

Basic Statistics using Excel

Weighted Portfolio



Problem Statement:

You have been asked to construct a portfolio of two stocks (SBI and ICICI) for maximum possible return, but with a 2.7% limit on the volatility of the portfolio, and based on the past two month's performance of the two stocks.

Getting Data



1. Download the daily data from finance.
2. Search for the quotes SBIN.NS and ICICI.NS.
3. Go to Historical Data, get last 45 working days data, and download to spreadsheet.
4. Copy the corresponding data in a single workbook.

Log Returns

1. Calculate the log return of each of the stocks.

Why Log Returns?

Time-additivity: consider an ordered sequence of n trades. A statistic frequently calculated from this sequence is the *compounding return*, which is the running return of this sequence of trades over time.

$$1 + r_i = \frac{p_i}{p_j} = \exp^{\log\left(\frac{p_i}{p_j}\right)}$$

$$\log(1 + r_i) = \log\left(\frac{p_i}{p_j}\right) = \log(p_i) - \log(p_j)$$

Compounded rate of Return:

$$(1 + r_1)(1 + r_2) \cdots (1 + r_n) = \prod_i (1 + r_i)$$

formula for calculating compound returns:

$$\sum_i \log(1 + r_i) = \log(1 + r_1) + \log(1 + r_2) + \cdots + \log(1 + r_n) = \log(p_n) - \log(p_0)$$

Standard Deviation and Covariance

Now that returns are calculated

2. Calculate the mean value of the corresponding returns. Mean value is seen as the reference point for understanding the stock's performance. As discussed in the previous lecture slides 31, 32 and 33, we will calculate the standard deviation and Covariance using the following formulas.

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{(n-1)}$$

Standard deviation is the volatility of the stock. It tells us how much the price can change over a given period of time.

Covariance measures how two variables move together. It measures whether the two move in the same direction (a positive covariance) or in opposite directions (a negative covariance)

Stepwise Procedure

1. First calculate the $X_i - X_m$.
2. Then calculate the $Y_i - Y_m$.
3. Multiply both columns elementwise.
4. Calculate the covariance by adding all the rows and dividing it by $n-1$ to account for Bessel's correction.
5. Now take the squares of the columns $X_i - X_m$ and $Y_i - Y_m$.
6. Calculate the standard deviation, by taking the square root of the sum of each of the two columns and dividing it by $n-1$.
7. Populate random portfolio weights.
8. From the previous session lecture slide 36, we can calculate the portfolio returns and portfolio standard deviation using the following

Portfolio Return and Variance

$$\text{Portfolio Return} = w*(R_A) + (1-w)*(R_B)$$

$$\text{Portfolio Variance} = w_A^2 * \sigma^2(R_A) + w_B^2 * \sigma^2(R_B) + 2*(w_A)*(w_B)*\text{Cov}(R_A, R_B)$$

Where: w_A and w_B are portfolio weights, $\sigma^2(R_A)$ and $\sigma^2(R_B)$ are variances and $\text{Cov}(R_A, R_B)$ is the covariance

Finally, plot the values to find the answer to your problem statement.

Statistics

The Random Walk Model

A Model for Stock Prices

- **Random Walk Model:** Today's price = yesterday's price + a change that is independent of all previous information. (It's a model, and a very controversial one at that.)
- Start at some known P_0 so $P_1 = P_0 + \Delta_1$ and so on.
- Price Change $\Delta_1 \sim k P_0$ where k is random number that follows the normal distribution.
 $k \sim N(\mu, \sigma)$

