Cross Currency Swap Theory & Practice - An Illustrated Step-by-Step Guide of How to Price Cross Currency Swaps and Calculate the Basis Spread

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Abstract

A Cross Currency Swap (CCS) is a financial instrument that allows investors to exchange a set of cashflow liabilities for an equivalent set in another currency, often USD. Investors trade CCS to secure cheaper funding, hedge FX exposures, manage liquidity risk and of course for speculative purposes.

In this paper we review the CCS product, its features and risks. We show how to price CCS and provide the mathematical formulae with examples & illustrations. Furthermore we outline how to calculate the CCS Basis Spread, which is how CCS are quoted in the financial marketplace.

Disclaimer

This paper has been based upon publicly available information and contains no copyrighted materials. The information contained is intended for illustrative purposes only.

Notation

CCS	An abbreviation for a Cross Currency Swap
ccy	An abbreviation for currency, used to indicate the trade leg currency
c^{CSA}	The CSA currency or collateral posting currency
c^{Reset}	The notional reset currency
c^{Val}	The valuation or pricing currency
DOM	The domestic or money currency
FOR	The foreign or asset currency
$f(t)^{FOR/DOM}$	Forward FX rate at time t for the foreign/domestic currency pair
l_j	The floating rate corresponding to the jth coupon Usually fixed in at the start of the coupon period
MM	An abbreviation for millions e.g. USD 1MM equals USD 1,000,000
m	The total number of float coupons
n	The total number of fixed coupons
N_t^{ccy}	The notional at time t for the of the CCS trade leg with currency ccy
$P(s,t)^{ccy-CSA}$	Discount factor for the ccy currency collateralized in the CSA currency, where s represents the pricing date which we often set as zero and $s < t$
$PV(\Omega_{ccy})$	The PV of the trade leg with currency ccy
$PV(Cpn, \Omega_{ccy})$	The PV of coupons only for trade leg with currency ccy
$PV(Exch, \Omega_{ccy})$	The PV of notional exchanges only for trade leg with currency ccy
$PV(Resets, \Omega_{ccy})$	The PV of notional resets only for trade leg with currency ccy
$PV(\Omega_{DOM})$	The PV of the domestic currency leg
$PV(\Omega_{FOR})$	The PV of the CCS foreign currency leg
$PV(\Omega_{Xccy})$	The PV of the Cross Currency Swap
s_{ccy}	The floating basis spread over Libor on the trade leg in currency ccy, quoted in basis points bps i.e. 1/100th of a percent
$s^{FOR/DOM}$	Spot FX rate for the foreign / domestic currency pair. This is the amount of domestic currency need to purchase 1 unit of the foreign currency
t_i	The time in years to the ith fixed coupon payment date
t_j	The time in years to the jth float coupon payment date
Ω_{ccy}	The trade leg with currency ccy
Ω_{DOM}	The domestic leg
Ω_{FOR}	The foreign leg
Ω_{Xccy}	The Cross Currency Swap
ϕ	An indicator function used to reflect trade direction: +1 for when receiving the foreign and paying domestic currency -1 for when receiving the domestic and paying foreign currency
$\Psi(t, ccy)$	The FX notional adjustment factor at time t to be applied to the trade leg with currency ccy
τ_i	Accrual period or year fraction of the ith fixed coupon
τ_j	Accrual period or year fraction of the jth floating coupon
$\mathbb{1}_{\{A\}}$	The kronecker delta function: 1 if A is true, 0 otherwise

Table 1: Notation

1 Cross Currency Swaps

A Cross Currency Swap (CCS) is a financial instrument, whereby cashflow liabilities in two different currencies are exchanged. This provides investors with a mechanism to switch borrowing to a more favourable currency, other uses include:

- To secure cheaper debt
- As a liquidity tool
- To hedge or reduce forward exchange rate calculations
- For speculative purposes

Firstly we present the CCS product and discuss the transaction risks, namely counterparty credit risk, FX and interest rate risk. Secondly We provide an example of a typical funding problem that can be solved using Cross Currency Swaps, which we use for illustrative purposes to discuss the CCS product and trade risk management features. Thirdly we demonstrate how to price cross currency swaps taking a step-by-step approach to pricing CCS and its components individually. Fourthly we draw attention to the fact that Cross currency swaps are quoted and referenced in the marketplace by their floating par spread or fixed par rate, which is the break-even Libor spread or fixed rate that makes a CCS trade price to par. We conclude by deriving and explaining how to calculate and quote CCS par-spreads and par-rates.

1.1 Funding Scenario

Suppose we have two counterparties A and B, with A being a European Investor with cheap access to EUR funding and B being a North-American Investor with cheap access to USD funding.

Both investors wish to borrow for overseas projects; the European investor wishes to borrow USD 1MM[1] and the North-American investor wishes to borrow EUR 0.875MM (equivalent of USD 1MM). However both investors have poor access to overseas funding, with disfavourable overseas borrowing rates.

One solution to the above problem would be to enter into a Cross Currency Swap, such as the 1Y CCS in section (2.1), whereby each party borrows the funds they require in their local markets where they have a competitive advantage. European party A would borrow EUR 0.875MM and North-American party B the FX equivalent amount of USD 1MM. Investors then enter a Cross Currency Swap whereby they exchange the notional principal borrowed and funding obligations.

European Party A receives USD and pays USD interest and North-American Party B receives EUR and pays EUR interest. On maturity date at the end of the transaction party A returns the USD 1MM USD borrowed and likewise party B returns EUR 0.875MM.

[1] As per market convention we use MM to denote a notional in millions.

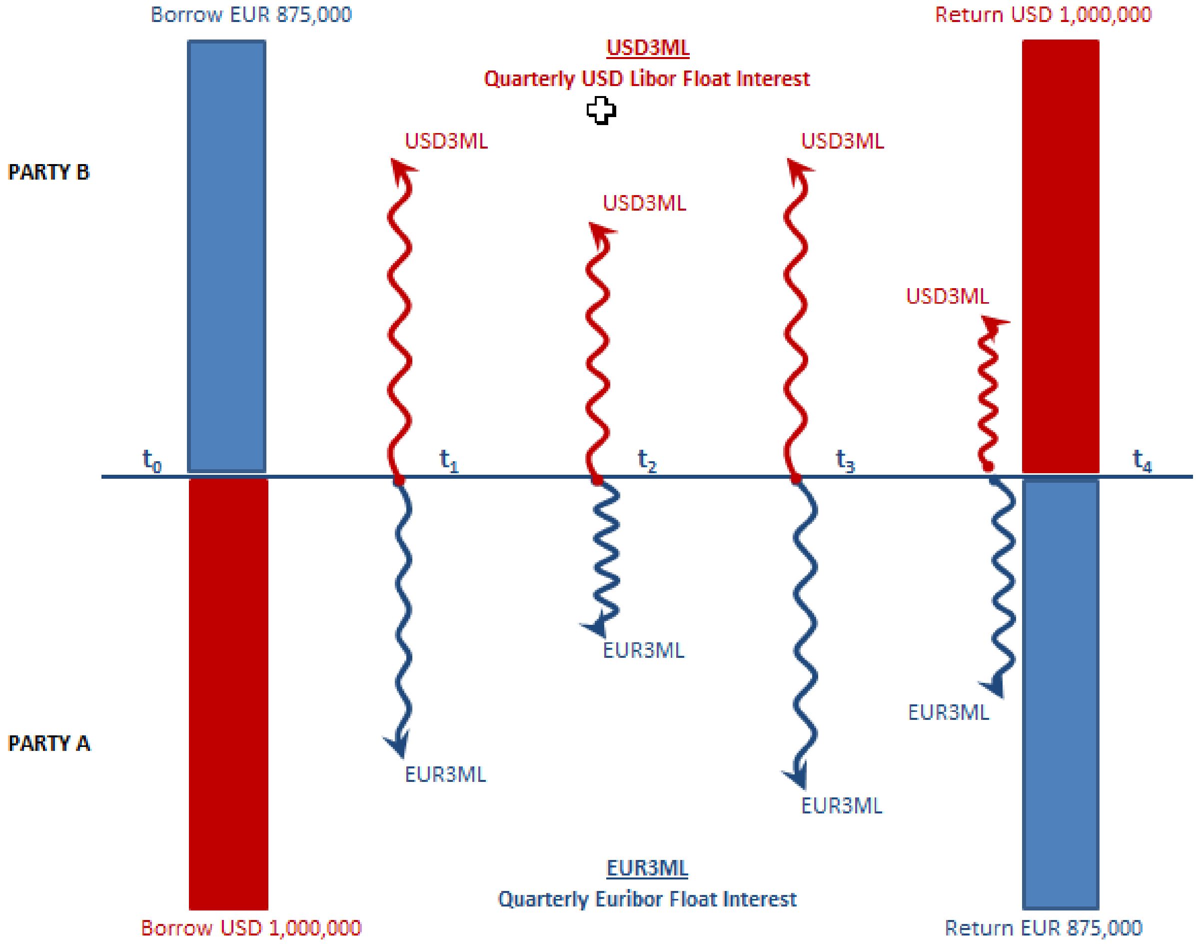

Figure 1: EUR/USD 1 Year Cross Currency Swap Cashflow Diagram

1.2 Cross Currency Swap Risks

In the above funding example investors entered a CCS transaction to satisfy overseas funding requirements at a more competitive funding levels. After entering the CCS transaction however there are market risks that need to be carefully and actively managed.

Over the life of the CCS transaction ceteris paribus, all things remaining equal, there are minimal risks. However fast moving and volatile markets present risk to CCS investors. The main sources of risk include:

1. Counterparty Default Risk
2. Credit Risk
3. FX Risk
4. Interest Rate Risk

The counterparty may default or the likelihood of default my increase presenting elevated levels of credit risk and XVA[2] and charges.

