, it becomes hard to decumulate it without a price concession that reflects the lower monitoring incentive by the blockholder.

Empirically, the presence of blockholders varies greatly from country to country. Blocks are commonplace in continental Europe and Asia but much less so in the United States and United Kingdom. Even in the United States, however, the involvement of large institutional shareholders increased dramatically with the advent of public pension fund activism during the mid-1980s and the greater role of mutual funds in equity investing. Davis and Yoo (2003) report that mutual funds manage over 20 percent of U.S. equity and hold sizeable blocks of 10 percent or more in many of the largest U.S. companies.

There is evidence that institutional investors do regard corporate governance as important to their investment decisions, but nevertheless tend to exit rather than voice their discontent (see section 10.6 for details). This aversion to exerting influence may be due partly to the U.S. short-swing trading rule, which obliges investors who actively participate in management decisions (or hold a stake of at least 5 percent) to hold their stake for at least six months, or if else disgorge any capital gains made on the in-and-out transaction. This deters institutions like mutual funds from activism, as they may experience unpredictable outflows that oblige them to liquidate shares.

Another reason for the lack of activism of institutional investors—even a frequent pro-management attitude—is conflict of interest: not opposing or even backing the managers of the companies in which they invest may enable them to get or keep lucrative contracts to run those companies' pension funds. Davis and Kim (2007) analyze proxy voting by U.S. mutual-funds in 2004 and find that aggregate pro-management votes at the fund family level are positively correlated with the magnitude of mutual funds' business ties to firms in their portfolio, even though the correlation does not emerge when examining voting behavior at specific firms: in other words, funds with strong business ties with portfolio firms tend to be less activist across the board; they do not necessarily cast their votes specifically in favor of client firm management.

A more fundamental question is whether shareholder activism actually prompts better corporate performance. The evidence is mixed. While some studies document positive short-term market price reactions to announcements **(p.360)** of certain kinds of activism, others find that large blockholders fail to improve the long-term operating or stock market performance of the targeted companies (for a survey, see Gillan and Starks, 2007). One reason for this mixed evidence may be that often activist shareholders intervene "behind the scenes," so their actions are not picked up by studies that rely on public information. A rare exception is the study by Becht et al.(2009), who obtained data from British Telecom's pension fund manager Hermes on its private engagements with management in companies targeted by its UK Focus Fund. They find that the fund intervenes predominantly through such private engagements and, in contrast with most previous studies of activism, they report that the Hermes fund considerably outperforms benchmarks and that its abnormally high returns are correlated with its interventions.

There is more agreement on the evidence regarding shareholder activism by hedge funds. These are more lightly regulated and have fewer conflicts of interests than other institutional investors. Most empirical studies support the view that their activism creates value for shareholders by improving the governance, capital structure decisions, and operating performance of target firms (for a survey, see Brav, Jiang, and Kim, 2009). Brav, Jiang, Thomas, and Partnoy (2008), using a large data set from 2001–2006, find that activist hedge funds in the United States attain success or partial success in two-thirds of the cases in which they intervene, and that the announcement of their activism is accompanied by an abnormal return of approximately 7 percent, with no reversal during the subsequent year, and is followed by higher payout, better operating performance, and greater CEO turnover.

Interestingly, Edmans, Fang, and Zur (2012) find that market liquidity increases hedge funds' propensity to acquire large blocks of shares, but that once the stake is acquired, liquidity increases the likelihood of hedge funds opting for "exit" rather than voice, exactly as predicted by the model here: greater liquidity makes it more likely that the blockholder will choose a Schedule 13Gs filing (passive investment) rather than 13Ds (active investment), especially if its manager's wealth is sensitive to the stock price.

10.4. Price Discovery, Corporate Investment, and Executive Compensation

Sections 10.2 and 10.3 examined how market liquidity affects two key aspects of corporate policy, namely investment and governance. But these are also affected by price discovery. An essential function of securities markets is discovering asset values by aggregating investors’ private signals about future cash flows. The information conveyed by market prices can then be used **(p.361)** by managers to guide to their investment decisions; this is the topic of section 10.4.1. If stock prices provide information about a company’s performance, they can also be used by shareholders to index compensation and give executives sharper incentives to maximize the company’s value, as will be shown in section 10.4.2.

10.4.1 Stock Prices and Investment Allocation

The information conveyed by stock market prices can be useful to firms’ managers and stakeholders more generally. For instance, managers may decide to pursue or terminate an investment plan (e.g., a major acquisition, a research and development project, or diversification into new products and markets) after observing the market’s reaction to its announcement. In line with this idea, Luo (2005) studies the case of merger announcements empirically and shows that managers use the stock price reaction to these announcements in deciding whether to cancel or consummate the deal. Or a sharp decline in stock prices may suggest to a financier that other investors have adverse information on the firm’s prospects and induce him raise the return required for new capital (Goldstein, Ozdenoren, and Yuan, 2009). **(p.362)**

Figure 10.4. Stock prices and investment decisions: time line

Box 10.1 Strategy Changes and Stock Price Reactions: HP'S Double U-turn

On August 19, 2011, the price of the computer company Hewlett Packard’s (HP) stock plunged 20.5 percent, after a decline of 6 percent the previous day. This followed an announcement by HP’s chief executive officer Leo Apotheker that he planned to spin off or sell the personal computer unit for which HP was universally known and refocus the company on software production with the planned $11 billion acquisition of Autonomy, Britain’s largest software producer. The new strategy was motivated by considerably larger profit margins in software than in personal computers. Clearly, the stock market did not react well to the announcement. On September 22, the company fired Mr. Apotheker and replaced him with Meg Whitman, former CEO of eBay. On October 27, HP announced that it had dropped the plan to spin off the personal computer unit, largely because it would have lost synergies related to bulk purchasing of components, thus significantly increasing costs. The price of HP shares rose by 22.6 percent from $22.80 on September 22 to $27.95 on November 30.

The possibility that managers' investment decisions may be determined by stock prices creates a complex interdependence between the stock market and the real economy. It is clear that real investment decisions affect stock prices, since the fundamental value of a security is the discounted value of its future cash flows. But if managers gather information from stock prices, causation may also run the other way. That is, stock prices will also affect investment decisions, hence future cash flows.

To capture this feedback from stock price to investment, consider the sequence of actions illustrated in figure 10.4. At date 0, a firm announces that it is considering a new investment project (e.g., an R&D program for a new chip or a move in a new product line as in the example of Box 10.1), which is equally likely to be of high quality (H) or low quality (L). The project will contribute a net gain $G > 0$ to the company's value (originally equal to V) if it is of high quality or else entail the loss of the investment I:

$$\Delta V = \begin{cases} G & \text{if the company invests and project is high-quality (H)}, \\ -I & \text{if the company invests and project is low-quality (L)}, \\ 0 & \text{if the company does not invest.} \end{cases}$$

It is assumed that, if no information about quality is available, the project is not viable: $G < I$, so that the project's expected net present value, $\frac{1}{2}G - \frac{1}{2}I$, is negative.

At date 1, after the announcement of this investment opportunity, the firm's shares are traded by investors, some of whom have valuable information about the quality of the project. As in Chapter 3, two potential types of traders place orders that are filled by risk-neutral, competitive, and uninformed market makers. With probability π, an informed risk-neutral speculator who knows whether the project's quality is H or L comes to the market, and makes a trade that maximizes his expected profit. With probability $1 - \pi$, a liquidity trader buys or sells one unit of stock with equal probabilities. The market makers price the stock at its expected value, conditional on the arrival of a buy or sell order.

(p.363) At the same time, with probability γ the firm's manager privately receives a report that enables him to establish the quality of the project; with probability $1 - \gamma$ he does not.

Thus, at date 2, when the decision to proceed with the project or not must be taken, the manager considers both his own information (if any) and the reaction of the stock market to the announcement. The issue is whether and how the information conveyed by trades on the stock market can help guide the investment decision.

To solve for the equilibrium, let us conjecture that informed traders buy a unit of the stock if they learn that the project is of high quality and sell a unit otherwise: later we will check that this strategy is indeed optimal in equilibrium.

