Cash-Settled Swaptions
A Review of Cash-Settled Swaption Pricing

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Abstract

In this paper we provide an outline of interest rate swaptions and how to price swaptions with different payoff or settlement types. Firstly we review the different settlement styles commonplace in financial markets. Secondly we review the swaption pricing formulae corresponding to each settlement type and review pricing considerations associated with each payoff.

Thirdly we review the pricing of swaptions with different choices of settlement style. We outline swaption par-yield pricing considerations and review the cash-annuity factor. We note that par-yield cash-settlement is not arbitrage-free, yet despite this par-yield settlement is a standard swaption payoff in European markets.

We emphasize par-yield settlement, since his method can be provide significant benefits and yet be quite opaque and often misunderstood. Par-yield settlement provides a simplified representation of the cash payoff of a swap, which is convenient for swaption payoff standardization, trade netting and the exchange clearing of swaptions. Trade netting can significantly reduce counterparty credit risk, associated XVA and capital charges, which can be substantial.

Finally we review how to extend the par-yield and cash-annuity definitions to allow the pricing of swaptions with stubs and irregular coupons.

Notation

The notation in table (1) will be used for pricing formulae.

A_N^{Fixed}	The swap fixed leg annuity scaled by the swap notional
A_N^{Float}	The swap float leg annuity scaled by the swap notional
b	The cost of carry, $b = r - q$
c	In the cash annuity formula, the total number of regular fixed leg coupons
C	Value of a European call option
C_N^{Fixed}	The swap fixed leg cash annuity term scaled by the swap notional
K	The strike of the European option
l	The Libor floating rate in % of an interest rate swap floating cashflow
m	In an interest rate swap, the total number of floating leg coupons
	In the cash annuity formula, the number of swap coupons per year
M_t	A tradeable asset or numeraire M evaluated at time t
n	In an interest rate swap, the total number of fixed leg coupons
	In the cash annuity formula, the swap tenor as an integer number of whole years
N_t	A tradeable asset or numeraire N evaluated at time t
N	The notional of an interest rate swap
$N(z)$	The value of the Cumulative Standard Normal Distribution
P	Value of a European put option
p^{Market}	The market par rate in % for a swap. This is the fixed rate that makes the swap fixed leg price match the price of the floating leg
$P(t,T)$	The discount factor for a cashflow paid and time T and evaluated at time t, where $t < T$
ϕ	A call or put indicator function, 1 represents a call and -1 a put option.
	In the case of swap 1 represents a swap to receive and -1 to pay the fixed leg coupons
q	The continuous dividend yield or convenience yield
r	The risk-free interest rate (zero rate)
r^{Fixed}	The fixed rate in % of an interest rate swap fixed cashflow
s	The Libor floating spread in basis points of an interest rate swap floating cashflow
S	For options the underlying spot value
σ	The volatility of the underlying asset
T	The time to expiry of the option in years
t	The pricing or valuation date of the trade
τ	The year fraction of a swap coupon or cashflow
V	Value of a European call or put option
X_T	The option payoff evaluated at time T

Table 1: Notation

Introduction

A swaption is an option contract that serves to provide the holder with the right to enter a forward starting swap at a fixed rate set today. Swaptions are quoted as N x M, where N indicates the option expiry in years and M refers to the underlying swap tenor in years. Hence a 1 x 5 Swaption would refer to 1 year option to enter a 5 year swap[1].

Swaptions are specified as $payer$ or $receiver$ meaning that one has the option to enter a swap to pay or receive the fixed leg of the swap respectively. Furthermore swaptions have an associated option style with the main flavours being European, American and Bermudan, which refer to the option exercise date(s), giving the holder the right to exercise at option expiry only, at any date up to and on discrete intervals up to and including option expiry respectively.

Another important swaption feature is the payoff specification which can be specified for cash or physical settlement, meaning that if exercised we can specify to enter into the underlying swap or receive the cash equivalent on expiry. In what follows we discuss the common swaption settlement types traded in financial markets and how the choice of settlement style impacts the pricing of European Swaptions.