We also have FX risk. On trade date in the example above investors were exchanging notionals of USD 1MM which was equivalent to EUR 0.875MM. FX rates may move adversely during the life of the trade. Perhaps USD 1MM will increase / decrease in value relative to EUR.

Similarly we have interest rate risk. Interest rates may increase resulting in elevated borrowing costs.

2 Features

In this section we review CCS product features and describe how each feature is evaluated and influences the trade pricing. The cross currency swap product has several interesting trade features, which can be tailored to mitigate and reduce credit and market risks. However market standard cross currency swaps typically trade as marked-to-market with float-float interest, meaning we are rebalancing trade notionals each coupon period to settle notional differences resulting from FX market movements and paying floating interest coupons on both trade legs.

Cross currency swaps are bilateral OTC contracts between two investors and are bespoke and customizable with the following features:

- Fixed or Floating Interest
- CSA[3] Collateral Posting
- Forward FX Rates
- Marked-to-Market
- Notional Resets

2.1 Cross Currency Example Trade

In this section we review the different trade features of cross currency swaps by considering a 1Y market-to-market EUR/USD CCS with trade details and cashflows as shown figures (2) and (3) below. This trade provides a potential solution to the funding problem presented in section (1.1) above. Throughout this paper will refer to this example to illustrate cross currency swap trade features and pricing.

[2]XVA is a collective term for credit charges called x Valuation Adjustments, where X can be Credit, Debit, Margin, Funding, Capital and other similar credit valuation adjustments.

[3]Credit Support Annex, see [3] for more information.

Cross Currency Swap, Ω_{Xccy}

TradeDate	Fri, 26-Oct-18	
Maturity (Years)	5Y	Wed, 25-Oct-23
Trade Notional	1,000,000	
Trade Currency	USD	
MtM	YES	
NotionalExchanges	YES	
Reset Currency	USD	USD
CSA Currency	USD	
Valuation Currency	USD	
SpotFX	1.14030	USD/EUR
LegCurrency	EUR	USD
LegNotional	876,962	1,000,000
PayOrReceive	PAY	RECEIVE
LegType	FLOATING	FLOATING
RateOrSpread (%)	0.00000%	0.00000%
FloatIndex	EUR EURIBOR 3M	USD LIBOR 3M
Frequency	QUARTERLY	QUARTERLY
LegResetsRequired	NO	YES
LegSpotFX	0.87696	1.14030
ValuationFXAdj	1.14030	1.00000
DaycountBasis	ACT/360	ACT/360
UseMarketSchedule	NO	NO

Figure 2: EUR/USD 1 Year Cross Currency Swap Trade

Leg1 - EUR Cashflows

	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				876,962			876,962	1.000000	876,962	1.1403	1,000,000
1	-876,962	Fri, 26-Oct-18	0.87696	0	0.00000%	-0.31695%	703	1.002365	704	1.1403	803
2	-876,962	Fri, 25-Jan-19	0.86980	0	0.00000%	-0.31644%	701	1.004182	704	1.1403	803
3	-876,962	Fri, 26-Apr-19	0.86287	0	0.00000%	-0.28931%	641	1.005926	645	1.1403	736
4	-876,962	Fri, 26-Jul-19	0.85560	-876,962	0.00000%	-0.22709%	-876,459	1.007807	-883,301	1.1403	-1,007,229

Leg2 - USD Cashflows

	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				-1,000,000			-1,000,000	1.000000	-1,000,000	1.0000	-1,000,000
1	1,000,000	Fri, 26-Oct-18	1.14030	-8,233	0.00000%	2.47475%	-1,977	0.994180	-1,966	1.0000	-1,966
2	1,008,233	Fri, 25-Jan-19	1.14969	-8,104	0.00000%	2.79581%	-979	0.988041	-967	1.0000	-967
3	1,016,337	Fri, 26-Apr-19	1.15893	-8,635	0.00000%	2.93764%	-1,088	0.981419	-1,067	1.0000	-1,067
4	1,024,972	Fri, 26-Jul-19	1.16878	1,024,972	0.00000%	3.05383%	1,032,884	0.974803	1,006,858	1.0000	1,006,858

Figure 3: EUR/USD 1 Year Cross Currency Swap Cashflows

2.2 Fixed or Floating Interest

In a CCS transaction there are two trade legs with coupons grouped by currency and described as a trade leg. Investors specify how they wish to borrow / lend funds i.e. paying fixed or floating interest. CCS trades are specified as either Float-Float, Fixed-Float or Fixed-Fixed referring to the investor interest rate preferences and market views.

In the example outlined in section (1.1) the transaction parties A and B were exchanging EUR and USD notionals and funding liabilities. The European party A is borrowing USD can choose to pay fixed or floating USD Libor interest and likewise the North-American party B is borrowing EUR and can chose to pay fixed or EUR Euribor interest.

Market standard CCS trades are typically Float-Float i.e. both parties pay floating interest however investors can chose to pay fixed instead of floating interest, which is tantamount to entering an Interest Rate Swap (IRS) to swap float for fixed cashflows.

Fixed rate interest payers (receivers) benefit when interest rates rise (fall) and Libor floating rate payers (receivers) benefit when interest rates fall (rise). In a CCS transaction investors are naturally exposed to and take a view on future interest rate movements when deciding whether to pay fixed or floating coupon interest.

2.3 CSA Collateral Posting

CCS transactions are OTC transactions dealt bilaterally between two counterparties. In the OTC markets investors are required to post collateral to cover liabilities from open trading positions, see [3] for a comprehensive summary. As outlined in [4] the yield curves used for pricing CCS should be adjusted to account for CSA collateral posting.

Collateral posting also has the effect of reducing the borrowing rate / increasing discount factors. Discount factors are denoted $P(s,t)$ with pricing date s, evaluation date t and $s \leq t$. A discount factor represents the value of a unit cashflow paid on payment date t evaluated on the pricing or valuation date s. We usually set $s = 0$ to indicate today as the valuation date. Discount factors have an associated cashflow currency C^{ccy} and CSA collateral currency C^{CSA}, hence for in this paper we write discount factors as,

$$P(0,t)^{C^{ccy}-C^{CSA}}$$

For a EUR cashflow to be paid in 1 year's time being part of a trade collateralized using a USD CSA we would denote a EUR 1Y discount factor as follows,

$$P(0,1Y)^{EUR-USDCSA}$$

and likewise a USD 1Y discount factor for a a USD cashflow paid in 1 year's time would be represented as,

$$P(0,1Y)^{USD-USDCSA}$$

2.4 Forward FX Rates

In the cross currency swap we are required to FX adjust our cashflows into a common valuation currency denoted C^{Val}. All PVs are converted into the valuation currency using spot FX rates i.e. the current market FX rate. The valuation currency is often USD.

FX rates are quoted as Foreign/Domestic, which we denote FOR/DOM, whereby the market quotes the amount of domestic currency that is required to purchase 1 unit of the foreign currency. Some market participants also refer to the pair as Asset/Money indicating the amount of money that must be paid to receive 1 unit of the asset.

For example a quote of EUR/USD 1.143 would indicate that we need to pay USD 1.143 to receive EUR 1 and likewise a quote of USD/EUR 0.875 indicates that we need to pay EUR 0.875 to receive USD 1. FX rates are often quoted to several decimal places called the pip size, with for example a pip size of 5 indicating that we quote to 5 decimal places, see [6] for further information.

When calculating FX Forward rates for cross currency swap calculations we assume FX forward invariance as outlined in [4]. This is to say that we can imply FX Forward rates from discount factors regardless of the collateral posted using the following relationship.

$$f(t)^{FOR/DOM} = s^{FOR/DOM}\left(\frac{P(0,t)^{FOR-CCYCSA}}{P(0,t)^{DOM-CCYCSA}}\right) \tag{1}$$

where $s^{FOR/DOM}$ is the spot rate and $f(t)^{FOR/DOM}$ is the forward FX rate at time t for the foreign / domestic currency pair denoted FOR/DOM and $CCYCSA$ is the CSA collateral posting currency.

The above relationship (1) assumes that for a fixed CSA we can replicate the forward FX at time t by borrowing domestic funds to purchase spot FX and then deposit the resulting foreign currency purchased until time t. We can imply the forward FX rate using (1) for any CSA collateralization. For example the foreign / domestic currency pair EUR/USD we have the following:

$$f(t)^{EUR/USD} = \begin{cases} s^{EUR/USD}\left(\frac{P(0,t)^{EUR-USDCSA}}{P(0,t)^{USD-USDCSA}}\right) & \textit{, if using USD CSA} \\ s^{EUR/USD}\left(\frac{P(0,t)^{EUR-EURCSA}}{P(0,t)^{USD-EURCSA}}\right) & \textit{, if using EUR CSA} \\ s^{EUR/USD}\left(\frac{P(0,t)^{EUR-JPYCSA}}{P(0,t)^{USD-JPYCSA}}\right) & \textit{, if using JPY CSA} \\ s^{EUR/USD}\left(\frac{P(0,t)^{EUR-AUDCSA}}{P(0,t)^{USD-AUDCSA}}\right) & \textit{, if using AUD CSA} \end{cases} \tag{2}$$

Considering the 1Y cross currency swap from section (2.1) we can derive the EUR/USD forward FX rates using formula (1) as shown below.