If the manager has gotten a private report, he needs look no further to decide whether to invest. The interesting case is when he has not received any such report, and so his only guide is the stock market. As in Chapter 3, if a buy order is placed, it executes at the ask a; a sell order executes at the bid b. Hence the transaction price p reveals the direction of trading interest. Depending on whether he observes a transaction at ask or at bid, the manager updates his estimate of the probability of the project being of quality H, using Bayes' Law:[3]

$$\Pr(H|p=a) = \frac{1+\pi}{2},$$
$$\Pr(H|p=b) = \frac{1-\pi}{2}.$$

If the manager observes a trade at the ask, then his updated expectation of the project's net present value is

(10.4)

$$\Pr(H|p=a)G + \left(1-\Pr(H|p=a)\right)(-I)$$
$$= \frac{1+\pi}{2}G - \frac{1-\pi}{2}I.$$

A transaction at the ask will persuade him to invest if this updated expected present value is positive, which requires that

(10.5)

$$\pi \geq \frac{I-G}{I+G}.$$

Recalling that π is the proportion of informed investors, inequality (10.5) states that the executive's decision is guided by the stock market only if the trading is informative enough. This inequality is more easily satisfied if I is very close to **(p.364)** G, that is, if the project is borderline in the first place, so that even a little bit of positive news can tip the balance. Naturally, if a transaction is observed at the bid, he elects not to invest: his prior estimate of the project's net present value was already negative by assumption, and his posterior estimate will be even lower.

What is the allocative value of the information from the stock market? Let us compare the outcome of the investment decision with and without a stock market. We assume that the informativeness condition (10.5) holds, because otherwise the stock market would clearly play no role.

In the absence of a stock market (that is, if the company is not listed), the manager will only invest with probability $\gamma/2$, namely if he receives private information about the project and if the information is positive. Therefore, ex ante the value of the firm, if privately held, is

(10.6)

$$V_{private} = V + \frac{\gamma}{2} G.$$

If the firm is listed, however, the manager can fine-tune his investment decision and invest even if he does not have private information, as long as he observes a trade at the ask price on the market. As before, he gets positive private information with probability $\gamma/2$, in which case he again invests gaining G. But when he does not receive private information, which occurs with probability $1 - \gamma$, he will observe a transaction at the ask with probability 1/2 and therefore invest. The net present value of this policy is given by equation (10.4), so that the ex ante value of the firm when publicly listed is

(10.7)

$$\begin{aligned} V_{public} &= V + \frac{\gamma}{2} G + \frac{1-\gamma}{2} \left(\frac{1+\pi}{2} G - \frac{1-\pi}{2} I \right) \\ &= \underbrace{V + \frac{\gamma}{2} G}_{V_{private}} + \underbrace{(1-\gamma) \frac{\pi}{2} G}_{\text{informational gain}} \underbrace{- (1-\gamma) \frac{1-\pi}{2} \frac{I-G}{2}}_{\text{loss from noise}}. \end{aligned}$$

Thus we can think of the total contribution of the stock market to the investment decision as consisting of two components: the gain from investing when the informed trader is present and provides a valuable positive signal by buying the stock, and the loss from overinvesting when the noise trader buys stock (in which case the investment on average is mistaken, given our assumption that $I \rangle G$). Clearly, the net effect is positive whenever the informativeness condition (10.5) is satisfied: the manager will only consider the stock price when it improves the firm's investment decision. And as one would expect, the net gain is increasing in the informativeness of trading, as measured by π.

Under our assumptions, the stock market encourages investment, insofar as it sometimes prompts the manager to invest even when without private **(p.365)** information. However, this feature of the model is not robust: if we were to reverse the assumption that on average the investment is not viable (i.e., if we assumed that $G \rangle I$), the stock market would act as a brake on investment, by prompting the manager to forgo investment when traders sell. We leave this case as an exercise to the reader (see exercise 2).

Having obtained the manager's equilibrium investment strategy, we can now compute the bid and ask prices that will prevail on the market in equilibrium. This problem is somewhat more subtle here than in Chapter 3, because market makers must take into account the relationship between market trading, the manager's investment decision, and the probability of the project's being high quality. Recall that market makers have no independent information and simply condition their quotes on whether there is a buy or a sell order. Computing the equilibrium bid and ask prices is also useful to check the validity of our initial conjecture that the informed speculator always finds it profitable to buy when he learns that the company's project is high quality, and to sell if it is low quality.

Since equilibrium bid and ask prices depend on the investment policy, they differ depending on whether the informativeness condition (10.5) holds or not. If it does, market makers know that the manager will invest either when he receives positive private information or when he observes a buy order. So, competitive market makers will set their ask price at

(10.8)

$$a = V + (1-\pi)\underbrace{\left[\frac{\gamma}{2}G + (1-\gamma)\frac{G-I}{2}\right]}_{\text{NPV if noise trader buys}} + \pi \underbrace{G}_{\substack{\text{NPV if}\\ \text{informed}\\ \text{buys}}} .$$

The term in square brackets is the net present value of the investment conditional on the order coming from a noise trader: with probability $\gamma/2$, the manager knows that the project is high quality and invests; with probability $\gamma/2$, he knows that it is low quality and does not invest; and with probability $1 - \gamma$, he has no private information and mistakenly takes guidance from the stock market, so that the project's expected value is $(G - I)/2$ (since the noise trade contains no real information). The last term in the expression is the value of the investment conditional on the buy order coming from an informed trader, namely G.

Conversely, a competitive market maker will set his bid price at

(10.9)

$$b = V + (1-\pi)\underbrace{\frac{\gamma}{2}G}_{\substack{\text{NPV if noise}\\ \text{trader sells}}} .$$

(p.366) If there is a sell order, the manager will only invest if he has positive private information. In this case, the sell order necessarily comes from a noise trader because the manager's information and the informed trader's signal never conflict. Hence, the market maker anticipates a successful investment (worth G) with probability $(1 - \pi)\gamma/2$ (the joint probability of a noise trade and positive private information).

How will the informed speculator react to the ask and bid prices in (10.8) and (10.9)? If he knows that the project is of high quality, and expects the manager to invest upon observing the speculator's own buy order, he will value the firm at $V + G$. Hence, by placing a buy order at the ask in (10.8) he will book the expected trading profit:

(10.10)

$$(V+G)-a=\frac{1-\pi}{2}\left[G+\ (1-\gamma)\,I\right]\Bigg\rangle 0.$$

Similarly, if he knows that the project is of low quality, the speculator will book a profit from a sell order at the bid price in (10.9). In this case, since the manager will not invest upon observing the speculator's own sell order, the firm will be worth V, so that a sell order implies the expected profit is

(10.11)

$$b-V=\ (1-\pi)\,\frac{\gamma}{2}\,G\Bigg\rangle 0.$$

It can also be readily shown that the informed speculator has no incentive to trade against his information (see exercise 3): for instance, he does not short the security when he knows that the investment is intrinsically sound.[4]

Hence, as initially conjectured, in equilibrium the speculator buys upon receiving good news about the project and sells otherwise. We leave it to the reader to show that the same applies if the informativeness condition (10.5) does not hold, so that the stock market does not affect the manager's investment decisions (see exercise 4).

To sum up, we have established that there is an equilibrium in which the informed speculator buys if he receives a positive signal about the manager's strategy but sells otherwise, and the manager implements his strategy if he receives good information about the project or if the stock market's reaction to the announcement of his strategy is positive, but not otherwise.

Using equations (10.8) and (10.9), one can compute the equilibrium bid-ask spread:

(10.12)

$$S=\pi G-\frac{(1-\pi)\,(1-\gamma)}{2}\,(I-G)\,.$$

(p.367) Clearly, S is increasing in π.[5] So, as in Chapter 3, an increase in the prevalence of informed trading not only increases the informational content of stock prices but also makes the market less liquid. This points to an interesting conclusion: informed trading π increases the stock market's ability to guide investment, as measured by the gain $V_{public} - V_{private}$ in (10.7), at the cost of reducing market liquidity, as measured by the bid-ask spread (10.12). This highlights the danger of considering liquidity as a universally desirable attribute of the stock market, and provides a counterexample to the view that liquidity increases asset values and investment, as suggested by the analysis in Chapter 9. In this model, more frequent informed trading reduces liquidity, but on average raises the stock price, because it improves the allocation of investment, as is attested by the fact that the average transaction price (the midprice, since buyers and sellers arrive with equal probabilities in this example) is increasing in π:

$$m = \frac{a+b}{2} = V + \frac{1}{2}\left\{\left[(1-\gamma)\,\pi + \gamma\right]G - \frac{(1-\pi)\,(1-\gamma)}{2}\,(I-G)\right\}.$$

In this model, the frequency of informed trading also increases the frequency of investment: if π is so low that it does not satisfy condition (10.5), the manager invests only with positive private information, that is, with probability $\gamma/2$; if instead π is above the threshold set by condition (10.5), he invests also when he sees buying in the stock market, that is, with probability 1/2.[6]

Measuring the extent to which managers "follow the market" is an active research area with potentially important implications, as it links the informational efficiency of the stock market to the real economy.[7] Using U.S. data, Chen, Goldstein, and Jiang (2007) find that firms' investment decisions respond positively to their stock price: if the price rises in one year, investment tends to increase more in the next year, consistent with the thesis that managers gather information from stock prices. Moreover, in accordance with the **(p.368)** model developed above, the sensitivity of investment to stock prices is greater for firms with more informative stock prices: firms that attract more informed investors (as measured by the PIN variable presented in Chapter 5) have a greater sensitivity of investment to prices.