1 Swaption Settlement Styles: Cash or Physical Settlement

Naturally traders agree upfront the option payoff and how to settle options when they expire in-the-money. Interest rate swaption payoffs are specified as part of each and every swaption transaction. The option payoff is referred to as the settlement type or style. Settlement can be either physical or cash. Physical-settlement indicates that the payoff involves an asset transfer and cash-settlement indicates a cash payment is made. There are several swaption settlement types referenced in the interest rate swaption market namely

1. Swap-Settlement (Physical)
2. Price-Settlement (Cash)
3. Par-Yield Settlement (Cash)

1.1 Physical-Swap Settlement

In the case of physical settlement also known as swap-settlement an actual swap is entered into giving a swaption payoff X_T of

$$X_T = A_N^{Fixed}(T)\left[\phi\left(p^{market} - K\right)\right]^+ \tag{1}$$

[1]Note the underlying 5 year swap in this case would be a forward starting swap, starting in 1 year with a tenor of 5 years and ending in 6 years from the contract spot date.

where ϕ is an indicator function set to 1 for a payer swaption that pay the fixed leg and -1 for a receiver swaption that receive the fixed leg of the underlying swap and having annuity factor $A_N^{Fixed}(T)$ defined as

$$A_N^{Fixed}(T) = N \sum_{i=1}^{n} \tau_i P(t, t_i) \tag{2}$$

with $t < t_i \ \forall i$

1.2 Cash-Price Settlement

With regards to swaptions with cash-settlement on exercise one receives a cash payment equal to the price or present value PV of the swap as observed at option expiry. Economically price and swap settlement are equivalent with both swaption types having a payoff given by (1) and (2) namely

$$X_T = A_N^{Fixed}(T) \left[\phi \left(p^{market} - K\right)\right]^+ \tag{3}$$

again with the annuity factor $A_N^{Fixed}(T)$ defined as

$$A_N^{Fixed}(T) = N \sum_{i=1}^{n} \tau_i P(t, t_i) \tag{4}$$

with $t < t_i \ \forall i$

This annuity factor A_N^{Fixed} being typically scaled by very large trade notionals N can be the subject of much controversy since this makes the payoff dependent on yield curve and discount calculation, which can vary significantly between market participants.

1.3 Cash Par-Yield Settlement

In European markets, a third variety of swaptions is commonplace, namely par-yield settlement. This is another form of cash-settlement, whereby an approximated cash payoff representing the cash value of the underlying swap is specified. The par-yield cash-payoff was designed to be simple, transparent and a reasonable approximation to the underlying swap cash value requiring no knowledge of yield curves and discount factors.

The standardisation of the payoff makes this choice of settlement ideal for exchange clearing and the netting of both cleared and bilateral transactions. This facilitates the reduction of counterparty credit risk, XVA exposures and associated capital charges.

Historically there have been several instances of swaption payoff disputes relating to the calculation of the cash-settlement payoff, where market participants could not agree on the price of the underlying swap, with price differences due to disagreement of bid-offer spreads, liquidity, supply and demand factors. However the main source of price discrepancies centred on yield curve calibration methodologies and discount factor calculations.

Yield curve construction is typically a complex and in-house proprietary process and not an exact science. Different market participants might use different yield curve calibration instruments, skew quotes to reflect their trading positions and use different curve construction methodologies, which would lead to different swap prices. Furthermore some traders could take advantage of this to transact at more favourable prices with unsuspecting and less sophisticated clients. The use of interest rate curves to determine the swaption payoff makes the payoff calculation less transparent and opaque.

To tackle these problems par-settlement was introduced with the payoff specified as follows

$$X_T = C_N^{Fixed}(T) \left[\phi \left(p^{market} - K\right)\right]^+ \tag{5}$$

with a cash annuity term $C_N^{Fixed}(T)$ independent of yield curve and discount factor calculations outlined below.

Cash Annuity Formula, $C_N^{Fixed}(T)$

The cash annuity $C_N^{Fixed}(T)$ provides an exact and transparent cash-payoff subject to market swap rates p^{market} being observable. The key and only component of the underlying swap payoff determined from yield curve discount factors is the annuity factor A_N^{Fixed}, which in the pay-yield cash settlement case is replaced with the cash-annuity factor C_N^{Fixed} and defined as

$$C_N^{Fixed}(T) = N \sum_{i=1}^{c} \tau_i D(T, t_i) \tag{6}$$

where c denotes the total number of whole regular fixed coupons and the coupon year fraction τ and the discount factor $D(T, t_j)$ are defined as

$$\begin{aligned} \tau &= \frac{1}{m} \\ D(T, t_i) &= \frac{1}{\prod_{j=1}^{i} \left(1 + p^{Market} \tau\right)} \end{aligned} \tag{7}$$

where m denotes the number of fixed coupons per year, p^{market} is the underlying swap rate observed at the swaption valuation date at time t and T represents the swaption expiry date. Note we have $T < t_i \; \forall i$.