ForwardFX EUR/USD

FixingDate	$s^{EUR/USD}$	$P(t_0, t_i)^{EUR\_USDCSA}$	$P(t_0, t_i)^{USD\_USDCSA}$	$f(t_i)^{USD/EUR}$
Fri, 26-Oct-18	1.14030	1.000000	1.000000	1.14030
Fri, 25-Jan-19	1.14030	1.002365	0.994180	1.14969
Fri, 26-Apr-19	1.14030	1.004182	0.988041	1.15893
Fri, 26-Jul-19	1.14030	1.005926	0.981419	1.16878

Figure 4: EUR/USD Forward FX Calculations

2.5 Marked-to-Market, MtM

At inception on CCS trade date the notional principals in different currencies are exchanged and later returned on CCS maturity date; this presents FX and counterparty default risk. To reduce these risks marked-to-market CCS require notional resets on every coupon period to rebalance FX exposures in the transaction, which also reduces the credit risk. Each coupon notional is rebalanced or restruck so that the trade leg notionals in each currency are equivalent. This is achieved by resetting the notionals to their FX equivalent values and settling the difference as a notional exchange. Naturally on the FX forward reset date the reset notional is fixed and is no longer sensitive to FX market movements.

In a marked-to-market CCS one trade leg will have a constant notional and the other trade leg will have the equivalent FX forward adjusted notional. The trade leg with the variable notional is called the reset leg. The change in notional is called the notional reset or notional exchange. Investors are required to settle this difference in notional on a cashflow by cashflow basis.

2.6 Notional Resets

Before executing a cross currency trade investors must specify the reset leg and currency. The non-reset leg will have a constant notional and the reset leg notional will vary with market FX Forward rates. The non-reset leg notional is projected daily until the FX forward fixing date upon which the notional is fixed converted into the reset leg notional at the prevailing Forward FX fixing rate in the market. This is done for every coupon notional on the reset leg.

In our example funding scenario in section (1.1) USD 1MM[4] was exchanged for EUR 0.875MM, based on an FX Spot exchange rate of USD/EUR of 0.875. At maturity the same notional amounts are exchanged regardless of where the FX is trading at maturity.

If at maturity FX rates fall to USD/EUR 0.700 the trade notional of USD 1MM is now worth EUR 0.7MM and has decreased in value by EUR 0.175MM. Therefore at maturity the European investor borrowing the USD 1MM can purchase USD 1MM in the open FX markets at maturity for EUR 0.7MM saving EUR 0.175MM compared the EUR 0.875MM initial outlay on trade date. On the flip-side the investor returning EUR is required to pay more in USD to purchase the same amount of EUR.

Over the lifetime of a non-MtM CCS trade as FX rates move one side of the transaction will gain in value due whilst the other depreciates. Parties can hedge this risk in the FX markets, however the CCS trade itself will incur XVA credit charges and require collateral posting to mitigate the credit risk exposure, see [3]. The credit charges can be substantial and therefore MtM CCS was introduced with Notional Resets to reset the FX risk.

MtM CCS trades have notional resets by default, whereby on every coupon period the FX difference in notional value is cash settled. This immunizes investors to long term FX risk. Notional resets are typically applied on the CCS trade leg denominated in the valuation currency C^{Val}.

[4]Here we use the market standard terminology of MM to denote notionals in millions.

Notional resets for the **foreign** currency leg are calculated by converting the initial **domestic** currency notional on trade date by the **foreign** coupon forward FX rate as follows,

$$N_{t_i}^{FOR} = N_{t_0}^{DOM} f(t)^{FOR/DOM} = \left(\frac{N_{t_0}^{FOR}}{s^{FOR/DOM}} \right) f(t)^{FOR/DOM} \tag{3}$$

giving,

$$N_{t_i}^{FOR} = N_{t_0}^{FOR} \left(\frac{f(t)^{FOR/DOM}}{s^{FOR/DOM}} \right) \tag{4}$$

Similarly notional resets for the **domestic** currency leg are calculated by converting the initial **foreign** currency notional on trade date by the **domestic** coupon forward FX rate.

$$N_{t_i}^{DOM} = N_{t_0}^{FOR} f(t)^{DOM/FOR} = \left(\frac{N_{t_0}^{DOM}}{s^{DOM/FOR}} \right) f(t)^{DOM/FOR} \tag{5}$$

leading to,

$$N_{t_i}^{DOM} = N_{t_0}^{DOM} \left(\frac{f(t)^{DOM/FOR}}{s^{DOM/FOR}} \right) \tag{6}$$

For illustration purposes using the trade example from section (2.1) we calculate the reset leg notionals and corresponding resets for the **foreign** USD leg of the trade as follows,

USD Notional Resets

	Notional^{EUR}	FXFixingDate	ForwardFX	Notional^{USD}	$\text{NotionalReset}^{USD}$
0					
1	876,962	Fri, 26-Oct-18	1.14030	1,000,000	-8,233
2	876,962	Fri, 25-Jan-19	1.14969	1,008,233	-8,104
3	876,962	Fri, 26-Apr-19	1.15893	1,016,337	-8,635
4	876,962	Fri, 26-Jul-19	1.16878	1,024,972	1,024,972

Figure 5: Example Calculation of Marked-to-Market Notional Resets

3 Pricing

Cross Currency Swap pricing notation can be quite challenging to write concisely to capture all transaction features. CCS comprise of the following components for which we proceed to explain the pricing taking a step-by-step approach,

- Trade Coupons - Notional Resets
- Trade Coupons - Conversion to the Valuation Currency
- Principal Notional Exchanges
- Notional Reset Exchanges

In what follows we proceed with a discussion on trade notation, which can be quite tricky to notate all the CCS trade dynamics accurately. Secondly we discuss FX forward calculations and a notional reset factor Ψ; we introduce the later for notional ease. Thirdly we continue to discuss the pricing of the coupon, notional exchange and reset components listed above. We conclude by presenting a fully generic pricing formulae for CCS pricing and par spread calculations.

3.1 Trade Notation, Ω

Cross currency swap notation is non-trivial with many trade parameters requiring specification. In this paper we simplify the notation by defining the collection of trade parameters as Ω_{Xccy}. Likewise we define the collection of CCS foreign currency and domestic currency leg parameters as Ω_{FOR} and Ω_{DOM} respectively.

This allows us to conveniently express trade features such as the following,

Notation	Feature
$\Omega_{Xccy} = MTM$	Marked-to-Market CCS
Ω_{ccy}	CCS trade leg denominated in currency ccy
$\Omega_{ccy} = \textit{Fixed}$	Fixed trade leg denominated in currency ccy
$\Omega_{ccy} = \textit{Float}$	Float trade leg denominated in currency ccy

which provides particularly useful CCS pricing notation,

Notation	Feature
$PV(\Omega_{Xccy} = MTM)$	PV of a Marked-to-Market CCS
$PV(\Omega_{FOR})$	PV of the foreign currency leg
$PV(\Omega_{DOM})$	PV of the domestic currency leg
$PV(\Omega_{FOR} = \textit{Fixed})$	PV of the fixed coupon foreign currency leg
$PV(\Omega_{DOM} = \textit{Float})$	PV of the float coupon domestic currency leg

3.2 Forward FX Rates

For the purposes of CCS pricing forward FX rates must be kept arbitrage free as outlined in [1] and [4]. As shown in section (2.4) we can imply the forward FX rate from curve discount factors and spot FX rates as follows,

$$f(t)^{\textit{FOR/DOM}} = s^{\textit{FOR/DOM}} \left(\frac{P(0,t)^{FOR-CSA}}{P(0,t)^{DOM-CSA}} \right) \tag{7}$$

where FOR indicates the foreign currency, DOM the domestic currency and CSA the CSA collateral posting currency. For a good review of the FX market we refer the reader to [5] and [6].

Remark: FX Quote Conventions
An FX spot quote is quoted as FOR/DOM whereby we specify how many domestic (DOM) currency units we must pay for 1 foreign (FOR) currency unit. So a quote of USD/EUR 0.875

indicates that USD 1 is equal to EUR 0.875. Likewise EUR/USD 1.142 indicates that EUR 1 is worth USD 1.142. As an alternative to Foreign/Domestic some market participants refer to the FX pair as Asset/Money, where we pay the money currency to receive 1 unit of the asset currency.

3.3 Notional Reset Factor, Ψ

In this paper we introduce new terminology namely the notional reset factor Ψ to facilitate notational brevity. The notional reset factor has a dual role to convert foreign cashflows into the valuation currency and also incorporates the notional reset feature as described in sections (2.4) and (2.6), which we expand upon below.

Cross currency swaps have two trade legs with cashflows in different currencies, however we are required to evaluate the price of all CCS cashflows in the valuation currency denoted C^Val. Cashflows not denominated in the valuation currency are required to converted into the valuation currency by the prevailing spot FX rate $s^{FOR/DOM}$. For a given coupon payment at time t with currency C^{Leg} we have,

$$\alpha(t, C^{Leg}) = \begin{cases} 1 & \text{, } \textit{if } C^{Leg} = C^{Val} \\ s^{FOR/DOM} & \text{, } \textit{if } C^{Leg} \neq C^{Val} \textit{ and } C^{Leg} = C^{FOR} \\ s^{DOM/FOR} & \text{, } \textit{if } C^{Leg} \neq C^{Val} \textit{ and } C^{Leg} = C^{DOM} \end{cases} \tag{8}$$

For MtM CCS we are required to apply a notional reset adjustment $\beta(t, C^{Leg})$ as outlined in section (2.6) and equations (4) and (6) for the foreign and domestic trade legs respectively. The marked-to-market notional reset adjustments are given by,

$$\beta(t, C^{Leg}) = \begin{cases} 1 & \text{, } \textit{if } C^{Leg} \neq C^{Reset} \\ \left(\frac{f(t)^{FOR/DOM}}{s^{FOR/DOM}}\right) & \text{, } \textit{if } C^{Leg} = C^{Reset} \textit{ and } C^{Leg} = C^{FOR} \\ \left(\frac{f(t)^{DOM/FOR}}{s^{DOM/FOR}}\right) & \text{, } \textit{if } C^{Leg} = C^{Reset} \textit{ and } C^{Leg} = C^{DOM} \end{cases} \tag{9}$$

For notional ease we combine the valuation adjustment $\alpha(t, C^{Leg})$ and the notional reset adjustment $beta(t, C^{Leg})$ into a single factor, which we call the notional reset factor $Psi(t, C^{Leg})$ as follows,

$$\Psi(t, C^{Leg}) = \underbrace{\alpha(t, C^{Leg})}_{\textit{Valuation Adj}} \underbrace{\beta(t, C^{Leg})}_{\textit{Notional Reset Adj}} \tag{10}$$

Note we sometimes use the notation $\Psi(t, C^{Leg})$ and $\Psi(t)^{C^{Leg}}$ interchangeably for brevity, e.g. $\Psi(t)^{EUR}$ or $\Psi(t)^{USD}$.