10.4.2 Stock Prices and Executive Compensation

A central theme in corporate finance is that, when control is entrusted to professional managers, interests of shareholders and management may conflict. These conflicts can take many forms. Managers may pursue objectives that are not in the shareholders' best interest: for instance, they may opt for an "easy life" rather than work hard to seek the best profit opportunities for the company; they may pursue unwarranted expansion (empire building) or carry out "pet projects" despite their low profitability, or they may give themselves unduly large salaries or perks, or make nepotistic appointments. It can be difficult or inconvenient for shareholders to monitor and restrain such opportunism, especially if ownership is dispersed, so that shareholders' stakes are too small to justify the costs of activism.

And even if shareholders could verify managers' behavior, it may be very hard to curtail their opportunism. Consider an executive who is seen to opt for an "easy life," as opposed to hard work: it may be very difficult for the shareholders to take such a manager to court and prove that he did not work as hard as possible. The reason is that "working hard" cannot be accurately described in a contract enforceable in court: it is impossible to foresee all the contingencies that can arise in managing the firm and specify what a "hardworking" manager must do in each. Indeed, in the United States the so-called "business judgment rule" gives managers considerable flexibility to act as they deem appropriate in the company's interest. In more technical jargon, even if shareholders could observe the manager's effort level, such effort may not be verifiable or legally enforceable. This generates a moral hazard (or agency) problem between shareholders and management.

While lawsuits may be ineffective, shareholders can mitigate this moral hazard and thus improve the company's performance by giving management financial incentives to work hard.[8] By indexing the manager's compensation to some measure of company performance (based on accounting variables or on stock prices), shareholders can induce the manager to exert effort—a practice known as "pay for performance." Holmstrom and Tirole (1993) contend **(p.369)** that linking executive pay to the price of their company's stock may be very effective in sharpening their incentives to exert effort. They argue that if shares are listed and publicly traded, speculators will gather information on the firm's performance and the stock price will thus at least partly reflect this information. The more precise the information conveyed by the firm's stock price, the more effective linking managerial compensation to it will be in eliciting effort, and hence the greater the enhancement of the firm's performance.

In this section, we capture this point in a simple setting where shareholders can choose whether the manager's compensation is to depend on the final value of the company (say, its long-term earnings or its terminal liquidation value) or to the price of the stock at some interim date, when investors observe the effort exerted. Specifically, as illustrated in figure 10.5, at date 0 shareholders design the compensation contract; at date 1 the manager chooses whether to exert effort at a personal cost c (a choice that is publicly observed, though not verifiable in court); at date 2, the shares are traded on the stock market at price P; finally, at date 3, the final value of the company V—say, its earnings—is either high (V^H) or low (V^L). The manager's effort increases the company's expected value: the probability that $V = V^H$ is $\underline{\theta}$ if the manager exerts no effort, $\bar{\theta} = \underline{\theta} + \Delta\theta$ if he exerts effort.

Thus, if he exerts effort, the manager increases the company's expected value by $\Delta\theta(V^H - V^L) = \Delta\theta\Delta V$. We assume that effort is efficient, i.e., that the effect on the company's expected final value is greater than the cost of the manager's effort: $\Delta V \rangle\ c/\Delta\theta$. In other words, shareholders would find it worthwhile to just compensate the manager for his effort if they could make sure that the manager does not shirk. For simplicity, we assume that the manager's

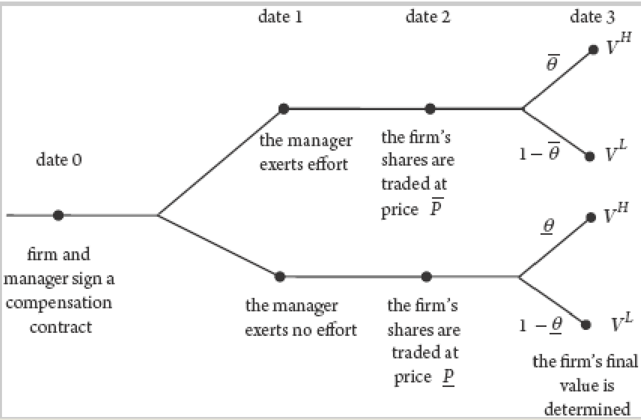


Figure 10.5. Stock prices and managerial compensation: time line

(p.370) reservation wage (i.e., what he could get from another employer) is zero and that he enjoys limited liability: he cannot be made to pay any penalty over and above his compensation.

Consider first the case in which the manager's pay can only be conditioned on the final value of the company (i.e., incentive pay can only be based on reported earnings: pay $w(v)$ takes one of two values w^H or w^L depending on the firm's final value). It is most economical for the owners to set the salary as low as possible in case of failure: $w^L = 0$. Incentive compatibility requires that the manager be given a high enough compensation w^H in case of success to induce him to exert effort:

(10.13)

$$\bar{\theta} w^H - c \geq \underline{\theta} w^H.$$

The lowest-cost pay scheme that elicits effort is thus:

(10.14)

$$w(v) = \begin{cases} w^H = c/\Delta\theta & \text{if} V = V^H, \\ w^L = 0 & \text{if} V = V^L. \end{cases}$$

The manager's participation constraint is satisfied, since this compensation package gives him a positive expected payoff $\bar{\theta} w^H - c = \underline{\theta} c/\Delta\theta > 0$, which is greater than his zero reservation wage. So shareholders end up paying the manager a "rent" to stimulate him to exert effort. This is due to the manager's limited liability: if shareholders could inflict a penalty when the company's value is low, they could avoid paying him this rent, since they could incentivize him while just compensating him for his effort.[9] Hence the company's expected value, at time 0, is

(10.15)

$$V_0 = \left[\bar{\theta} V^H + \left(1 - \bar{\theta}\right) V^L\right] - \bar{\theta} w^H = V^L + \bar{\theta} \left(\Delta V - \frac{c}{\Delta\theta}\right).$$

So far, it has been assumed that, although the company's shares are publicly traded, shareholders do not take the stock price into account in determining the manager's pay. But, at date 2, the stock price will be higher when the manager exerts effort than when he does not since investors observe the manager's effort.[10] Suppose now that the manager's compensation can be conditioned on the stock price rise associated with his effort: he can be paid a salary $\bar{w}$ if the stock price signals that the manager exerted effort at date 1, and zero otherwise. **(p.371)** Hence, at date 2, the company's stock price is:

(10.16)

$$P = \begin{cases} \overline{P} = \underline{\theta} V^H + (1 - \underline{\theta}) V^L + \Delta\theta \Delta V - \overline{w} & \text{if the manager exerts effort,} \\ \underline{P} = \underline{\theta} V^H + (1 - \underline{\theta}) V^L & \text{if the manager shirks.} \end{cases}$$

The incentive constraint now is

(10.17)

$$\overline{w} - c \geq 0.$$

Hence, shareholders optimally offer the following compensation package to the manager:

(10.18)

$$w(P) = \begin{cases} \overline{w} = c & \text{if} P = \overline{P}, \\ \underline{w} = 0 & \text{if} P = \underline{P}. \end{cases}$$

Under this compensation scheme, from expression (10.16), $\bar{P} - \underline{P} = \Delta\theta\Delta V - c$. This difference is positive: as conjectured, the company's stock price is higher when the manager is seen to exert effort ($\bar{P} \rangle \underline{P}$).