As can be seen from the above the coupon year fraction τ is approximate and constant for every coupon period with no consideration for daycount basis, holiday calendars and business day conventions. The discount factor $D(T, t_i)$ is a discretely compounded discount factor independent of the yield curve calculation, which relies on treating the underlying swap rate p^{market} as a discount yield.

The simplistic form of the cash annuity standardizes the underlying swap payoff making par-yield settlement ideal for exchange clearing and trade netting both on a cleared and bilateral basis.

Note this representation of the cash-annuity formula in contrast to formula quoted in standard literature and shown below supports swaps tenors that are not denominated in whole years. However this formulation does not support swaps with irregular coupons or stubs.

An alternative yet equivalent cash annuity formula is often quoted in standard market literature, which we outline below for completeness.

Alternative Cash Annuity Formula, $C_N^{Fixed}(T)$

Standard market literature often quotes the cash annuity formula evaluated at option expiry T as follows, supporting swaps quoted with tenors in whole years only.

$$C_N^{Fixed}(T) = N \sum_{i=1}^{nm} \frac{\frac{1}{m}}{\left(1 + \frac{1}{m} S(t)\right)^i} \tag{8}$$

or equivalently

$$C_N^{Fixed}(T) = N \frac{1}{S(t)} \left(1 - \frac{1}{\left(1 + \frac{1}{m} S(t)\right)^{nm}}\right) \tag{9}$$

where n denotes the swap tenor in years with $n \in \mathbb{Z}^+$, m the number of fixed coupons per year and $S(t)$ is the underlying swap rate observed at time t.

The above formulation calculates the value of the cash-annuity at option expiry time T. Note that when pricing swaptions prior to expiry we need to discount the cash-annuity payoff to valuation date t where $t < T$ and hence multiply the cash annuity by $P(t, T)$.

Clearly the standard cash-annuity formula only supports swaps with tenors denominated in whole years and furthermore does not support swaps with irregular coupons or stubs, which is intentional in order to standardize the underlying swap, facilitate exchange clearing, trade netting and mitigate settlement disputes.

2 Swaption Pricing

In this section we inspect the swaption pricing formulae, paying close attention to the impact of par-yield cash settlement on swaption pricing.

2.1 Pricing with Physical-Swap Settlement

As outlined in our swaption pricing and derivation paper [8] European interest rate swaps traded with physical- or swap-settlement have an arbitrage free price, which can be priced using Black-76 closed-form analytical formula scaled by the interest rate swap fixed leg annuity $A_N^{Fixed}(t)$ namely

European Swaption Price with Physical and Cash-Price Settlement

Assuming a lognormal Black-76 volatility we price European interest rate swaptions with physical-swap or cash-price settlement as follows:

$$\begin{aligned} V(t) &= \phi A_N^{Fixed}(t)\textit{Black-76}(p^{Market}, K, (T - t), \sigma(K, T), r{=}0) \\ &= \phi A_N^{Fixed}(t)\left(p^{Market}N(\phi d_1) - KN(\phi d_2)\right) \end{aligned} \tag{10}$$

where

$$d_1 = \frac{ln(p^{Market}/K) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{(T-t)}}$$

and with

$$d_2 = d_1 - \sigma\sqrt{(T-t)}$$

and where ϕ is an indicator function with value 1 for a payer swaption and -1 for a receiver swaption.

Remark: Underlying Swaps with floating Libor Spreads
In the case that our underlying swap has a Libor spread on the float leg, as shown in [8], we modify the strike replacing K with K' where $K' = K - s\left(\frac{A_N^F loat(T)}{A_N^{Fixed}(t)}\right)$

2.2 Pricing with Cash-Price Settlement

European interest rate swaps with cash-price settlement have an identical price as in the physical-swap settlement case. The option holder is entitled to the cash equivalent of the underlying swap referenced in the physical swap-settlement case. The cash equivalent price is dependent on the yield curve used for valuation of the underlying swap, see equation (10), which is sometimes open to dispute.

2.3 Pricing with Cash Par-Yield Settlement

In the cash-settlement or par-yield settlement case the swaption payoff in equation (5) is a deterministic function of the swap rate, which is a reasonable approximation of the swap present value. Market practitioners derive the pricing formula for par-yield settlement in the same way as that of the physical-settled swaption and by directly substituting and replacing the A_N^{Fixed} annuity term in equation (10) with the cash-annuity approximation $C_N^{Fixed}(S(T))$.