Remark: Forward Starting CCS
Note for forward starting CCS we would typically adjust the notional reset expression $\beta(t, C^{Leg})$ by replacing the spot term $s^{DOM/FOR}$ with the forward FX rate as observed at the trade start date $f(t_0)^{DOM/FOR}$.

Again as an illustration using the trade example from section (2.1) we calculate the notional scaling factor $\Psi(t, C^{Leg})$ as follows,

Notional Scaling Factor, ψ(t,EUR)

	Notional	$s^{USD/EUR}$	$f(t)^{USDEUR}$	α(t,EUR)	β(t,EUR)	Psi(t,EUR)	NotionalAdj
0	876,962						
1	876,962	1.1403	1.1403	1.1403	1.0000	1.1403	1,000,000
2	876,962	1.1403	1.1403	1.1403	1.0000	1.1403	1,000,000
3	876,962	1.1403	1.1403	1.1403	1.0000	1.1403	1,000,000
4	876,962	1.1403	1.1403	1.1403	1.0000	1.1403	1,000,000

Notional Scaling Factor, ψ(t,USD)

	Notional	$s^{USD/EUR}$	$f(t)^{USDEUR}$	α(t,USD)	β(t,USD)	Psi(t,USD)	NotionalAdj
0	1,000,000						
1	1,000,000	1.1403	1.1403	1.0000	1.0000	1.0000	1,000,000
2	1,000,000	1.1403	1.1497	1.0000	1.0082	1.0082	1,008,233
3	1,000,000	1.1403	1.1589	1.0000	1.0163	1.0163	1,016,337
4	1,000,000	1.1403	1.1688	1.0000	1.0250	1.0250	1,024,972

Figure 6: Example Calculation of the Notional Reset Factor, Ψ

3.4 Coupons

Typically cross currency swaps are traded as float-float instruments. This is to say that the foreign and domestic trade legs pay floating interest respectively. However investors can chose to pay fixed or floating interest. We present the pricing formulae for fixed and floating interest coupons below.

Fixed rate coupons can be priced as follows,

$$PV(Cpn, \Omega_{ccy}{=}\mathit{Fixed}, r_{ccy}) = \sum_{i=1}^{n} N_{t_0}^{ccy} \Psi(t_i)^{ccy} r_{ccy} \tau_i P(0, t_i)^{ccy-CSA} \tag{11}$$

Similarly we can price Libor floating rate coupons as,

$$PV(Cpn, \Omega_{ccy}{=}\mathit{Float}, s_{ccy}) = \sum_{j=1}^{m} N_{t_0}^{ccy} \Psi(t_j)^{ccy} (l_j + s_{ccy}) \tau_j P(0, t_j)^{ccy-CSA} \tag{12}$$

In full generality for a given CSA agreement we price CCS coupons as,

$$PV(Cpn, \Omega_{ccy}) = \begin{cases} \sum_{i=1}^{n} N_{t_0}^{ccy} \Psi(t_i)^{ccy} r_{ccy} \tau_i P(0, t_i)^{ccy-CSA} & \text{, if } \Omega_{ccy} = \text{Fixed} \\ \sum_{j=1}^{m} N_{t_0}^{ccy} \Psi(t_j)^{ccy} (l_j + s_{ccy}) \tau_j P(0, t_j)^{ccy-CSA} & \text{, if } \Omega_{ccy} = \text{Float} \end{cases} \tag{13}$$

where $N_{t_0}^{ccy}$ denotes the trade notional on trade date.

Therefore for a Float-Float EUR/USD cross currency swap Ω_{Xccy} with a USD CSA we would evaluate the present value for the EUR leg Ω_{EUR} coupons as,

$$PV(Cpn, \Omega_{EUR}{=}\mathit{Float}, s_{EUR}) = \sum_{j=1}^{m} N_{t_0}^{EUR} \Psi(t_j)^{EUR} (l_j + s_{EUR}) \tau_j P(0, t_j)^{EUR-USDCSA} \tag{14}$$

and likewise the for the USD leg Ω_{USD},

$$PV(Cpn, \Omega_{USD}{=}Float, s_{USD}) = \sum_{j=1}^{m} N_{t_0}^{USD} \Psi(t_j)^{USD} (l_j + s_{USD}) \tau_j P(0, t_j)^{USD-USDCSA} \quad (15)$$

Considering the USD 1MM EUR/USD Float-Float MtM CCS trade example[5] in section (2.1) and using equations (14) and (15) produces the following cashflow and present value results,

Leg1 - EUR Coupons (C^{Val} = USD, C^{Reset} = USD, C^{CSA} = USD)

	Notional	SpotFX	ForwardFX	ValuationAdj	ResetAdj	NotionalAdj	YearFraction	FloatRate, l	Spread, s	Coupon	DiscFactor	CouponPV	Annuity
	N^{EUR}	$f(t_0)^{EUR/USD}$	$f(t)^{EUR/USD}$	α	β	ψ	τ	l	s^{EUR}	Cpn	$P(t_0,t)^{EUR-USDCSA}$	PVlet	PV01
1	-876,962	0.87696	0.87696	1.14030	1.00000	1.1403	0.2528	-0.31695%	0.00000%	801	1.002365	803	253,376
2	-876,962	0.87696	0.86980	1.14030	1.00000	1.1403	0.2528	-0.31644%	0.00000%	800	1.004182	803	253,835
3	-876,962	0.87696	0.86287	1.14030	1.00000	1.1403	0.2528	-0.28931%	0.00000%	731	1.005926	736	254,276
4	-876,962	0.87696	0.85560	1.14030	1.00000	1.1403	0.2528	-0.22709%	0.00000%	574	1.007807	579	254,751
											Total	**2,920**	**1,016,238**

Leg2 - USD Coupons (C^{Val} = USD, C^{Reset} = USD, C^{CSA} = USD)

	Notional	SpotFX	ForwardFX	ValuationAdj	ResetAdj	NotionalAdj	YearFraction	FloatRate, l	Spread, s	Coupon	DiscFactor	CouponPV	Annuity
	N^{USD}	$f(t_0)^{USD/EUR}$	$f(t)^{USD/EUR}$	α	β	ψ	τ	l	s^{USD}	Cpn	$P(t_0,t)^{USD-USDCSA}$	PVlet	PV01
1	1,000,000	1.14030	1.14030	1.00000	1.00000	1.0000	0.2528	2.47475%	0.00000%	6,256	0.994180	6,219	251,307
2	1,000,000	1.14030	1.14969	1.00000	1.00823	1.0082	0.2528	2.79581%	0.00000%	7,125	0.988041	7,040	251,811
3	1,000,000	1.14030	1.15893	1.00000	1.01634	1.0163	0.2528	2.93764%	0.00000%	7,547	0.981419	7,407	252,134
4	1,000,000	1.14030	1.16878	1.00000	1.02497	1.0250	0.2528	3.05383%	0.00000%	7,912	0.974803	7,713	252,562
											Total	**28,379**	**1,007,813**

Figure 7: Example Coupon Calculations for 1Y EUR/USD MtM CCS

Remark: Negative Interest Rates
Note that in the above example shown in figure (7) we receive both the EUR and USD coupons. The EUR leg of the transaction pays floating Euribor interest, however since Euribor rates are negative this means we receive the coupons instead of paying them. Whilst this may seem rather odd, this is indeed correct and the norm for currencies and markets with negative interest rates such as EUR and JPY.

3.5 Notional Exchanges

Notional exchanges for a CCS trade Ω_{Xccy} are simply the upfront borrowing of funds and the subsequent return of funds on a trade leg or currency basis. Additionally notional exchanges need to be spot FX converted into the valuation currency.

We capture the spot FX valuation adjustment(s) via the notional reset parameter Ψ as outlined in section (3.3). Note here we exclude notional exchanges arising from Notional Resets and discus this feature separately.

[5] This is shorthand to say we have a Marked-to-Market Cross Currency Swap with USD 1,000,000 notional with Floating EUR and Floating USD coupons.