The key point is that, by linking the manager's pay to the stock's performance, shareholders can incentivate the manager without leaving any rent to him: they just compensate him for his effort, so his net payoff is zero. The reason is that in this model the stock price is a more precise (in fact, here perfect) signal of the effort than reported earnings. As a result, with stock-based compensation the expected value of the company at date 0 will be greater than under earnings-based compensation:

(10.19)

$$V'_0 = \left[\bar{\theta}V^H + \left(1 - \bar{\theta}\right)V^L\right] - \overline{w} = V^L + \left(\bar{\theta}\Delta V - c\right),$$

which exceeds the value in equation (10.15). The gain in value is $V'_0 - V_0 = c\underline{\theta}/\Delta\theta$. Thus the increase in expected value is proportional to $\underline{\theta}$, the likelihood that the company will underperform when the manager shirks, and to c, the private cost of effort to the manager. Intuitively, a higher value of either one of these parameters worsens the problem of moral hazard and so increases the desirability of indexing the manager's compensation to stock prices. In exercise 6, the reader is invited to solve for the optimal compensation scheme when investors do not necessarily observe the manager's effort.

While this model shows that stock-based compensation can benefit shareholders by sharpening managerial incentives, this type of scheme also has some drawbacks. One is pointed out by Holmstrom and Tirole (1993) themselves: stock prices are a reliable device for managerial discipline only if they are informative about the company's future performance, but the greater informativeness of stock prices comes at the cost of lower liquidity, because the informational rents of speculators are gained at the expense of uninformed investors. If these investors are rational, at first they will be willing to buy the company's shares only at a lower price—they will require an illiquidity discount. **(p.372)** As shown in section 9.2.4 of Chapter 9, more informed trading raises this illiq-uidity premium and so lowers the initial price that the security will command. This factor works in the opposite direction from that considered so far: while stock price informativeness tends to increase the company's initial value by enabling shareholders to better discipline managers, the related reduction in market liquidity tends to reduce value. The familiar tension between price discovery and market liquidity resurfaces. Holmstrom and Tirole (1993) conclude that companies will have to trade off the beneficial impact of the increase in executive effort against the implied illiquidity discount.

But more recently, other disadvantages of stock-based (and option-based) managerial compensation have emerged: while it does induce effort, it may also lead managers to divert valuable resources to misrepresent performance (Goldman and Slezak, 2006). This danger has been shown to be empirically relevant by a number of studies, which detect a positive correlation between managerial incentive pay and accounting fraud (see section 10.6). Last but not least, in the context of the 2007–09 financial crisis, stock- and option-based compensation was indicted by some observers as responsible for excess risk-taking by bank managers (Bebchuk, Cohen, and Spamann, 2010), although the evidence on this point is not yet decisive (Fahlenbrach and Stulz, 2009).

10.5. Corporate Policies and Market Liquidity

We have seen that a liquid security market can increase corporate investment and that the informational content of stock prices can improve firms' investment decisions and help them to provide incentives to management. This suggests that companies may wish to enhance the liquidity of their stock or the informativeness of its price. And companies can in fact affect these variables by a variety of decisions.

Clearly, the most basic decision in this respect is whether or not to go public. But even after this decision is made, this is not the end of the story: after an IPO on its domestic stock market, a company may decide to cross-list on other exchanges. Alternatively, it can choose from the start to list on multiple exchanges, via a global IPO. The determinants and effects of these choices are discussed briefly in section 10.5.1.

Public listing, however, is no guarantee that the shares will be actively traded, with narrow bid-ask spreads and informative prices. To achieve this, a company can act on three other fronts. First, it can support the liquidity of its shares directly by paying designated market makers to trade in its stock and maintain a tight bid-ask spread: Section 10.5.2 discusses the effects of these policies on liquidity and company value. Second, a company can affect the liquidity of its **(p.373)** securities by disclosing detailed, precise and timely value-relevant accounting data and by encouraging financial analysts to evaluate and convey them to investors, as discussed in section 10.5.3. Third, it can affect the liquidity and informativeness by adjusting its issuance of shares and debt securities, hence its leverage—the key parameter of a firm's capital structure. Alternatively, it can tweak the design of its securities so as to vary their price sensitivity to information on its performance. These issues are discussed in section 10.5.4.

10.5.1 Listing and Cross-listing

Liquidity is the most obvious benefit from listing a company's shares on a public market: the trading platform for its stock acts as a coordination device, where potential buyers and sellers can meet and trade, instead of searching informally for a counterparty. Per se, this liquidity gain expands the company's shareownership base to investors who would otherwise shun it, for fear of being unable to liquidate the shares at a reasonable price in case of need. In fact, many investors may not even be aware of a company's existence until it lists. As highlighted by Merton (1987), investors' portfolio choices are constrained by their limited awareness of which securities are available for investment.

Hence, going public should enable companies not only to reduce the cost of equity financing—thanks to the lower liquidity premium (see Chapter 9)—but also to access a larger pool of external funds, by expanding their shareholder base. These effects are further amplified by the fact that public companies typically disclose much more information than private ones do (section 10.5.3). In the United States, young companies tend to use their improved post-IPO access to external equity to increase investment, while more mature companies use it mainly to pay down debt (Mikkelson, Partch, and Shah, 1997); in Italy, instead, newly listed companies use the funds mainly to reduce leverage, and negotiate lower interest rates with their banks (Pagano, Panetta, and Zingales, 1998).

The benefits of going public extend beyond liquidity and access to finance. An IPO enables the controlling shareholders to divest some of their shares and diversify their portfolios (Pagano, 1993b; and Chemmanur and Fulghieri, 1999). It also allows the company to use the stock price as an input into managerial decisions (section 10.4.1) and can improve output market performance by enhancing the firm's visibility and reputation.

However going public also entails costs. Some are straightforward: the underwriting fees charged by the investment banks that assist the company in listing and placement, stock exchange listing fees, and the costs of complying with disclosure requirements. Other costs are more subtle, involving the negative **(p.374)** fallout of disclosure due to greater visibility to competitors and tax authorities (section 10.5.3). Since some of these costs have a large fixed cost component, it is not surprising that many small companies refrain from going public. Pagano, Panetta, and Zingales (1998) report that company size is the single most important determinant of the probability of going public.

Most companies that go public do so via domestic IPOs, in the country where they are headquartered. Yet they can still increase their market access further by cross-listing their shares on foreign exchanges. Less frequently, companies undertake their IPO directly in a foreign market or simultaneously at home and abroad—a so-called global IPO. The benefits and costs associated with cross-listing are similar but not identical to those associated with the first listing discussed above.

On the benefit side, cross-listing may increase liquidity and access to new investors; indeed, companies tend to cross-list in more liquid and larger exchanges (Pagano et al., 2001). However, in contrast to an IPO, a cross-listing creates an additional market for the same stock. Hence, in theory, it might reduce the liquidity of the home market, or fail to produce a very active foreign market for the stock. In other words, the fragmentation of trading induced by cross-listing may reduce total liquidity (see Chapter 7, section 7.2)—

an issue that has been addressed by a number of empirical studies (see the references in section 10.6).

The effect of cross-listing on the cost of equity capital appears to vary with company's country of origin: Non-U.S. companies from countries with relatively poor shareholder protection tend to get positive stock price reactions when they announce that they will cross-list in the United States, while no such reaction is observed for cross-listings by U.S. companies (Karolyi, 1998). Cross-listing also enhances increased visibility and reputation: actually, of 305 cross-listed European companies, 57 percent reported that the most important benefits of a foreign listing are increased visibility and prestige (Bancel and Mittoo, 2001).

This suggests that some benefits of cross-listing stem specifically from the wide range of choices of where to cross-list, while for the original IPO, listing on the domestic exchange is the default option. Hence, cross-listing can be used strategically to "bond" the company to the rules of an investor-friendly foreign jurisdiction or to benefit from the prestige and visibility of an established stock exchange. Indeed, companies tend to cross-list in countries with better investor protection (Pagano et al., 2001)—a choice generally rewarded by the market via a lower cost of capital or equivalently a "cross-listing premium." Consistent with the bonding hypothesis, the premium of companies that cross-list in the United States is positively correlated with measures of improvements in corporate governance (see the survey by Karolyi, 2006, and the references in section 10.6).

(p.375) Another strategic benefit of cross-listing is to access investors who are best at evaluating its business plans or its technology. Hence, a cross-listing can be used to enhance the informativeness of stock prices to managers and thereby firm value, as is shown by Foucault and Gehrig (2008). For instance, a firm that plans to launch a product in the U.S. market knows that American investors are well placed to judge its potential. One way for the firm to obtain credible feedback from U.S. consumers is to cross-list in the United States, thus encouraging local investors to trade its stock and contribute to price discovery. U.S. investors may also have greater expertise in evaluating the business of high-tech firms, such as software or biotech companies, which have a particularly strong presence in the U.S. economy.