However since the traditional physical-settlement valuation in (10) requires the annuity to be evaluated at validation date t instead of option expiry T with $t < T$ we have to further discount the cash-annuity to time t scaling the cash-annuity term by $P(t, T)$ as outlined below.

European Swaption Price for Cash Par-Yield Settlement

Assuming a lognormal Black-76 volatility we price European interest rate swaptions with cash par-yield settlement as follows:

$$\begin{aligned} V(t) &= \phi P(t,T) C_N^{Fixed}(T) \textit{Black-76}(p^{Market}, \textit{K, (T - t), } \sigma\textit{(K, T), r=0}) \\ &= \phi P(t,T) C_N^{Fixed}(T) \left(p^{Market} N(\phi d_1) - K N(\phi d_2) \right) \end{aligned} \tag{11}$$

where

$$d_1 = \frac{ln(p^{Market}/K) + \frac{1}{2}\sigma^2 (T-t)}{\sigma\sqrt{(T-t)}}$$

and with

$$d_2 = d_1 - \sigma\sqrt{(T-t)}$$

and where ϕ is an indicator function with value 1 for a payer swaption and -1 for a receiver swaption.

The cash-annuity term is calculated as

$$C_N^{Fixed}(T) = N \sum_{i=1}^{c} \tau_i D(T, t_i) \tag{12}$$

where c is the total number of regular whole fixed coupons and where the coupon year fraction τ and the discount factor $D(T, t_i)$ are defined as

$$\begin{aligned} \tau &= \frac{1}{m} \\ D(T, t_i) &= \frac{1}{\prod_{j=1}^{i} (1 + p^{Market}\tau)} \end{aligned} \tag{13}$$

where m denotes the number of fixed coupons per year and p^{market} is the underlying swap rate observed at the swaption valuation date at time t and T represents the swaption expiry date. Note we have $T < t_i \ \forall i$.

Remark: Underlying Swaps with floating Libor Spreads
In the case that our underlying swap has a Libor spread on the float leg, as shown in [8], we modify the strike replacing K with K' where $K' = K - s\left(\frac{A_N^F loat(T)}{A_N^{Fixed}(t)}\right)$

3 Cash Annuity Pricing Considerations & Accuracy

There are pros and cons of using cash par-yield settlement. Whilst the swaption payoff is standardized and decoupled from the yield curve we have introduced complications into swaption pricing. We have a very clear picture and precise understanding of the swaption payoff, but we now have an approximate pricing formula. We have a precise understanding of our swaption at option expiry T, but an approximate price on all valuation dates t prior to expiry.

In deriving the swaption price with cash par-yield settlement it was assumed that we could replace the annuity factor with the cash-annuity factor. However as outlined in swaption pricing paper [8] the derivation of the swaption pricing formula relied on the evaluation of the expected swaption payoff using the Martingale Representation Theorem, which guaranteed an arbitrage-free price.

The requirement of the Martingale Representation Theorem was that the annuity measure $\mathbb{Q}_A$ must correspond to a tradeable instrument in order to allow replication of the option payoff and guarantee no-arbitrage. However in the particular case of the cash-annuity, which is a cash approximation of the underlying swap, there is no tradeable instrument and therefore our pricing formula is not arbitrage-free and an approximation.

We acknowledge the limitations of the cash annuity in that we can only replicate option payoffs on underlying swaps without stubs, since the formulation of the par-yield payoff assumes the underlying swap tenor with no consideration of stub coupons taken into account. Swap coupon periods are assumed to be identical with constant and equal year fractions calculated as $1/m$, with m representing the number of coupons per year. Hence an annual coupon is assumed to be of length exactly equal to 1.0 year, semi-annual coupons have length 0.5 years, quarterly coupons 0.25 years etc. with no consideration for daycount basis, holiday calendars and business day conventions. The cash payoff has be simplified and approximates the underlying swap with moderate accuracy.

Furthermore swaption pricing with par-yield settlement does not enforce arbitrage relationships such as put-call parity in that a long receiver swaption and a short payer swaption is no longer a standard forward starting swap. For a review on the accuracy of the swaption price when using cash par-yield settlement we refer the reader to a dedicated paper on the subject, see "Cash-Settled Swaptions: How Wrong Are We?" [9].

Despite the pricing drawbacks there are significant benefits to trading swaptions with par-yield settlement, namely the payoff simplification and standardization facilitates exchange clearing, trade netting and mitigates settlement disputes on option exercise. Trade netting is possible in a cleared and bilateral capacity. This can significantly reduce counterparty credit risk, XVA exposures and capital charges, which can be substantial in magnitude.