Mathematically we price notional exchanges as follows,

$$PV(Exch, \Omega_{ccy}) = \underbrace{N_{t_0}^{ccy} \Psi(t_n)^{ccy} P(0,t_n)^{ccy-CSA}}_{\text{Final Notional Exchange}} - \underbrace{N_{t_0}^{ccy} \Psi(t_0)^{ccy} P(0,t_0)^{ccy-CSA}}_{\text{Upfront Notional Exchange}} \tag{16}$$

In the case of our EUR/USD CCS trade from section (2.1) and applying equation (16) to each trade leg individually we value the EUR leg notional exchanges as follows,

$$\begin{aligned} PV(Exch, \Omega_{EUR}) = \; & N_{t_0}^{EUR} \Psi(t_n)^{EUR} P(0,t_n)^{EUR-USDCSA} \\ & - N_{t_0}^{EUR} \Psi(t_0)^{EUR} P(0,t_0)^{EUR-USDCSA} \end{aligned} \tag{17}$$

and likewise the USD leg exchanges as,

$$\begin{aligned} PV(Exch, \Omega_{USD}) = \; & N_{t_0}^{USD} \Psi(t_n)^{USD} P(0,t_n)^{USD-USDCSA} \\ & - N_{t_0}^{USD} \Psi(t_0)^{USD} P(0,t_0)^{USD-USDCSA} \end{aligned} \tag{18}$$

Applying equations (17) and (18) to our example trade from section (2.1) leads to following results,

Leg1 - EUR Notional Exchanges C^{Val} = USD C^{Reset} = USD C^{CSA} = USD

	Initial Notional	SpotFX	ForwardFX	ValuationAdj	ResetAdj	NotionalAdj	NotionalExchange	DiscountFactor	PV
	$N(t_0)^{EUR}$	$f(t_0)^{EUR/USD}$	$f(t)^{EUR/USD}$	α	β	ψ	Exchange	$P(t_0,t)^{EUR\text{-}USDCSA}$	PVlet
0	-876,962	0.87696	0.87696	1.14030	1.00000	1.14030	1,000,000	1.000000	1,000,000
1	-876,962	0.87696	0.87696	1.14030	1.00000	1.14030	0	1.002365	0
2	-876,962	0.87696	0.86980	1.14030	1.00000	1.14030	0	1.004182	0
3	-876,962	0.87696	0.86287	1.14030	1.00000	1.14030	0	1.005926	0
4	-876,962	0.87696	0.85560	1.14030	1.00000	1.14030	-1,000,000	1.007807	-1,007,807
								Total	-7,807

Leg2 - USD Notional Exchanges C^{Val} = USD C^{Reset} = USD C^{CSA} = USD

	Initial Notional	SpotFX	ForwardFX	ValuationAdj	ResetAdj	NotionalAdj	NotionalExchange	DiscountFactor	PV
	$N(t_0)^{USD}$	$f(t_0)^{USD/EUR}$	$f(t)^{USD/EUR}$	α	β	ψ	Exchange	$P(t_0,t)^{USD\text{-}USDCSA}$	PVlet
0	1,000,000	1.14030	1.14030	1.00000	1.00000	1.00000	-1,000,000	1.000000	-1,000,000
1	1,000,000	1.14030	1.14030	1.00000	1.00000	1.00000	0	0.994180	0
2	1,000,000	1.14030	1.14969	1.00000	1.00823	1.00823	0	0.988041	0
3	1,000,000	1.14030	1.15893	1.00000	1.01634	1.01634	0	0.981419	0
4	1,000,000	1.14030	1.16878	1.00000	1.02497	1.02497	1,024,972	0.974803	999,145
								Total	-855

Figure 8: Example Notional Exchange Calculations for 1Y EUR/USD MtM CCS

3.6 Notional Resets

Notional resets are a marked-to-market CCS feature described in section (2.6). The notional reset compensates CCS investors for adverse movements in FX forward rates. The notional reset feature applies to one trade leg only, namely the leg with currency C^{Reset}. The reset currency is specified a priori as part of the CCS transaction and is often chosen to be the valuation currency giving $C^{Reset} = C^{Val}$.

One could say that the notional reset captures the FX forward performance of the reset leg notional. As outlined in section (3.3) we capture this feature on a trade leg basis both the spot FX valuation adjustment $\alpha(t, C^{Leg})$ and the FX forward reset performance adjustment $\beta(t, C^{Leg})$ as a single factor $\Psi(t, C^{Leg})$, which we call the notional reset factor and sometimes write as $\Psi(t)^{C^{Leg}}$ for brevity.

We calculate the both the notional reset for each coupon and the valuation currency adjustment using the notional reset factor, see equations (4) and (6) for the foreign and domestic currencies respectively. This gives,

$$N_{t_i}^{Ccy} = N_{t_0}^{Ccy}\Psi(t_i, Ccy) \tag{19}$$

Remark: Notional Reset Performance
Note that notional FX resets for each coupon at time t_i are referenced relative to the initial trade notional N_{t_0}.

On the reset leg the notional varies from coupon to coupon. If the leg notional increases in value the leg holder returns difference to the counterparty and vice versa if the notional decreases in value.

We evaluate the CCS reset leg using indicator or Kronecker delta functions to reflect that reset amounts are only exchanged for marked-to-market CCS and only on the reset currency C^{Reset} trade leg.

$$\begin{aligned} PV(\text{Reset}, \Omega_{ccy}) &= \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{ccy=C^{Reset}\}}\sum_{j=1}^{m-1}\left(N_{t_j}^{ccy} - N_{t_{j+1}}^{ccy}\right)P(0,t_j)^{ccy-CSA} \\ &= \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{ccy=C^{Reset}\}}\sum_{j=1}^{m-1}\left(N_{t_0}^{ccy}\Psi(t_j, ccy) - N_{t_0}^{ccy}\Psi(t_{j+1}, ccy)\right)P(0,t_j)^{ccy-CSA} \\ &= \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{ccy=C^{Reset}\}}\sum_{j=1}^{m-1}N_{t_0}^{ccy}\Big(\Psi(t_j, ccy) - \Psi(t_{j+1}, ccy)\Big)P(0,t_j)^{ccy-CSA} \end{aligned} \tag{20}$$

where $\mathbb{1}_{\{A\}}$ is the Kronecker delta function, namely $\mathbb{1}_{\{A\}} = \begin{cases} 1 & \text{, if A is true} \\ 0 & \text{, o.w.} \end{cases}$

Remark: Final Notional Reset
Note that since we have notional exchanges at the start and end of the CCS contract there is no exchange of the reset amount on the final trade coupon, since this is already accounted for in the final notional exchange.

Finally note that we can choose to take the indicator functions outside of the present value function as follows for notational ease.

$$\mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{FOR=C^{Reset}\}}PV(\text{Reset}, \Omega_{ccy}) \tag{21}$$

When calculating the trade resets for the trade example in section (2.1) using equation (20) we have the following results,

Leg1 - EUR Notional Resets C^{Val} = USD C^{Reset} = USD C^{CSA} = USD

	Initial Notional	SpotFX	ForwardFX	ValuationAdj	ResetAdj	NotionalAdj	Indicator(MtM)	Indicator(C^{Reset})	ResetAmount	DiscountFactor	PV
	$N(t_0)^{EUR}$	$f(t_0)^{EUR/USD}$	$f(t)^{EUR/USD}$	α	β	ψ	$1(\Omega$=MtM$)$	$1(C^{Reset})$	Reset	$P(t_0,t)^{EUR\text{-}USDCSA}$	PVlet
0	-876,962	0.87696	0.87696	1.14030	1.00000	1.14030	1	0	0	1.000000	0
1	-876,962	0.87696	0.87696	1.14030	1.00000	1.14030	1	0	0	1.002365	0
2	-876,962	0.87696	0.86980	1.14030	1.00000	1.14030	1	0	0	1.004182	0
3	-876,962	0.87696	0.86287	1.14030	1.00000	1.14030	1	0	0	1.005926	0
4	-876,962	0.87696	0.85560	1.14030	1.00000	1.14030	1	0	0	1.007807	0
										Total	**0**

Leg2 - USD Notional Resets C^{Val} = USD C^{Reset} = USD C^{CSA} = USD

	Initial Notional	SpotFX	ForwardFX	ValuationAdj	ResetAdj	NotionalAdj	Indicator(MtM)	Indicator(C^{Reset})	ResetAmount	DiscountFactor	PV
	$N(t_0)^{USD}$	$f(t_0)^{USD/EUR}$	$f(t)^{USD/EUR}$	α	β	ψ	$1(\Omega$=MtM$)$	$1(C^{Reset})$	Reset	$P(t_0,t)^{USD\text{-}USDCSA}$	PVlet
0	1,000,000	1.14030	1.14030	1.00000	1.00000	1.00000	1	1	0	1.000000	0
1	1,000,000	1.14030	1.14030	1.00000	1.00000	1.00000	1	1	-8,233	0.994180	-8,185
2	1,000,000	1.14030	1.14969	1.00000	1.00823	1.00823	1	1	-8,104	0.988041	-8,007
3	1,000,000	1.14030	1.15893	1.00000	1.01634	1.01634	1	1	-8,635	0.981419	-8,474
4	1,000,000	1.14030	1.16878	1.00000	1.02497	1.02497	1	1	0	0.974803	0
										Total	**-24,666**

Figure 9: Example Notional Reset Calculations for 1Y EUR/USD MtM CCS

3.7 Trade Pricing

We are now fully equipped to price CCS trades having explained all the Cross Currency Swap trade features in section (2) and having the specifically explained how to price the CCS coupons, notional exchanges and resets in sections (3.4), (3.4) and (3.4) respectively.

Firstly we note that a CCS trade can be priced by evaluating the trade legs individually.

$$PV(\Omega_{Xccy}) = \phi\Big[PV(\Omega_{FOR}) - PV(\Omega_{DOM})\Big] \tag{22}$$

Expanding the trade legs Ω_{FOR} and Ω_{DOM} into their constituent parts gives,

$$\begin{aligned} PV\left(\Omega_{Xccy}\right) = \phi\Big[& PV\Big(\text{Cpn}, \Omega_{FOR}\Big) + PV\Big(\text{Exch}, \Omega_{FOR}\Big) + PV\Big(\text{Resets}, \Omega_{FOR}\Big) \\ & - PV\Big(\text{Cpn}, \Omega_{DOM}\Big) - PV\Big(\text{Exch}, \Omega_{DOM}\Big) - PV\Big(\text{Resets}, \Omega_{DOM}\Big)\Big] \end{aligned} \tag{23}$$

We can evaluate (23) using the present value expressions from (13), (16) and (20) representing the trade coupons, exchanges and resets respectively, see the appendix for the pricing formulae for Float-Float, Fixed-Float and Fixed-Fixed CCS.