The evidence on cross-listings and foreign IPOs is consistent with the thesis that companies seek to list where investors are better positioned to value their shares. Pagano, Röell, and Zechner (2002) document that the foreign sales of European companies that cross-listed between 1986 and 1997 tended to expand afterwards, and that U.S. exchanges attracted mainly cross-listings of high-tech and export-oriented companies. The same motivations appear to apply to most companies that choose to make their original IPO outside their domestic market. Caglio, Weiss Hanley, and Marietta-Westberg (2011) document that foreign IPOs tend to involve high-tech firms strongly oriented towards foreign markets, and originate from countries with low capital market development and poor disclosure requirements.

10.5.2 Designated Market Makers

As was explained in section 10.2, companies whose shares are more liquidin secondary market trading fetch a higher price at the IPO stage (Ellul and Pagano, 2006): the fact that investors appear to value post-IPO liquidity suggests that firms can benefit from policies that improve the liquidity of their stock. And in fact, IPO underwriting banks sometimes act as market makers to boost the new stock's liquidity in the months following the offering (see Ellis, Michaely, and O'Hara, 2000). Firms pay their underwriter an additional fee for this service, which attests the fact that liquidity is particularly valuable at the IPO stage.

But firms can enter into this type of arrangement even after the IPO: they can—and often do—hire a market-maker to maintain a liquid market in their stock, by keeping the spread below some agreed level, for instance. Such designated market makers (DMMs) have been increasingly popular in LOB markets: they have appeared in France, Germany, Italy, the Netherlands, Sweden, and Norway, where they operate with aggressive limit orders on the buy and sell side of the book and thereby increase liquidity to incoming orders. **(p.376)** In this sense, their operation is completely different from that of the NYSE specialists, whose last-mover advantage has such potential drawbacks, as the possibility of cream skimming, which might harm liquidity (section 6.3.3 of Chapter 6).

Indeed, the evidence shows that DMMs have increased market liquidity—as measured by spreads and other indicators—in all the markets where they have been introduced (see section 10.6 for the relevant references). The studies by Menkveld and Wang (2011) for the Netherlands and by Skjeltorp and Ødegaard (2011) for Norway also show that the introduction of a DMM is associated with a reduction in liquidity risk, as measured by the "liquidity betas" proposed by Acharya and Pedersen (2005), described in Chapter 9. These studies conclude that the introduction of a DMM corresponds to statistically and economically significant abnormal returns, which vary considerably across markets. The average yearly cumulative abnormal return associated with their announcement ranges from 1 percent in Oslo to 3.5 percent in Amsterdam, 5 percent in Paris and 7 percent in Stockholm. In any event, this is concurrent evidence that liquidity is valued by the market, well after the IPO stage.

Since hiring a DMM is a corporate decision, one wonders which companies choose to spend money to enhance market liquidity and what type of benefit is obtained. The study by Skjeltorp and Ødegaard (2011) on DMMs in the Oslo stock market is enlightening. It shows that only relatively illiquid firms choose to hire a DMM (see figure 10.6). The histogram in panel A shows the frequency distribution of the relative bid-ask spread for the companies that do not have a DMM in a given year; panel B displays that information for firms that do hire a DMM, both the year before and the year after the hire. Comparing the three plots, we see that the most liquid firms do without a DMM—probably because they are confident that their bid-ask spreads are narrow anyway—and for companies that do hire a DMM, the distribution of spreads is distinctly shifted to the left.

This study also sheds light on why companies hire a DMM, showing that the decision is affected by the likelihood that the firm will interact with the capital markets in the future. Firms that choose to hire a DMM have greater growth opportunities (as measured by their Tobin's *Q*) and are more likely to raise external equity after the appointment of the DMM. Presumably they introduce a DMM precisely because they plan to issue more shares in the future and hope to do so at better prices.

10.5.3 Disclosure Policy

As observed in section 10.5.1, committing to a high degree of corporate disclosure is part and parcel of the decision to go public. First, at the IPO stage a **(p.377)**

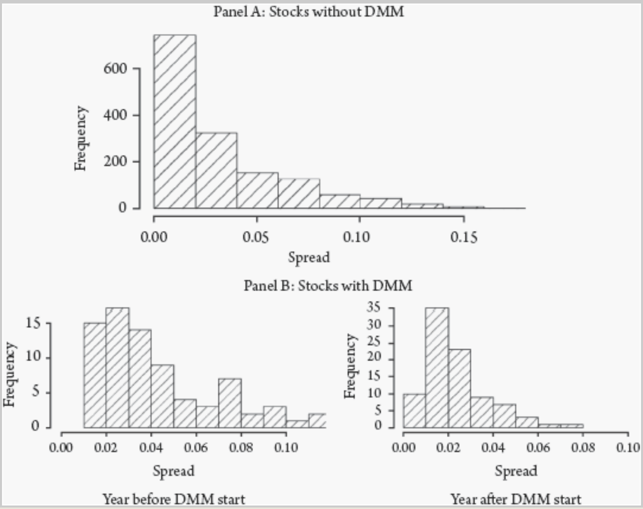


Figure 10.6. Frequency distribution of bid-ask spread for stocks without DMM and with DMM (figure 2 in Skjeltorp and Ødergaard, 2011)

company must file a prospectus, which provides certified balance sheet data, as well as information about ownership and control, business plans and expected performance, reasons for going public, and so on. Second, from that moment onward, the company commits to periodic disclosure of certified balance-sheet data and regular earnings announcements. Third, blockholders commit to disclose information about trades that imply a substantial change in their stake in the company and on related-party transactions (e.g., deals with important suppliers or customers).

However, if the shareholders want, they can opt for even more disclosure than required by the domestic stock market or regulator. For instance, they can cross-list in a market with more stringent requirements or better enforcement against insider trading, or in a jurisdiction with stronger shareholder protection. All of these courses—see the evidence on the "bonding hypothesis" in section 10.5.1—enable companies to secure a lower cost of capital. But cross-listing is not the only way to commit to better disclosure: a company can do so by having its accounts certified by reputable auditing companies, by adopting international rather than national accounting standards, **(p.378)** or by committing to regular meetings with analysts to set out the company's prospects.

Insofar as they mitigate adverse selection problems, all these disclosure mechanisms should help firms to raise cheaper and more abundant equity capital not only in the IPO but also in subsequent seasoned equity offerings. Indeed, there is empirical evidence that firms that choose greater transparency tend to attract more funding from investors and have a lower cost of capital. Since greater public disclosure reduces the rents from private information, it should also increase market liquidity, which itself lowers the cost of capital, as shown in Chapter 9.

The liquidity-promoting effect of disclosure is documented by several studies: when firms switch to an accounting regime that features better disclosure, their stocks typically become more liquid. This effect is confirmed by crosscountry evidence. Lang, Lins, and Maffett (2009) show that firms that choose a higher degree of transparency (as measured by less earnings management, better accounting standards, higher quality auditors, more analyst following, and more accurate analyst forecasts) have narrower bid-ask spreads and fewer non-trading days. They also demonstrate that this increased liquidity is associated with lower cost of capital and higher valuation. Further evidence on the relationship between transparency, liquidity, and access to capital markets comes from what happens to companies when they lose all of their analysts' coverage (figure 5.2 of Chapter 5). Their liquidity suffers compared to a control sample of similar companies (Ellul and Panayides, 2011), and their investment drops significantly (Derrien and Kecskés, 2011), suggesting that their access to capital diminishes or their cost of capital increases. Section 10.6 reports further evidence on these issues.

If disclosure has such beneficial effects for firms, why is it that not all firms opt for maximum disclosure? The fact is that disclosure also has costs, and the trade-off between costs and benefits differs between companies and countries, depending on a variety of circumstances. The first, and most obvious, cost of transparency is the expense of disseminating the information in a credible way to investors (section 10.5.1): listing fees, underwriting commissions, auditing fees, regulation compliance costs, and so on. For instance, compliance with the disclosure rules of a foreign jurisdiction sometimes requires switching to a different accounting system. Biddle and Saudagaran (1989) and Saudagaran and Biddle (1992) report that stringent disclosure requirements deter the listing of foreign companies, and the companies surveyed by Bancel and Mittoo (2001) place them among the chief disadvantages of a cross-listing.