4 Cash Annuity Stub Extension

In this section we consider how to extend the cash-annuity $C_N^{Fixed}(T)$ to support swaps with coupon stubs. Swaps with irregular front and end coupons are said to have coupon stubs. There are 4 kinds of stub-type namely Short-Start, Short-End, Long-Start and Long-End as outlined in the below example.

Stub-Type Example

Considering the example of a 2Y1M fixed-float swap on EUR 6M Libor each of the four stub types would indicate a swap with a tenor of 2 years and 1 month referencing EUR 6 month Libor with coupons bearing the following characteristics.

- **Short-Start**
  A short 1m coupon at the swap start followed by 4 regular coupons of 6m each
- **Short-End**
  4 regular coupons of 6m each followed by short coupon of 1m at the swap end
- **Long-Start**
  A long 7m coupon at the swap start followed by 3 regular coupons of 6m each
- **Long-End**
  3 regular coupons of 6m each followed by a long coupon of 7m at the swap end

Short Stubs: Short-Start and Short-End Stub Types
When the stub is *short* the stub is simply placed at the start or end of the swap as shown in the below swap coupon time-line, where we use red to denote the short stub coupon.

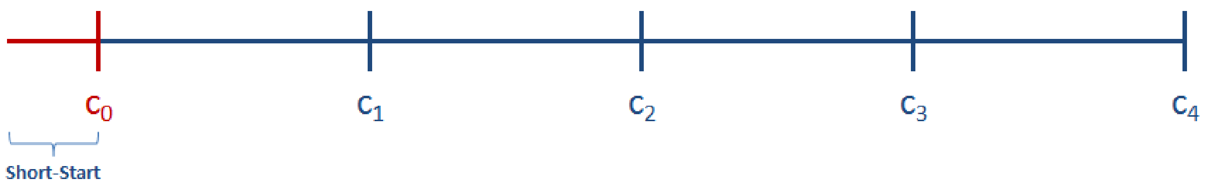

Figure 1: Short-Start Stub Illustration

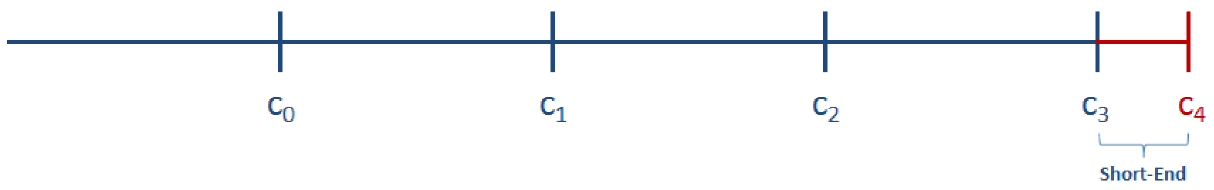

Figure 2: Short-End Stub Illustration

Long Stubs: Long-Start and Long-End Stub Types
When a stub is *long* the irregular coupon is combined with the adjacent coupon to make a long coupon as shown in the below swap coupon time-line. We use red to indicate the stub coupon and grey to indicate which regular coupon has been combined with the stub to form a long coupon.

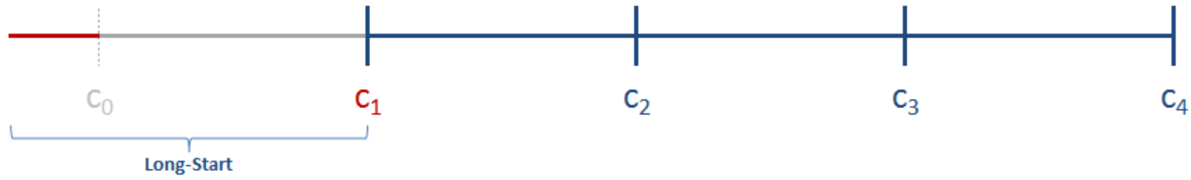

Figure 3: Long-Start Stub Illustration

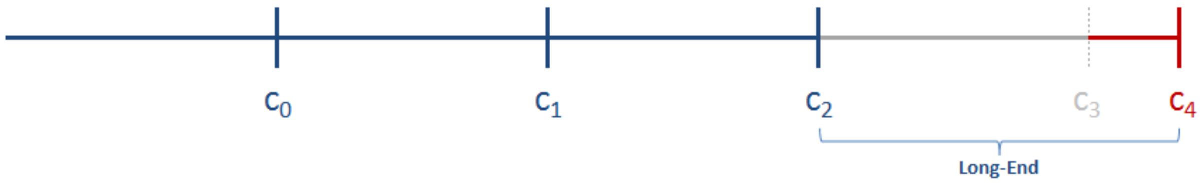

Figure 4: Long-End Stub Illustration

To support swaps with stub coupons we propose that the cash-annuity be modified as below. Firstly let us define the discount factors for both regular and stub coupons.

