PRM-02 Summary Document

Profitability analysis using risk-adjusted return measures

Risk-adjusted return measures/ratios quantify the performance of a strategy as the excess return (over a benchmark or risk free rate) per unit of risk. Different ratios are computed based on which benchmark rate we take and how we measure the risk.

For example, consider the **Sharpe ratio**, in which the benchmark return is the risk free rate and the measure of risk is the standard deviation:

Sharpe ratio = $\frac{(\mu - r)}{\sigma}$
Where,
r = risk free rate,
μ= mean return and
σ= the standard deviation of returns.

Another popular risk metric is the **Sortino ratio**, which is a modified version of Sharpe ratio wherein the standard deviation of negative returns is considered. It is the portfolio returns minus risk-free returns divided by downside (negative) standard deviation.

Sortino ratio = $\frac{(\mu - MAR)}{\sigma d}$
Where,
MAR = Minimum acceptable return (can also be the risk free rate),
μ= mean return and
σd= the standard deviation of negative returns.

Then, we have relative risk metrics such as the **Treynor ratio** which use a measure of risk relative to some benchmark. In the case of Treynor ratio, this risk measure is the Beta of the asset, which measures the volatility of a stock/portfolio in relation to the market. A portfolio with a beta greater than 1 is considered to be more volatile than the market; while a beta less than 1 means less volatility. Beta values are not bounded like the correlation values.

Treynor ratio = $\frac{(\mu - r)}{Beta}$
Where,
r = risk free rate,
μ= mean return and
$Beta$= measures the volatility of a stock/portfolio in relation to the market

Finally, we have tail risk metrics like the **VaR and Expected Shortfall**, which quantify the risk in case of tail/extreme events.

VaR measures the potential loss in the value of a risky asset over a defined period for a given confidence interval.

For example, if the VaR for an asset is at 10 million at 99% percent confidence level, then it means that in 1% of cases the loss will exceed the VAR amount. However, one should note that VaR does say anything about the size of the losses within this 1%.

Expected Shortfall (ES) is computed by taking a weighted average between the VaR value and the losses exceeding VaR. CVaR is seen as an extension of VaR and is considered superior to VaR as it also gives the expected size of loss in case of a tail event.

Some other popular ratios used for quantifying risk-adjusted performance are:

- Omega ratio
- Calmar ratio
- Up capture/Down capture ratios
- RAROC

<u>Position sizing</u>

Kelly criterion (or Kelly strategy or Kelly bet), is a formula that comes from the world of gambling and is used to determine the optimal theoretical size for a bet.

Under the Kelly criterion, the optimal fraction 'f' of total wealth that should be put on each bet, that maximises the wealth in the long run is given by:

$$f = \frac{(pb-aq)}{ab}$$

Where,
a= units lost on betting 1 unit
b= units gained on betting 1 unit
p=probability of win
q=probability of loss =1-'p'

In the world of financial markets, the above formula is adapted for a continuous time framework with multiple possible outcomes for each trade. So for a trading strategy with mean return μ and variance of returns σ^2 , the optimal fraction to be traded for each trade is given by:

$$f = \frac{(\mu-r\ \ )}{\sigma^2}$$

Where,
r = risk free rate
μ= mean return and
σ= the standard deviation of returns

A drawback of the Kelly criterion is that it does not take the drawdowns into consideration and that is why many traders bet with a fraction of the value of optimal ‘f’ suggested by the Kelly criterion or consider an advance model like the leveraged space theory which incorporates drawdown into the model.