Brazilian DI1 Interest Rate Futures

Saïd Business School, University of Oxford, UK

Nicholas Burgess

nburgessx@gmail.com

March 2022

Introduction

DI1 futures are referenced against one-day interbank deposits (CDI) in the Brazilian onshore market. They have an underlying rate R computed as the average of 1D CDI rates compounded daily on a Bus/252 day count basis, where Bus/252 is equivalent to Act/252.

Futures are short interest rate instruments; being long a DI1 future is tantamount to being short CDI interest rates. In a DI1 futures contract we purchase a future cash flow of BRL 100,000 in advance to lock-in current interest rates and to protect against adverse market movements. This cash flow is discounted using CDI rates, which gives the quoted cash settlement price.

If on day 1 the discount factor is 0.90 say then the settlement price is BRL 90,000. Over the life of the futures contract when rates fall we profit and if rates climb we suffer losses[1]. Futures are exchange traded so any profit or loss is cash settled on a daily basis. At contract maturity we receive the full BRL 100,000 purchased.

Unit Price

100,000

Tick Size

0.01

Daily Compounded Rate, R

$$R(t,T) = \frac{\left(\prod_{t=1}^{T}[1+CDI_t]^{d_t/252}-1\right)}{\left({}^{D}/_{252}\right)}$$

where $D = \sum_t d_t$ the total number of interest days and d_t = number of days each individual 1D CDI rate is applicable; every weekday counts for 1 day of interest except for Friday which counts for 3 days to account for interest payable on Friday and to carry the position over the weekend.

[1] The DI1 discount factor, DF has an inverse relationship with the CDI rate, R with DF = $1/(1+R)^{\tau^{252}}$. Now as the DI1 Future Price = 100,000*DF should CDI rates fall the discount factor increases and we profit. Similarly if CDI rates rise the DF decreases and we suffer losses. Due to the inverse relationship between the futures price and the CDI rate, we consider a long futures position equivalent to being short CDI rates.

Capitalization Factor

The capitalization growth factor computes the future value of a cash flow. It computes the growth of a cash flow based on compounded DI interest rates with simple compounding and a day count fraction τ determined using a Bus/252 basis. We round the capitalization factor **to 7 decimal places** so that futures prices denominated in BRL 100,000 scale to 2 decimal places giving a tick size of 0.01.

$$\text{CapFac} = \text{round}(\ (1 + \text{R})^{\tau^{252}}, 7\ )$$

Discount Factor

The discount factor computes the present value of a future cash flow. It is the inverse of the capitalization growth factor. It is also rounded **to 7 decimal places** so that futures prices denominated in BRL 100,000 scale to 2 decimal places giving a tick size of 0.01.

$$\text{DF} = 1 / (1 + R)^{\tau^{252}} = \text{round}(\ 1 / \text{CapFac}, 7\ )$$

Traded Price

DI1 futures represent BRL 100,000 discounted by the interest rate R. The futures traded price (TP) is the unit price scaled by the discount factor and quoted **to 2 decimal places**.

$$\text{TP} = \text{round}(\ 100{,}000 * \text{DF}, 2\ )$$

Futures Tickers

The futures instrument ticker comprises of product + month + year e.g. DI1 + F + 23 = DI1F23, indicating DI1 Futures for delivery Jan 2023. The futures month Codes are as follows,

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Letter	F	G	H	J	K	M	N	Q	U	V	X	Z

Expiry Date

The first business day of the contract month. For example, a DI1Z23 (December 2023) future would expire on the first business day of December 2023.

Variation Margin - Daily Profit & Loss

Futures are cash settled on exchange with daily P&L calculated as below. The variation margin computes the daily change in a trade's present value. The calculation is slightly different for new and outstanding positions.

For New Positions

For new positions we compare the present value difference when valuing the trading position using the current price (TP) and the initial traded price P0.

$$\text{P\&L} = \text{Number of Contracts} * (\text{TP}(t) - \text{P0})$$

$$\text{where Initial Price} = P0 = 100{,}00/(1 + i\%)^{n/252}$$

For Outstanding Positions

For existing positions also compute the daily change in present value, however we firstly we scale yesterday's price by the cap factor to compute it's present value before valuing the trading position using the current price and the yesterday's price.

$$\text{P\&L} = \text{Number of Contracts} * (\text{TP}(t) - \text{TP}(t\text{-}1) \times \text{CF}(t\text{-}1))$$

$$\text{where Cap Factor, CF(t-1)} = (1 + DI(t-1)\%)^{1/252}$$

Example

A trader buys 100 DIF23 contracts traded at 10.00% DI rate. The B3 exchange[2] performs cash settlement on DI prices not rates, so B3 will convert the DI rate to a DI price rounded to 2 decimal places. What is the traded price and futures position?

Expiry

First business day of January 2023 (F23)

Cap Factor

$$\text{CapFac} = (1 + 10\%)^{150/252} = 1.0583724$$

[2] Formerly Bolsa de Valores, Mercadorias e Futuros (BVMF)

Discount Factor

$$\mathrm{DF} = {}^{1}/_{\mathrm{CapFac}} = 0.9448470$$

Traded Price

The initial trading price is quoted using 100,000 pricing units and capitalization factor specified determined by the traded interest rate i% with n days to expiry, where n as the number of days between the trade date and the day preceding expiry. The cap factor and discount factors are rounded to 7 decimal places so that our unit traded price is quotes to 2 decimal places.

TP = 100,000 * DF = BRL 94,484.70

Futures Position

We are long futures and short DI, we benefit if DI rates decrease and suffer losses if DI increase. The futures postion FP is as follows,

$$\begin{aligned} \mathrm{FP} &= \text{Number of Contracts} * \mathrm{TP} \\ &= 100 * 94{,}484.70 \\ &= \mathrm{BRL}\ 9{,}448{,}470.00 \end{aligned}$$

References

DI1 Contract Specifications
B3 Exchange
https://www.b3.com.br/en_us/products-and-services/trading/interest-rates/one-day-interbank-deposit-futures.htm

DI1 Technical Specifications
B3 Exchange
http://www.bmf.com.br/bmfbovespa/pages/contratos2/pdf/IDfutures.pdf