Modified Discount Factors for Swaps with Stub Coupons

Regular Coupon Discount Factors
The ith regular fixed coupon of length $\tau = \frac{1}{m}$ can be discounted with discrete compounding and interpreting the swap rate p^{Market} as the discount yield as follows

$$D(T, t_i) = \frac{1}{\prod_{j=1}^{i} (1 + p^{Market} \tau)} \tag{14}$$

Stub Coupon Discount Factors
Likewise stub coupons of length τ' in years can also be discounted in the same manner as (14) adjusting for the stub coupon length as follows

$$D(T, \tau')^{Stub} = \frac{1}{(1 + p^{Market} \tau')} \tag{15}$$

Secondly using the notation τ and τ^i to denote the length in years of a regular swap coupon and a stub respectively and having defined the discount factors as above allows us the modify the cash annuity formula for stub coupons as follows.

Modified Cash Annuity Terms for Swaps with Stub Coupons

Cash Annuity for Short-Start Stubs
In this case we have a short front stub as shown in figure (1)

$$C_N^{Fixed}(T) = \underbrace{N\tau' D(T,\tau')^{Stub}}_{\textit{Short Stub}} + N\sum_{i=1}^{c}\tau D(T,t_i)\ \underbrace{D(T,\tau')^{Stub}}_{\textit{Disc Fact Adjustment}} \tag{16}$$

We insert the stub coupon to the front of the annuity formula and make a stub discount factor adjustment to subsequent coupons to discount regular coupons to the front stub start date, which is the swap start date.

Cash Annuity for Short-End Stubs
In this case we have a short end stub as shown in figure (2)

$$C_N^{Fixed}(T) = N\sum_{i=1}^{c}\tau D(T,t_i) + \underbrace{N\tau' D(T,t_c)D(T,\tau')^{Stub}}_{\textit{Short Stub}} \tag{17}$$

We append the stub coupon to the end of the annuity formula and discount it to the swap start date.

Cash Annuity for Long-Start Stubs
In this case we have a long front stub as shown in figure (3)

$$C_N^{Fixed}(T) = \underbrace{N(\tau+\tau')D(T,\tau+\tau')^{Stub}}_{\textit{Long Stub}} + N\sum_{i=2}^{c}\tau D(T,t_i)\ \underbrace{D(T,\tau')^{Stub}}_{\textit{Disc Fact Adjustment}} \tag{18}$$

We combine the stub coupon with the first regular coupon and make a stub discount factor adjustment to subsequent coupons to discount regular coupons to the the swap start date.

Cash Annuity for Long-End Stubs
In this case we have a long front stub as shown in figure (4)

$$C_N^{Fixed}(T) = N\sum_{i=1}^{c-1}\tau D(T,t_i) + \underbrace{N(\tau+\tau')D(T,t_{c-1})D(T,\tau+\tau')^{Stub}}_{\textit{Long Stub}} \tag{19}$$

We combine the stub coupon with the last regular coupon and make a discount factor adjustment to final coupon to discount it to the swap start date correctly.

5 Conclusion

In summary in this paper we have looked at the different types of swaption payoffs commonly used in interest rate swaption markets. We emphasised the cash par-yield settlement type, which is the market standard settlement type in Europe. This settlement method both simplifies and standardizes the swaption payoff making it easier to clear and net trades and it doing so potentially reduces XVA capital charges, albeit at the same time introducing pricing error and bias.

We looked how to price European swaptions with the different settlement styles quoted in the market. We reviewed the cash-annuity approximation which is an integral part of the par-yield cash-settlement payoff. In reviewing the different settlement types we considered the impact of the cash annuity on swaption pricing.

Finally we looked at the limitations of the standard par-yield and cash annuity formulas, which only allow the pricing of swaptions with underlying swaps with regular whole coupons with no consideration for swaps with stub coupons. We relaxed this constraint and also consider how to price swaptions with stubs by making a slight modification the cash annuity formula.

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