A second, and less obvious, cost stems from the non-exclusive nature of information disclosure: once information about a firm's business plans or R&D projects is known to investors, it becomes known to competitors as well, who **(p.379)** may exploit it to appropriate the firm's profit opportunities. This should be a particularly acute concern for high-tech, innovative firms—and especially the more successful ones: Campbell (1979) and Yosha (1995) suggest that firms with projects whose value depends on confidentiality may prefer bilateral financing arrangements rather than going public. Regulation may further exacerbate this concern. In the United States, the Regulation Fair Disclosure (known as Reg FD) prevents public companies from disclosing information selectively to certain groups of investors.

A third, closely related cost arising from the non-exclusive nature of financial disclosure is diminished ability to evade or elude taxes: for instance, if the company provides more detailed accounting information to investors, it also reveals it to the tax authorities, especially if it is incorporated in a country that prescribes "book-tax conformity" (i.e., that bans reporting different earnings to tax authorities and investors). So in choosing accounting transparency, firms must trade off the benefits of access to more abundant and cheaper capital against the cost of a greater tax burden. Ellul et al. (2011) find that cross-country company-level data are consistent with the existence of this trade-off. First, investment and access to finance are correlated positively with accounting transparency and negatively with tax pressure. Second, transparency is negatively correlated with tax pressure: firms choose less transparency where the implied tax cost is higher, particularly in sectors where they are less dependent on external finance, so that the implied cost in terms of foregone investment is lower.

Finally, disclosing more information forces investors to work harder to appraise the pricing implications, and these information processing costs— whether in terms of personal effort or of the expense on additional financial advice—must be compensated by higher returns. Worse, if some investors are more sophisticated than others (e.g., better trained at pricing securities), disclosing more detailed information about companies or securities may increase the informational advantage of sophisticated investors over unsophisticated ones. In this case, paradoxically, disclosure may *exacerbate*, rather than mitigate, adverse selection problems. In other terms, the thesis that transparency reduces information asymmetries rests on the important assumption that all investors are equally good at processing the additional information. Failing this assumption, disclosing more information may lead unsophisticated investors to leave the market or to require a larger return on their investment to overcome the expected losses from investing in markets where they are disadvantaged. Thus, issuers may prefer to provide only coarse information for complex securities when information is particularly costly for many investors to process. Pagano and Volpin (2012), who make this point with reference to the asset-backed securities (ABS) created by the securitization process, also observe that while opacity may reduce adverse selection at the issue stage, it **(p.380)** creates the danger that adverse selection may reappear in secondary trading, if at that stage sophisticated investors are still able to dig out the undisclosed information.[11]

Some readers may wonder how the notion of "market transparency" extensively analyzed in Chapter 8 relates to the transparency that companies achieve via the disclosure policies discussed here. The two notions obviously refer to quite different "objects": Chapter 8 refers to the market trading process—how much is known about past trades, incoming orders, and existing quotes on a company's security. The disclosure policies considered here refer to a company's fundamental value, insofar as this can be gauged from its accounts, business plans, ownership, control, and so on. Bearing this in mind, we may call this "fundamental transparency." In any case, both forms of transparency, though different, can reduce adverse selection and increase liquidity, and both are therefore often rewarded by investors via higher issue prices or more generous funding. These two forms of transparency can be considered as substitutes, in the sense that an issuer who wishes to achieve a certain target level of liquidity and cost of capital may opt for greater corporate disclosure (more “fundamental transparency”) to compensate for the opacity of the trading process (less “market transparency”).

10.5.4 Capital Structure

Throughout this book the payoff of a stock has been taken to be a random variable v, equal to the fundamental value of the firm that issued the stock (standardizing for simplicity the number of shares to one). Hence, the rate of return offered by the company, $r = v/p - 1$, coincides with the return on its stock. This is appropriate if the firm is entirely equity financed. But if it has also issued some debt securities, the future cash flow v must be split between shareholders and debtholders in proportion to their holdings, and the return on equity will differ from the total return on the company’s assets. Denoting the initial value of the company’s equity by E and that of its debt by D, the total return on assets is the weighted average of the return to its equity and debt components:

$$r = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D.$$

(p.381) Hence, the return on the company’s stock is

$$r_E = \frac{E+D}{E} r - \frac{D}{E} r_D.$$

This expression shows that the more highly leveraged the firm, the greater the volatility of a shareholder’s rate of return: a 1 percent change in the firm’s total return r will determine a $\frac{E+D}{E}$ percent move in its stock return r_E, if the return on debt r_D stays constant. News about the firm’s future cash flow disproportionately affects the payoff to shareholders, because—if debt is and remains risk free—it leaves the rate of return to debtholders unaffected. Hence, the riskiness of stocks is not determined only by the riskiness of fundamentals, but also by the firm’s choice of its leverage ratio D/E: more generally, capital structure determines how the final value is split among different securities, such as equity, debt, hybrids, and options, and therefore the stochastic characteristics of the returns to each of these securities.

This has an important implication for liquidity. If some investors have better access to news about fundamentals (or are better at processing it), the asymmetry of information will create illiquidity only for information-sensitive securities, such as stocks, and not for safe ones, such as debt. Gorton and Pennacchi (1990) show that firms can mitigate the trading losses associated with informational asymmetry by designing securities with cash flows that are insensitive to private information and so can be safely bought by uninformed investors who want them only for their liquidity needs. Firms have an incentive to issue such safe securities, because they will be very liquid and, therefore, ex ante their buyers require no liquidity premium to hold them. Informed investors will instead concentrate their trades on information-sensitive securities, because these are the only ones from which they can hope to earn informational rents.

Of course, if the investment needs of uninformed investors are completely satisfied by the issuance of safe corporate debt by companies, then there are no stock market rents to be obtained by informed investors, because such rents arise from the trading losses incurred by the uninformed. If uninformed investors' wealth exceeds the amount of safe debt that companies can issue, in equilibrium they must hold and possibly trade some stocks. And the more leveraged the company, the more information sensitive and therefore the more illiquid these stocks will be. In this case, even the issuance of corporate debt does not completely shelter uninformed investors from the need to hold some illiquid securities and pay some transaction costs. This will restore some rents for informed investors and make stocks attractive for them.

However, as section 10.4.1 points out, the illiquidity of stocks comes with a potentially countervailing benefit, namely, the improved price discovery implied by informed trading. And, since greater leverage implies that stocks **(p.382)** are more information-sensitive, by increasing their leverage firms can increase the attractiveness of their stock to informed traders. If the proportion π of informed traders is not constant, but varies with informational rents, greater leverage will translate into a higher value of π, hence both higher trading costs for uninformed investors (equation (10.12)) and greater overall firm value (equation 10.7).

This suggests that in choosing its degree of leverage, and therefore the extent to which it wants to attract informed trading, a firm may seek to balance the higher trading costs that it thereby inflicts on its uninformed investors–and thus the illiquidity premium that it will have to pay, as argued in Chapter 9[12]–against the benefits of better price discovery. The extent to which a company wishes to leverage and thus attract informed trading will be all the more limited if uninformed traders' participation in the market is sensitive to the bid-ask spread: in response to more informed trading, they may curtail or abandon their trading altogether and thus reduce the profits available to informed traders, which by equations (10.10) and (10.11), are increasing in $1 - \pi$. Thus, information collection will be discouraged and there is a limit to how high π, and with it, stock price informativeness, can be made.

The trade-off between liquidity and the informativeness of stock prices is also affected by the value of information for firms' decisions: Chang and Yu (2010) argue that when information is not very important to improving operating decisions, a firm should limit its debt issuance, thereby reducing the information-sensitivity of its stock price. In this way, the firm discourages acquisition of private information, which reduces the illiquidity premium associated with adverse selection. This may explain why many firms issue no or modest amounts of debt, which is one of the main puzzles in corporate finance, given the compelling tax advantages of leveraging up to the hilt.

10.6. Further Reading

Stock market development, investment and growth.

There is a vast literature on the relationship between financial market development, investment, and growth, which is surveyed by Pagano (1993a), Demirguc-Kunt and Levine (2001) and Levine (2005), among others. Besides Henry (2000b), other studies of the effects of stock market liberalization are Bekaert, Harvey and **(p.383)** Lundblad (2005), who show that equity market liberalization is associated with a subsequent average annual real economic growth of about 1%, and that this effect is robust to controls for capital account liberalization and other simultaneous reforms, and Gupta and Yuan (2009), who document industries that are more externally dependent and face better growth opportunities grow faster following liberalization. Moreover, Levine and Zervos (1998b) find that such policies are associated with an increase in stock market liquidity, and Henry (2000a) and Bekaert and Harvey (2000) document that they coincide with a jump in stock prices and a drop in the cost of equity capital.