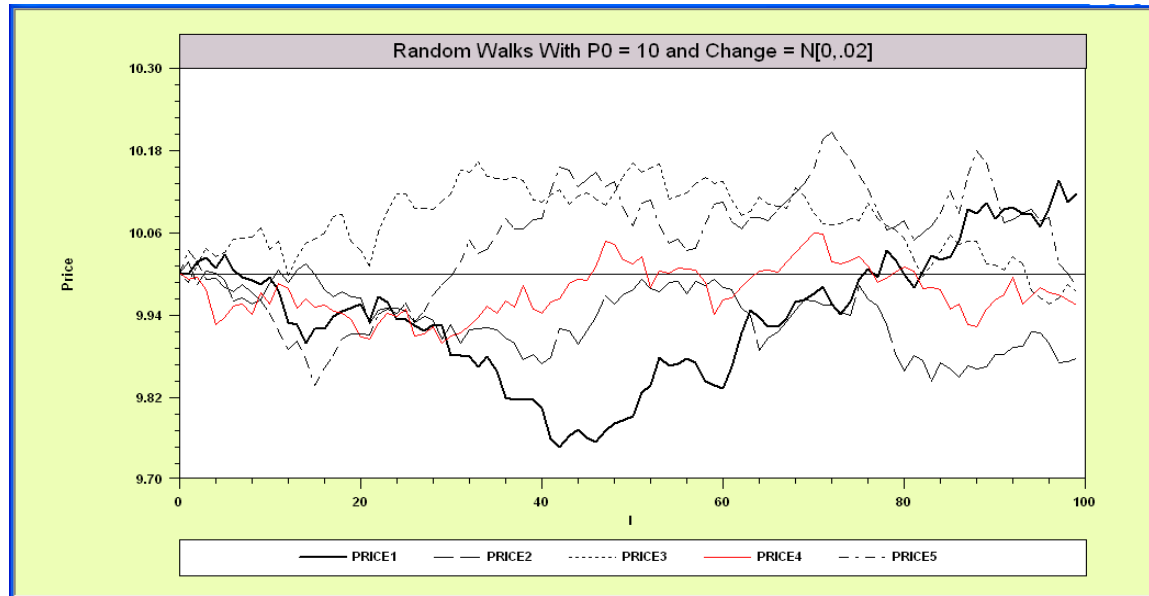
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- $P_1 = P_0 + k P_0$

Random Walk Simulations

$$P_t = P_{t-1} + \Delta_t, t = 1, 2, \dots, 100$$

Example: $P_0 = 10$, k is normally distributed with $\mu=0$, $\sigma=0.02$



Annual and Daily volatilities

Variance is additive

Daily Volatility (σ) = 1%

Daily Variance (σ^2) = $(1\%)^2$

Annual Variance (σ^2) = Variance_{day1} + Variance_{day2} + +
Variance_{day252}

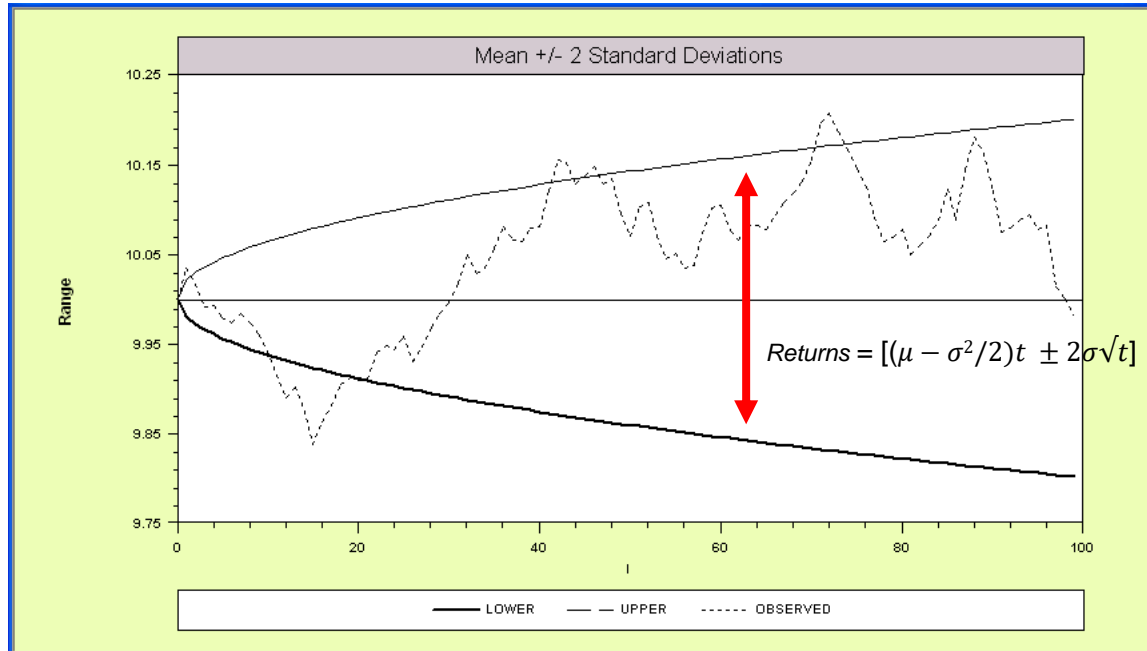
Annual Variance = $252 * \text{Variance}_{1 \text{ day}}$