For completeness pricing the trade example in section (2.1) using equation (23) gives the following trade and breakdown PVs,

CCS Trade, Ω_{xccy}	
$PV(\Omega_{xccy})$	-2,029
$PV(\Omega_{EUR})$	-4,887
$PV(\Omega_{USD})$	2,858

Breakeven - Par Spreads, s	
ParSpread, s_{EUR}	-0.19967%
ParSpread, s_{USD}	0.20134%

Leg1 Breakdown: Ω_{EUR}	
$PV(Cpn, \Omega_{EUR})$	2,920
$PV(Exch, \Omega_{EUR})$	-7,807
$PV(Resets, \Omega_{EUR})$	0
$PV01(\Omega_{EUR})$	-1,016,238

Leg2 Breakdown: Ω_{USD}	
$PV(Cpn, \Omega_{USD})$	28,379
$PV(Exch, \Omega_{USD})$	-855
$PV(Resets, \Omega_{USD})$	-24,666
$PV01(\Omega_{USD})$	1,007,813

Figure 10: Example Trade Price Calculations in USD for 1Y EUR/USD MtM CCS

3.8 Par Spread Calculations

Market Standard Float-Float MtM Cross currency swaps are quoted in the swaps market not in present value or PV terms but rather by their par spreads. The CCS par spread in basis points[6] is the breakeven spread to be applied to the chosen floating leg to price the CCS to zero or par. For standard CCS with a USD trade leg the par spread is usually quoted on the non-USD trade leg.

Noting that for floating coupons we can rearrange the coupon PV calculation formula from (13) as follows,

$$\begin{aligned}
PV(Cpn, \Omega_{ccy}{=}Float, s_{ccy}) &= \sum_{j=1}^{m} N_{t_0}^{ccy}\Psi(t_j)^{ccy}(l_j + s_{ccy})\tau_j P(0,t_j)^{ccy-CSA} \\
&= \underbrace{\sum_{j=1}^{m} N_{t_0}^{ccy}\Psi(t_j)^{ccy} l_j \tau_j P(0,t_j)^{ccy-CSA}}_{\textit{Floating Coupons with No Spread}} + \underbrace{\sum_{j=1}^{m} N_{t_0}^{ccy}\Psi(t_j)^{ccy} s_{ccy}\tau_j P(0,t_j)^{ccy-CSA}}_{\textit{Spread Component}} \\
&= \sum_{j=1}^{m} N_{t_0}^{ccy}\Psi(t_j)^{ccy} l_j \tau_j P(0,t_j)^{ccy-CSA} + s_{ccy}\underbrace{\sum_{j=1}^{m} N_{t_0}^{ccy}\Psi(t_j)^{ccy}\tau_j P(0,t_j)^{ccy-CSA}}_{\textit{Annuity or PV01}} \\
&= PV(Cpn, \Omega_{ccy}{=}Float, s_{ccy}{=}0) + s_{ccy}A_N(\Omega_{ccy})
\end{aligned} \tag{24}$$

which gives,

$$\underbrace{PV(Cpn, \Omega_{ccy}{=}Float)}_{\textit{Float Leg}} = \underbrace{PV(Cpn, \Omega_{ccy}{=}Float, s_{ccy}{=}0)}_{\textit{Float Coupons with No Spread}} + \underbrace{s_{ccy}A_N(\Omega_{ccy})}_{\textit{Float Spread}} \tag{25}$$

[6] A basis point is $(\frac{1}{100})$th of a percentage point.

where s_{ccy}=0 indicates that we are setting the Libor spread s to zero and $A_N(\Omega_{ccy})$ denotes the Annuity[7] scaled by the trade notional, for detailed information on annuity factors see [2].

Substituting equation (25) for the foreign leg floating coupons in (23) gives,

$$\begin{aligned} PV\left(\Omega_{Xccy}\right) = \phi\Big[& PV(\textit{Cpn}, \Omega_{FOR}\text{=}\textit{Float}, s_{FOR}\text{=}0) + s_{FOR}A_N(\Omega_{FOR}) \\ & + PV\Big(\text{Exch}, \Omega_{FOR}\Big) + PV\Big(\text{Resets}, \Omega_{FOR}\Big) \\ & - PV\Big(\text{Cpn}, \Omega_{DOM}\Big) - PV\Big(\text{Exch}, \Omega_{DOM}\Big) - PV\Big(\text{Resets}, \Omega_{DOM}\Big)\Big] \end{aligned} \tag{26}$$

now for par spreads we know the CCS trade PV has zero value therefore we have,

$$\begin{aligned} 0 = \phi\Big[& PV(\text{Cpn}, \Omega_{FOR}, s_{FOR}\text{=}0) + s_{FOR}A_N(\Omega_{FOR}) \\ & + PV\Big(\text{Exch}, \Omega_{FOR}\Big) + PV\Big(\text{Resets}, \Omega_{FOR}\Big) \\ & - PV\Big(\text{Cpn}, \Omega_{DOM}\Big) - PV\Big(\text{Exch}, \Omega_{DOM}\Big) - PV\Big(\text{Resets}, \Omega_{DOM}\Big)\Big] \end{aligned} \tag{27}$$

which we rearrange for the par spread s_{FOR} to get:

$$\begin{aligned} -\phi s_{FOR}A_N(\Omega_{FOR}) &= \phi\Big[PV(\text{Cpn}, \Omega_{FOR}\text{=}\textit{Float}, s_{FOR}\text{=}0) + PV\Big(\text{Exch}, \Omega_{FOR}\Big) + PV\Big(\text{Resets}, \Omega_{FOR}\Big) \\ & \quad - PV\Big(\text{Cpn}, \Omega_{DOM}\Big) - PV\Big(\text{Exch}, \Omega_{DOM}\Big) - PV\Big(\text{Resets}, \Omega_{DOM}\Big)\Big] \\ &= PV\left(\Omega_{Xccy}, s_{FOR}\text{=}0\right) \end{aligned} \tag{28}$$

noting that the ϕ terms cancel leads to an expression for the par spread s_{FOR} as,

$$s_{FOR} = -\Big(\frac{PV\left(\Omega_{Xccy}, s_{FOR} = 0\right)}{A_N(\Omega_{FOR})}\Big) \tag{29}$$

leading to a general expression for any CCS par spread s_{ccy} of

$$s_{ccy} = -\Big(\frac{PV\left(\Omega_{Xccy}, s_{ccy} = 0\right)}{A_N(\Omega_{ccy})}\Big) \tag{30}$$

This is to say that we can evaluate the CCS par spread by pricing a CCS with no spread i.e. s_{ccy}=0 on the target leg and divide by the annuity factor to get the solution.

Remark: Compounded Coupons
For the case where we have compounded coupons i.e. quarterly coupon accruals paying semi-annual the par spread evaluation is non-trivial. Many practitioners resort to solver and optimization methods to calculate par spreads in these circumstances.

[7] The annuity is also known as the PV01 and is the present value of a fixed leg in a swap with a unit fixed rate i.e. 100%.

We can verify the above result using the trade example from section (2.1) as follows,

CCS Trade, Ω_{xccy}

PV(Ω_{xccy})	-2,029
PV(Ω_{EUR})	-4,887
PV(Ω_{USD})	2,858

Breakeven - Par Spreads, s

ParSpread, s_{EUR}	-0.19967%
ParSpread, s_{USD}	0.20134%

Leg1 Breakdown: Ω_{EUR}

PV(Cpn, Ω_{EUR})	2,920
PV(Exch, Ω_{EUR})	-7,807
PV(Resets, Ω_{EUR})	0
PV01(Ω_{EUR})	-1,016,238

Leg2 Breakdown: Ω_{USD}

PV(Cpn, Ω_{USD})	28,379
PV(Exch, Ω_{USD})	-855
PV(Resets, Ω_{USD})	-24,666
PV01(Ω_{USD})	1,007,813

Figure 11: Example Trade Price and Breakdown Calculations for 1Y EUR/USD MtM CCS

3.9 Par Rate Calculations

Similarly when calculating the CCS fixed par rate p for a fixed-fixed or fixed-float CCS it is helpful to consider the fixed leg composition from equation (23).

We know that for an individual CCS trade leg,

$$PV\left(\Omega_{ccy}\right) = \phi\Big[PV\Big(\text{Cpn}, \Omega_{ccy}\Big) + PV\Big(\text{Exch}, \Omega_{ccy}\Big) + PV\Big(\text{Resets}, \Omega_{ccy}\Big)\Big] \tag{31}$$

and for a fixed coupon rate of zero $r = 0$ we have that,

$$\begin{aligned} PV\left(\Omega_{ccy}{=}\mathit{Fixed}, \text{r=0}\right) &= \phi\Big[\underbrace{PV\Big(\text{Cpn}, \Omega_{ccy}{=}\mathit{Fixed}\Big)}_{\text{Coupon PV = 0 since r = 0}} + PV\Big(\text{Exch}, \Omega_{ccy}\Big) + PV\Big(\text{Resets}, \Omega_{ccy}\Big)\Big] \\ &= \phi\Big[PV\Big(\text{Exch}, \Omega_{ccy}\Big) + PV\Big(\text{Resets}, \Omega_{ccy}\Big)\Big] \end{aligned} \tag{32}$$

giving us a convenient way to decouple the fixed coupon leg pricing from the CCS trade pricing,

$$\underbrace{PV\left(\Omega_{Xccy}{=}\mathit{Fixed}\right)}_{\text{CCS Trade}} = \underbrace{PV\left(\text{Cpn}, \Omega_{ccy}{=}\mathit{Fixed}\right)}_{\text{Fixed Coupons from ccy leg}} + \underbrace{PV\left(\Omega_{Xccy}, r_{ccy} = 0\right)}_{\text{CCS Trade with } r_{ccy} = 0} \tag{33}$$

equivalently for the fixed leg of the CCS we can to decouple the fixed coupons from the notional exchanges and notional resets as follows,

$$\underbrace{PV\left(\Omega_{ccy}{=}\mathit{Fixed}\right)}_{\text{Fixed Leg}} = \underbrace{PV\left(\text{Cpn}, \Omega_{ccy}{=}\mathit{Fixed}\right)}_{\text{Fixed Coupons}} + \underbrace{PV\left(\Omega_{ccy}{=}\mathit{Fixed}, r_{ccy} = 0\right)}_{\text{Fixed Exchanges \& Resets}} \tag{34}$$