Informational effects of market prices on firms.

Bond, Edmans and Goldstein (2011) offer a good survey of the literature on the real effects that financial markets have on firms' real decisions via the information provided by security prices.

Shareholder activism.

Several authors have analyzed the positive externality created by a large shareholder's monitoring, and the implied holdout problem. Shleifer and Vishny (1986) show that if a takeover bidder starts with zero initial holdings, he has no incentive to acquire shares and become an activist shareholder. Huddart (1993) and Admati, Pfleiderer, and Zechner (1994) analyze this externality in models where the potentially activist shareholder is risk-averse. Admati and Pfleiderer (2012) show that exit and voice are not always mutually exclusive, since a large shareholder may discipline managers by threatening exit on the basis of private information. Stoughton and Zechner (1997) and Pagano and Röell (1998) analyze models in which the stake of the activist blockholder is determined endogenously at the IPO stage. On the empirical front, McCahery, Sautner, and Starks (2010) show that the majority of institutional investors in the United States and the Netherlands, when dissatisfied with a company's governance, opt for exiting, although they also frequently use their voice. Parrino, Sias, and Starks (2003) also document the preference for exit over voice by U.S. institutional investors.

Managerial compensation, incentives, and fraud.

The literature on executive compensation is vast. A good number of models have shown that performance-based compensation in the form of stock and options can enhance the incentive to exert effort and take risk (see, for instance, Smith and Stulz, 1985; Hall and Murphy, 2000; and Dittmann and Maug, 2007). But more recent work highlights the possible distortions of such managerial incentives: Bergstresser and Philippon (2006), Burns and Kedia (2006), Kedia and Philippon (2009) and Peng and Röell (2008) demonstrate that high-powered incentive schemes (especially options) are positively correlated with proxies for accounting fraud, such as discretionary accruals, fraud accusations, accounting restatements and class action litigation.

(p.384) Cross-listing.

Karolyi (1998) surveys the evidence on stock price behavior around cross-listings and finds that the effect differs across companies; even when initially positive, the effect is often dissipated in the subsequent year. On balance, non-U.S. companies listing in the United States have positive cumulative abnormal returns (Foerster and Karolyi, 1999) and experience a reduction in the cost of capital (Karolyi, 1998). Foerster and Karolyi (1999) show that the stock prices of cross-listed companies rise more significantly when the expansion of the shareholder base is more pronounced. Foerster and Karolyi (1999), Kadlec and McConnell (1994), and Miller (1999) show that the price reaction to a cross-listing is positively correlated with the increase in the shareholder base.

Several studies document that cross-listing is associated with an increase in liquidity, in the form of lower bid-ask spreads and more trading: see Kadlec and McConnell (1994); Noronha, Sarin, and Saudagaran (1996); Foerster and Karolyi (2000); and Smith and Sofianos (1997). However, cross-listing may not always enhance liquidity, due to the potentially offsetting impact of market fragmentation, as in the models of Pagano (1989), Chowdhry and Nanda (1991), and Madhavan (1995). Indeed the evidence on the emerging markets indicates that cross-listing in the United States tends to depress domestic trading. Domowitz, Glen, and Madhavan (1998) show that the home market liquidity of Mexican companies decreases upon their issuance of American depository receipts; they relate this effect to the poor information linkages between the two markets. This is consistent with Levine and Schmukler (2006), who find that companies in emerging economies experience a drop in domestic trading when they cross-list, and with Halling et al.(2008), who document that a cross-listing has a positive effect on home market liquidity for companies in developed countries, but a negative one in emerging countries.

On the bonding hypothesis, Doidge (2004) ties the premium from cross-listing in the United States to the cross-listing's ability to reduce the voting premium, and therefore produces direct evidence on its effect on the controlling shareholders' ability to extract private benefits and increase the firm's ability to take advantage of growth opportunities. Doidge, Karolyi, and Stulz (2004) document that in 1997 foreign companies listed in the United States had a Tobin's *Q* ratio 16.5 percent higher than that of firms from the same country not listed in the United States. This difference is statistically significant and persists even after controlling for several firm and country characteristics. They also find that growth opportunities are valued more highly for firms listed in the United States especially for firms from countries with less investor protection. Doidge, Karolyi, and Stulz (2009) show that in spite of the greater compliance costs associated with the Sarbanes-Oxley Act in the United States, the listing **(p.385)** premium associated with a New York listing persists, while no such premium is associated with a London listing.

Designated market makers (DMM).

Nimalendran and Petrella (2003) show that the introduction of a specialist-based market-making mechanism for thinly traded Italian Stock Exchange securities was associated with a reduction in execution costs. Venkataraman and Waisburd (2007) find that the introduction of a DMM by relatively illiquid liquid firms listed on the Paris Bourse improves market quality and that its announcement is associated with a 5 percent cumulative abnormal return. They also show that DMM are mainly hired by younger and smaller firms and by those with less volatile stocks. Hengelbrock (2008) studies the effect of the introduction of DMM for thinly traded stocks on the Xetra platform of the Deutsche Börse, and finds that while quoted and effective spreads narrow when trading with one or two designated sponsors, further increases in the number of specialists do not necessarily result in greater liquidity. Anand, Tanggaard, and Weaver (2009) show that DMM introduction in Sweden increases liquidity, produces a 7 percent cumulative abnormal price run-up, increases volume, and leads DMMs to trade more in the stocks that they contract for. Menkveld and Wang (2011) study the introduction of DMM in the Amsterdam stock market, when Euronext allowed listed firms to hire DMMs on October 29, 2001, and find that for seventy-four stocks that hired a DMM, liquidity increased, liquidity risk dropped, and there was an average cumulative abnormal return of 3.5 percent. Interestingly, DMMs participate in more trades and incur a trading loss on high quoted-spread days (days when their constraint is likely to bind). Finally, Bessembinder, Hao, and Zheng (2012) show that firms can benefit from hiring a DMM as this is a way to make the liquidity for their stock higher than it would be in a competitive market.

Transparency, liquidity, and access to capital.

Leuz and Wysocki (2008) survey empirical work on the relationship between transparency and firms' access to capital markets. This literature consistently shows that increased disclosure is associated with greater stock market liquidity and lower cost of capital.

The idea that information disclosure may exacerbate adverse selection problems, discussed in the text with reference to Pagano and Volpin (2012), is also present in the theoretical work by Dang, Gorton, and Holmstrom (2010) and Morris and Shin (2012). In the same line of argument, Kim and Verrecchia (1994) show that earnings announcements decrease market liquidity if they allow sophisticated traders to increase their informational advantage over other traders. The same argument is used by Goel and Thakor (2003) to rationalize earnings smoothing: to maintain a liquid market for their stocks, companies will smooth earnings so as to reduce the informational rents of sophisticated **(p.386)** investors. Di Maggio and Pagano (2011) analyze the relationship between financial disclosure ("fundamental transparency") and "market transparency" in a model where investors have different information processing costs and securities are sold via a search market.

Empirically, the practice of selling securities exclusively in bundles, rather than separately, may be considered as one instance in which opacity increases liquidity: Kavajecz and Keim (2005) show that asset managers lower trading costs by 48 percent via "blind auctions" of stocks, whereby they auction a set of trades as a package to potential liquidity providers, without revealing the identities of the securities in the package to the bidders. Similarly, Vickery and Wright (2010) document that mortgage-backed securities issued by U.S. public agencies (agency MBS) are extremely opaque when placed via the “to-be-announced” market, where MBS sellers specify only a few basic characteristics of the security to be delivered, but are extremely liquid.

Capital structure, liquidity, and stock price informativeness.

Gorton and Pennacchi (1990) argue that information-insensitive securities such as debt are more liquid. This thesis is also central to the model of Dang, Gorton, and Holmstrom (2010). The idea that increasing the information sensitivity of the firm’s securities can be beneficial because it elicits information collection by sophisticated investors is formalized by Boot and Thakor (1993) and Fulghieri and Lukin (2001).