Annual Volatility = $\text{Sqrt}[(\text{Daily Variance}) * 252]$

or

Volatility of time period ,T = (Vol. of 1 time period) * $\text{sqrt}(T)$

Using the Empirical Rule to Formulate an Expected Range



Monte Carlo Simulation



Problem Statement:

You are required to predict the one month Close prices of a stock(in this case SBI), using Monte Carlo Simulation, and report the 95% confidence VaR value for this one month period.(Use all Historical Data available online)

Introduction



The modern version of the Markov Chain Monte Carlo method was invented in the late 1940s by Stanislaw Ulam, while he was working on nuclear weapons projects at the Los Alamos National Laboratory. Monte Carlo simulation is used to model possible movements of asset prices, in this case using Excel. The formula for predicting next day's price is given by:

$$\text{next day's price} = \text{today's price} * e^{(\text{drift} + \text{random value})}$$

The stock price model we developed in the Random Walk Model is as follows:

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma k \sqrt{\Delta t} \quad \text{.....} \quad (1)$$

where:

ΔS is the change in the stock price S in Δt (a small time interval t), and k has a standard normal distribution (i.e., a normal distribution with a mean of zero and standard deviation of 1.0).

Eq. (1) shows that:

$$\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma^2 \Delta t) \quad \text{.....} \quad (2)$$

Monte Carlo Simulations

Consider a stock that has a volatility of 20% per annum and has an expected return of 10% per annum. Say the price of the stock is at \$100 at Day 0.

1. Daily std dev can be calculated from the annual std dev.:

$$\text{Daily std dev.} = 20\% / \text{sqrt}(252)$$

2. Daily Returns can be calculated from the annual returns.:

$$\text{Daily returns} = 10\% / (252)$$

3. Simulated returns for Day 1 (from equation 2) is:

$$\text{NORMINV}(\text{RAND}(), \text{Daily returns}, \text{daily std dev.})$$

4. Price on Day 1 is:

$$(\text{Price on Day 0}) * \exp(\text{NORMINV}(\text{RAND}(), \text{Daily returns}, \text{daily std dev.}))$$

5. From Price on nth Day can be calculated by repeating the steps 3 and 4:

$$(\text{Price on Day } n-1) * \exp(\text{NORMINV}(\text{RAND}(), \text{Daily returns}, \text{daily std dev.}))$$

Assumptions of Monte Carlo

- The simulation only represents probabilities and not certainty
- The same process could be run by simulating the returns and then predicting the VaR price
- More the simulations, stable is the outcome
- Take average of the prices below the 95% confidence price to get the Expected Shortfall (ES).

Bollinger Bands

Bollinger Bands calculation consists of 3 simple steps.

1. Middle Bollinger Band. It is the moving average of the closing price.

For example to calculate a 20 period moving average, add the closing prices of a security for 20 consecutive days and divide that value by 20.

2. Upper Bollinger Band. To calculate the Upper Bollinger Band you calculate the Moving Average of the Close and add Standard Deviations to it.

For example the upper band formula would be $MOV_{20} + (2 * 20 \text{ Standard Deviation of Close})$.

3. Lower Bollinger Band. To calculate the Lower Bollinger Band you calculate the Moving Average of the Close and subtract Standard Deviations from it.

For example the lower band formula would be $MOV_{20} - (2 * 20 \text{ Standard Deviation of Close})$.

Q & A