If we assume that the fixed leg is the foreign currency leg FOR and substitute equation (33) into (22) we get,

$$\begin{aligned} PV\Big(\Omega_{Xccy}=\textit{Fixed}\Big) = \phi\Big[&\underbrace{PV\Big(\text{Cpn}, \Omega_{FOR}=\textit{Fixed}\Big)}_{\textit{Foreign Fixed Coupons}} + \underbrace{PV\Big(\Omega_{FOR}=\textit{Fixed}, r_{FOR}=0\Big)}_{\textit{Foreign Exchanges \& Resets}} \\ &- \underbrace{PV\Big(\Omega_{DOM}\Big)}_{\textit{Domestic Leg}}\Big] \end{aligned} \tag{35}$$

now for par swaps the total trade PV is zero giving,

$$0 = \phi\Big[PV\Big(\text{Cpn}, \Omega_{FOR}=\textit{Fixed}\Big) + PV\Big(\Omega_{FOR}=\textit{Fixed}, r_{FOR}=0\Big) - PV\Big(\Omega_{DOM}\Big)\Big] \tag{36}$$

leading to the below expression,

$$\begin{aligned} -\phi PV\Big(\text{Cpn}, \Omega_{FOR}=\textit{Fixed}\Big) &= \phi\Big[\underbrace{PV\Big(\Omega_{FOR}=\textit{Fixed}, r_{FOR}=0\Big) - PV\Big(\Omega_{DOM}\Big)}_{\textit{Xccy PV with fixed rate } r_{FOR}=0}\Big] \\ -\phi\sum_{i=1}^{n} N_{t_0}^{FOR}\Psi(t_i)^{FOR} r_{FOR}\tau_i P(0,t_i)^{FOR-CSA} &= \phi PV\Big(\Omega_{Xccy}, r_{FOR}=0\Big) \\ -\phi r_{FOR}\underbrace{\sum_{i=1}^{n} N_{t_0}^{FOR}\Psi(t_i)^{FOR}\tau_i P(0,t_i)^{FOR-CSA}}_{\textit{Annuity or PV01}} &= \phi PV\Big(\Omega_{Xccy}, r_{FOR}=0\Big) \\ -\phi r_{FOR} A_N(\Omega_{FOR}) &= \phi PV\Big(\Omega_{Xccy}, r_{FOR}=0\Big) \\ r_{FOR} A_N(\Omega_{FOR}) &= -PV\Big(\Omega_{Xccy}, r_{FOR}=0\Big) \end{aligned} \tag{37}$$

rearranging and denoting the par rate or break-even fixed rate $r^{FOR} = p^{FOR}$ gives,

$$p^{FOR} = -\left(\frac{PV\Big(\Omega_{Xccy}, r_{FOR}=0\Big)}{A_N(\Omega_{FOR})}\right) \tag{38}$$

leading to a general expression for par rate as the present value CCS trade with a fixed rate of zero divided by the annuity of the target leg.

$$p^{ccy} = -\left(\frac{PV\Big(\Omega_{Xccy}, r^{ccy}=0\Big)}{A_N(\Omega_{ccy})}\right) \tag{39}$$

We can verify the above result using a fixed-float CCS trade example by pricing a CCS with a zero fixed rate and scaling by the annuity factor as follows,

Cross Currency Swap, Ω_{Xccy}

TradeDate	Fri, 26-Oct-18	
Maturity (Years)	1Y	Sat, 26-Oct-19
Trade Notional	1,000,000	
Trade Currency	USD	
MtM	YES	
NotionalExchanges	YES	
Reset Currency	USD	USD
CSA Currency	USD	
Valuation Currency	USD	
SpotFX	1.14030	USD/EUR
LegCurrency	EUR	USD
LegNotional	876,962	1,000,000
PayOrReceive	PAY	RECEIVE
LegType	FIXED	FLOATING
RateOrSpread (%) (1)	0.00000%	0.00000%
FloatIndex		USD LIBOR 3M
Frequency	QUARTERLY	QUARTERLY
LegResetsRequired	NO	YES
LegSpotFX	0.87696	1.14030
ValuationFXAdj	1.14030	1.00000
DaycountBasis	ACT/360	ACT/360
UseMarketSchedule	NO	NO

ParRate

SwapPV	-4,950
PV01	-1,016,238
ParRate, p (2)	-0.48705%

Cross Currency Swap, Ω_{Xccy}

TradeDate	Fri, 26-Oct-18	
Maturity (Years)	1Y	Sat, 26-Oct-19
Trade Notional	1,000,000	
Trade Currency	USD	
MtM	YES	
NotionalExchanges	YES	
Reset Currency	USD	USD
CSA Currency	USD	
Valuation Currency	USD	
SpotFX	1.14030	USD/EUR
LegCurrency	EUR	USD
LegNotional	876,962	1,000,000
PayOrReceive	PAY	RECEIVE
LegType	FIXED	FLOATING
RateOrSpread (%)	-0.48705% (3)	0.00000%
FloatIndex		USD LIBOR 3M
Frequency	QUARTERLY	QUARTERLY
LegResetsRequired	NO	YES
LegSpotFX	0.87696	1.14030
ValuationFXAdj	1.14030	1.00000
DaycountBasis	ACT/360	ACT/360
UseMarketSchedule	NO	NO

Par Price Check	LEG1: EUR	LEG2: USD
LegPV	-2,858	2,858
SwapPV	0 (4)	USD

Figure 12: Example Fixed-Float CCS Example Trade with Par Rate Calculation

4 Conclusion

In summary we reviewed the cross currency swap CCS product providing a typical funding scenario that could be resolved using cross currency swaps. Secondly we looked at the risks associated with CCS products, namely counterparty credit risk, FX risk & interest risk and considered the CCS trade features that allow us to reduce and mitigate such risks. Thirdly we reviewed a CCS example trade and discussed the individual components of cross currency swaps and how to price each of them. Taking a step-by-step approach we presented the mathematical formulae, examples and illustrations to help understand the pricing of each individual component of the CCS instrument. Fourthly we discissed how to price cross currency swaps and presented complete pricing formulae.

Finally we mention that cross currency swaps are quoted in the marketplace as a par spread or par rate. This is the break-even Libor spread or fixed rate that makes the trade price to par. To conclude we show how par-spread and par-rate quotes are calculated for CCS trades and provide examples.

References

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Appendix 1

With this paper we provide an example Excel pricing workbook for illustration purposes to demonstrate Cross Currency Swap pricing and the formulae used. Kindly email the author to receive a copy.

Cross Currency Swap, Ω_{Xccy}

TradeDate	Fri, 26-Oct-18	
Maturity (Years)	5Y	Wed, 25-Oct-23
Trade Notional	1,000,000	
Trade Currency	USD	
MtM	YES	
NotionalExchanges	YES	
Reset Currency	USD	USD
CSA Currency	USD	
Valuation Currency	USD	
SpotFX	1.14030	USD/EUR
LegCurrency	EUR	USD
LegNotional	876,962	1,000,000
PayOrReceive	PAY	RECEIVE
LegType	FLOATING	FLOATING
RateOrSpread (%)	0.00000%	0.00000%
FloatIndex	EUR EURIBOR 3M	USD LIBOR 3M
Frequency	QUARTERLY	QUARTERLY
LegResetsRequired	NO	YES
LegSpotFX	0.87696	1.14030
ValuationFXAdj	1.14030	1.00000
DaycountBasis	ACT/360	ACT/360
UseMarketSchedule	NO	NO

Prices / ParSpreads	LEG1: EUR	LEG2: USD
LegPV	-24,992	15,686
SwapPV	-9,306	USD
LegPV* (s=0)	-24,992	15,686
NotionalExchanges	-9,939	-138,817
PV01	-5,113,948	5,076,346
ParRateOrSpread (%)	-0.18197%	0.18332%