10.7. Exercises

1. Trading disclosure and shareholder activism.

Consider the blockholder’s decision whether to intervene or not in the company’s management at date 1 in the model in section 10.3, and suppose that regulation forces him to disclose his intention to exit before trading, so that if he sells the market anticipates that no improvement will occur ($\mu_1 = V$). Show that in this case activism is guaranteed: the probability of intervention is $\pi = 1$.

2. Stock market as guide to investment.

In the model analyzed in section 10.4.1, we assumed that $G < I$ and found that on average the stock market encourages investment. Consider here the complementary case where $G > I$:

a. Show that in this case the stock market deters inappropriate investment. Explain the intuition behind this result.
b. Assume that the market is informative enough for the stock price to affect investment and derive the new expression for the increase in firm’s ex-ante value resulting from the presence of a stock market.

(p.387) 3. No price manipulation by speculators.

In the context of the model of section 10.4.1, verify that in equilibrium the informed speculator does not have an incentive to trade against his information, that is, to sell on good news or buy on bad news. Assume that the informativeness condition (10.5) holds.

4. Bid and ask prices when investment does not react to stock prices.

In the model analyzed in section 10.4.1, assume that the informativeness condition (10.5) does not hold.

a. Compute the equilibrium bid and ask prices, and show that in this case too, in equilibrium the informed speculator will buy upon receiving positive information and sell otherwise.
b. Compute the bid-ask spread and find out whether in this case it is increasing in the frequency of informed trading π.
c. Compare this bid-ask spread with that given by equation (10.12) under the

informativeness condition (10.5).
d. Finally, compute the midprice and see whether it depends on the frequency of informed trading π, explaining why.

5. Bid and ask prices when investment must be chosen before stock trading.

Consider the following change in the model analyzed in section 10.4.1: assume that, even though the informativeness condition (10.5) holds, the manager cannot react to the stock price because stock trading takes place once the investment decision has already been irrevocably made. (Technically, we are reversing the order of the trading and investment stages of the game along the time line.) Assuming that the manager's investment decision is private and that market participants cannot observe it before trading, derive the ask and bid prices under this new assumption, and establish whether the market is more or less liquid than in the model of section 10.4.1, by comparing the bid-ask spread that you have computed with that of equation (10.12). Explain the intuition behind your finding.

6. Incentive pay when stock price is imperfectly informative

Consider an extension of the model of section 10.4.2 in which the stock market is informed (i.e., able to observe the manager's effort) only with frequency π, and is uninformed with the complementary probability $1 - \pi$.

a. Show that if shareholders write an incentive contract conditional on the stock price only, the compensation that they will have to pay is inversely related to π (the proportion of time that the stock market observes effort). [Hint: in your analysis you may assume that the stock **(p.388)** market rationally expects the manager to exert effort if his pay contract respects his incentive constraint.]
b. (Harder problem) Assuming that $\pi < 1$, investigate whether shareholders can obtain a better outcome by conditioning pay $w(P, v)$ on *both* the interim stock price *and* the final value of the firm. What would the pay contract look like?

Notes:

(1.) By assumption, $G - C/\phi > 0 > -S_1$, so intervention will always occur with positive probability, from equation (10.3).

(2.) Edmans (2009) points out an additional reason why liquidity and monitoring by large shareholders are complementary, rather than anthitetical. Since in a liquid market a large shareholder has low exit costs after bad investment decisions, his trading decisions can convey more information to the market about the quality of managerial decisions and so reinforce the discipline to which managers are subject. By the same token, liquidity enhances the signal conveyed by the blockholder's retention of his stake when the firm books low earnings: precisely because he could sell at little cost, his decision not to do so signals that the fundamental value of the firm is sound, and keeps the stock price well aligned with fundamentals. Edmans also shows that when the blockholder chooses his stake optimally, greater market liquidity induces the blockholder to choose a larger stake, which makes the stock price more informative about the long-run payoff of the firm and forces management to avoid short-termist investment behavior.

(3.) These two probabilities can be obtained from equations (3.16) and (3.17), which describe the updating of the dealer's beliefs in Chapter 3. Setting the prior probability of the stock being high value $\theta_{t-1} = 1/2$ yields the updated probabilities $\theta_t^+ = \Pr(H|p = a)$ and $\theta_t^- = \Pr(H|p = b)$.

(4.) This type of manipulation can happen, however in more complex models with feedback from the stock market to investment decisions, as shown by Goldstein and Gümbel (2008). In this case, instead of contributing to the efficient allocation of corporate investment, the signals issued by stock market prices would impair it.

(5.) The equilibrium spread in (10.12) is positive, since we assume that condition (10.5) holds, that is, $\pi \geq \frac{I-G}{I+G}$ and, by the same token, $1 - \pi \leq \frac{2G}{I+G}$. Substituting these two expressions for π and $1 - \pi$ into equation (10.5), we find that $S \geq \gamma G(I - G)/(I + G)$, which is non-negative, so that $S \geq 0$ for any π that satisfies condition (10.5).

(6.) The argument developed so far has an important limitation: it takes the frequency of informed trading as a given parameter π, whereas in reality it could be endogenous. In markets where there is more trading by uninformed investors, speculators can obtain higher returns on their information, and so have more incentive to acquire information on the quality of firms' investment projects. Since more uninformed trading also tends to increase liquidity (as in Kyle's model), changes in it may result in a positive correlation between liquidity and the frequency of informed trading, in contrast with the tradeoff between liquidity and the allocative value of prices described in the text.

(7.) Empirical studies on this issue include Durnev, Morck, and Yeung (2004), Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), and Foucault and Frésard (2012).

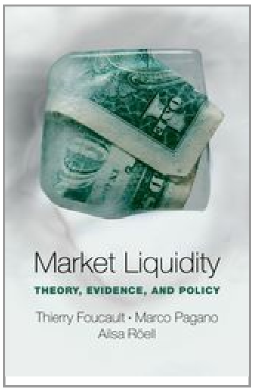
(8.) Other governance mechanisms that can realign the executive's incentives with those of shareholders are the risk of termination (i.e., managerial turnover) or career concerns (performance-based advancement).

(9.) Inflicting a penalty on an underperforming manager is equivalent to paying him a negative salary. If shareholders could pay a negative salary $w^L < 0$ to a manager for poor performance, they could meet his incentive constraint by paying him $w^H = c/\Delta\theta + w^L$ if $V = V^H$ and w^L if $V = V^L$. But now they can also reduce his wages to just barely meet his participation constraint by setting his average wage, net of effort cost, to zero: $\bar{\theta} w^H + (1 - \bar{\theta}) w^L = c$, so as to just compensate him for the cost of effort. Hence, the optimal manager's compensation would be $w^H = c(1 - \theta)/\Delta\theta > 0$ if $V = V^H$ and $w^L = -c\bar{\theta}/\Delta\theta < 0$ if $V = V^L$.

(10.) The difference is $\Delta\theta\Delta V - c > 0$. The proof is left to the reader as an exercise.

(11.) Another potentially harmful effect of disclosure is that when it refers to the ownership and control structure of the firm, it may discourage the accumulation of large blocks of shares and so hinder shareholder activism (section 10.3). Of course, this effect is harmful for companies only insofar as shareholder activism is value-increasing.

(12.) Concretely, equation (10.9) shows that if any original investor selling the stock receives a lower (bid) price, π is higher (a price lower than the average if the firm were private, by equation (10.6)). Equation (10.8) shows that any new uninformed investor wishing to replace him would need to pay more, π would be higher. These anticipated trading costs will push the price of the stock down.



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Thierry Foucault, Marco Pagano, and Ailsa Roell

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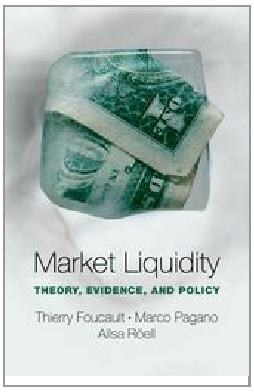

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Thierry Foucault, Marco Pagano, and Ailsa Roell

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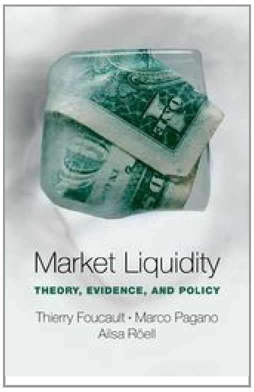

Market Liquidity: Theory, Evidence, and Policy

Thierry Foucault, Marco Pagano, and Ailsa Roell

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