Leg1 - EUR Cashflows

	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				876,962			876,962	1.000000	876,962	1.1403	1,000,000
1	-876,962	Fri, 26-Oct-18	1.00000	0	0.00000%	-0.31695%	703	1.002365	704	1.1403	803
2	-876,962	Fri, 25-Jan-19	1.00000	0	0.00000%	-0.31644%	701	1.004182	704	1.1403	803
3	-876,962	Fri, 26-Apr-19	1.00000	0	0.00000%	-0.28931%	641	1.005926	645	1.1403	736
4	-876,962	Fri, 26-Jul-19	1.00000	0	0.00000%	-0.22709%	503	1.007807	507	1.1403	579
5	-876,962	Sat, 26-Oct-19	1.00000	0	0.00000%	-0.13634%	302	1.009467	305	1.1403	348
6	-876,962	Sat, 25-Jan-20	1.00000	0	0.00000%	-0.05021%	111	1.010835	113	1.1403	128
7	-876,962	Sat, 25-Apr-20	1.00000	0	0.00000%	0.02216%	-49	1.011997	-50	1.1403	-57
8	-876,962	Sat, 25-Jul-20	1.00000	0	0.00000%	0.08249%	-183	1.013047	-185	1.1403	-211
9	-876,962	Sun, 25-Oct-20	1.00000	0	0.00000%	0.13501%	-299	1.013962	-303	1.1403	-346
10	-876,962	Sun, 24-Jan-21	1.00000	0	0.00000%	0.19845%	-440	1.014734	-446	1.1403	-509
11	-876,962	Sun, 25-Apr-21	1.00000	0	0.00000%	0.27912%	-619	1.015295	-628	1.1403	-716
12	-876,962	Sun, 25-Jul-21	1.00000	0	0.00000%	0.37754%	-837	1.015577	-850	1.1403	-969
13	-876,962	Mon, 25-Oct-21	1.00000	0	0.00000%	0.48748%	-1,081	1.015536	-1,097	1.1403	-1,251
14	-876,962	Mon, 24-Jan-22	1.00000	0	0.00000%	0.58832%	-1,304	1.015210	-1,324	1.1403	-1,510
15	-876,962	Mon, 25-Apr-22	1.00000	0	0.00000%	0.67584%	-1,498	1.014663	-1,520	1.1403	-1,733
16	-876,962	Mon, 25-Jul-22	1.00000	0	0.00000%	0.74980%	-1,662	1.013957	-1,685	1.1403	-1,922
17	-876,962	Tue, 25-Oct-22	1.00000	0	0.00000%	0.81171%	-1,799	1.013132	-1,823	1.1403	-2,079
18	-876,962	Tue, 24-Jan-23	1.00000	0	0.00000%	0.87156%	-1,932	1.012214	-1,956	1.1403	-2,230
19	-876,962	Tue, 25-Apr-23	1.00000	0	0.00000%	0.93160%	-2,065	1.011157	-2,088	1.1403	-2,381
20	-876,962	Tue, 25-Jul-23	1.00000	-876,962	0.00000%	0.99282%	-879,163	1.009939	-887,901	1.1403	-1,012,474

Leg2 - USD Cashflows

	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				-1,000,000			-1,000,000	1.000000	-1,000,000	1.0000	-1,000,000
1	1,000,000	Fri, 26-Oct-18	1.14030	-8,233	0.00000%	2.47475%	-1,977	0.994180	-1,966	1.0000	-1,966
2	1,008,233	Fri, 25-Jan-19	1.14969	-8,104	0.00000%	2.79581%	-979	0.988041	-967	1.0000	-967
3	1,016,337	Fri, 26-Apr-19	1.15893	-8,635	0.00000%	2.93764%	-1,088	0.981419	-1,067	1.0000	-1,067
4	1,024,972	Fri, 26-Jul-19	1.16878	-8,886	0.00000%	3.05383%	-974	0.974803	-949	1.0000	-949

Figure 13: Example Excel Cross Currency Swap Workbook

Appendix 2

Float-Float CCS

For a Float-Float CCS expanding equation (23) and plugging in the component expressions from (13), (16) and (20) gives,

$$
\begin{aligned}
PV\left(\Omega_{Xccy}\right) = \phi\Big[& \underbrace{\sum_{j=1}^{m} N_{t_0}^{FOR}\Psi(t_j)^{FOR}(l_j + s_{FOR})\tau_j P(0,t_j)^{FOR-CSA}}_{\textit{Foreign Float Coupons}} \\
& + \Big(\underbrace{N_{t_0}^{FOR}\Psi(t_m)^{FOR}P(0,t_m)^{FOR-CSA}}_{\textit{Foreign Final Exchange}} - \underbrace{N_{t_0}^{FOR}\Psi(t_0)^{FOR}P(0,t_0)^{FOR-CSA}}_{\textit{Foreign Upfront Exchange}} \Big) \\
& + \mathbb{1}_{\{\Omega = MtM\}}\mathbb{1}_{\{FOR = C^{Reset}\}} \underbrace{\sum_{j=1}^{m-1} N_{t_0}^{FOR}\Big(\Psi(t_j)^{FOR} - \Psi(t_{j+1})^{FOR}\Big)P(0,t_j)^{FOR-CSA}}_{\textit{Foreign Notional Resets}} \\
& - \underbrace{\sum_{j=1}^{m} N_{t_0}^{DOM}\Psi(t_j)^{DOM}(l_j + s_{DOM})\tau_j P(0,t_j)^{DOM-CSA}}_{\textit{Domestic Float Coupons}} \\
& - \Big(\underbrace{N_{t_0}^{DOM}\Psi(t_m)^{DOM}P(0,t_m)^{DOM-CSA}}_{\textit{Domestic Final Exchange}} - \underbrace{N_{t_0}^{DOM}\Psi(t_0)^{DOM}P(0,t_0)^{DOM-CSA}}_{\textit{Domestic Upfront Exchange}} \Big) \\
& - \mathbb{1}_{\{\Omega = MtM\}}\mathbb{1}_{\{DOM = C^{Reset}\}} \underbrace{\sum_{j=1}^{m-1} N_{t_0}^{DOM}\Big(\Psi(t_j)^{DOM} - \Psi(t_{j+1})^{DOM}\Big)P(0,t_j)^{DOM-CSA}}_{\textit{Domestic Notional Resets}} \Big]
\end{aligned}
$$

Fixed-Float CCS

For a Fixed-Float CCS expanding equation (23) and plugging in the component expressions from (13), (16) and (20) gives,

$$
\begin{aligned}
PV\left(\Omega_{Xccy}\right) = \phi\Big[& \underbrace{\sum_{i=1}^{n} N_{t_0}^{FOR} \Psi(t_i)^{FOR} r_{FOR} \tau_i P(0,t_i)^{FOR-CSA}}_{\textit{Foreign Fixed Coupons}} \\
& + \Big(\underbrace{N_{t_0}^{FOR} \Psi(t_n)^{FOR} P(0,t_n)^{FOR-CSA}}_{\textit{Foreign Final Exchange}} - \underbrace{N_{t_0}^{FOR} \Psi(t_0)^{FOR} P(0,t_0)^{FOR-CSA}}_{\textit{Foreign Upfront Exchange}} \Big) \\
& + \mathbb{1}_{\{\Omega=MtM\}} \mathbb{1}_{\{FOR=C^{Reset}\}} \underbrace{\sum_{i=1}^{n-1} N_{t_0}^{FOR} \Big(\Psi(t_i)^{FOR} - \Psi(t_{i+1})^{FOR} \Big) P(0,t_i)^{FOR-CSA}}_{\textit{Foreign Notional Resets}} \\
& - \underbrace{\sum_{j=1}^{m} N_{t_0}^{DOM} \Psi(t_j)^{DOM} (l_j + s_{DOM}) \tau_j P(0,t_j)^{DOM-CSA}}_{\textit{Domestic Float Coupons}} \\
& - \Big(\underbrace{N_{t_0}^{DOM} \Psi(t_m)^{DOM} P(0,t_m)^{DOM-CSA}}_{\textit{Domestic Final Exchange}} - \underbrace{N_{t_0}^{DOM} \Psi(t_0)^{DOM} P(0,t_0)^{DOM-CSA}}_{\textit{Domestic Upfront Exchange}} \Big) \\
& - \mathbb{1}_{\{\Omega=MtM\}} \mathbb{1}_{\{DOM=C^{Reset}\}} \underbrace{\sum_{j=1}^{m-1} N_{t_0}^{DOM} \Big(\Psi(t_j)^{DOM} - \Psi(t_{j+1})^{DOM} \Big) P(0,t_j)^{DOM-CSA}}_{\textit{Domestic Notional Resets}} \Big]
\end{aligned}
$$

Fixed-Fixed CCS

Finally for a Fixed-Fixed CCS expanding equation (23) and plugging in the component expressions from (13), (16) and (20) gives,

$$
\begin{aligned}
PV\left(\Omega_{Xccy}\right) = \phi\Big[& \underbrace{\sum_{i=1}^{n} N_{t_0}^{FOR}\Psi(t_i)^{FOR}(l_i + s_{FOR})\tau_i P(0,t_i)^{FOR-CSA}}_{\textit{Foreign Fixed Coupons}} \\
& + \Big(\underbrace{N_{t_0}^{FOR}\Psi(t_n)^{FOR}P(0,t_n)^{FOR-CSA}}_{\textit{Foreign Final Exchange}} - \underbrace{N_{t_0}^{FOR}\Psi(t_0)^{FOR}P(0,t_0)^{FOR-CSA}}_{\textit{Foreign Upfront Exchange}} \Big) \\
& + \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{FOR=C^{Reset}\}} \underbrace{\sum_{i=1}^{m-1} N_{t_0}^{FOR}\Big(\Psi(t_i)^{FOR} - \Psi(t_{i+1})^{FOR}\Big)P(0,t_i)^{FOR-CSA}}_{\textit{Foreign Notional Resets}} \\
& - \underbrace{\sum_{i=1}^{n} N_{t_0}^{DOM}\Psi(t_i)^{DOM}(l_i + s_{DOM})\tau_i P(0,t_i)^{DOM-CSA}}_{\textit{Domestic Fixed Coupons}} \\
& - \Big(\underbrace{N_{t_0}^{DOM}\Psi(t_n)^{DOM}P(0,t_n)^{DOM-CSA}}_{\textit{Domestic Final Exchange}} - \underbrace{N_{t_0}^{DOM}\Psi(t_0)^{DOM}P(0,t_0)^{DOM-CSA}}_{\textit{Domestic Upfront Exchange}} \Big) \\
& - \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{DOM=C^{Reset}\}} \underbrace{\sum_{i=1}^{n-1} N_{t_0}^{DOM}\Big(\Psi(t_i)^{DOM} - \Psi(t_{i+1})^{DOM}\Big)P(0,t_i)^{DOM-CSA}}_{\textit{Domestic Notional Resets}} \Big]
\end{aligned}
$$