Option Pricing

Option pricing models can be very mathematically complex. But that is the *model*. The *situation* that the model is formalizing doesn't have to be described in a mathematical way. A lot of knowledge can come from thinking about what the models need to describe, before the math is used. Wile it is certainly possible to trade options without knowing anything about pricing or greeks, it will be more confusing.

The most important thing to take from pricing theory is that option values do not depend on the expected return of the stock. This seems counter-intuitive. If calls increase in value as the stock goes up, shouldn't they be worth more if we expect that to happen? But realizing this was not the case was the great breakthrough of the Black-Scholes-Merton model. The important part of the model is, "Imagine we have a hedged portfolio consisting of a call and some short stock." The rest is just math.

We won't be doing that math, but we can see the general effect by working through a very simple model, where the stock moves to one of two possible values in one time step. Let's say the stock starts at \$100 and can move to either \$150 or \$80.

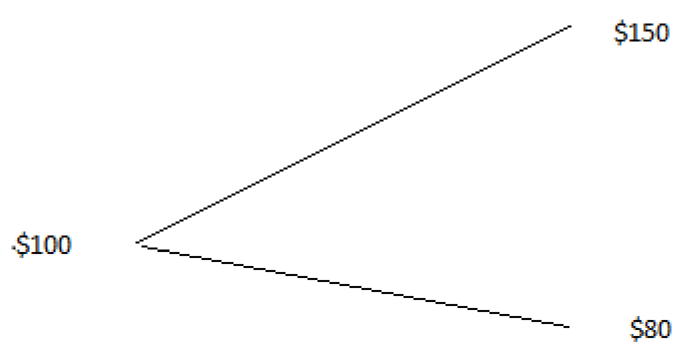

Figure One: The stylized stock path.

What is the \$100 strike call worth? We form a hedged portfolio that is short one option and long h units of stock (h is currently unknown). Because this is risk-free, this portfolio must be worth the same when the stock is \$150 or \$80. We also know that if we are in the \$80 state, the call is worthless and in the \$150 state it is worth \$50. That is,

$$150h - 50 = 80h - 0$$

Solving this gives h= 5/7. We also now see that the value of the portfolio in either final state is \$80 x 5/7= \$57.14. Finally, a hedged portfolio won't make or lose money over a time-step, so the initial portfolio is also worth \$57.14. That is,

$$100 \times \frac{5}{7} - C = 57.14$$

We solve this and get a call value of $14.29.

The most important thing isn't what is in these equations. The important thing is what isn't: the probability of the stock going up or down. We don't even know what these probabilities are. Equivalently, we have no idea (and no need to know) the expected final value of the stock. By creating a hedged portfolio, the stock's return is irrelevant. However, the *spread* between the final states does matter. This is as close as a one-step model can get to volatility.

Technically, this is called using *risk-neutral* probabilities. All option pricing is built on this idea. Unfortunately, it hasn't reached a lot of pundits yet. You will often hear statements like, "the options' market is implying this spread has a 5% chance of finishing in-the-money". These people are wrong, and their other opinions should be appropriately discounted. The options' market is making no real-world probability predictions at all. Real world probabilities depend on drift (return) and the options market has no opinion about that.

The Greeks

"Greeks" is a collective term for the derivatives (in the mathematical sense) of the option price, that describe the options' dependency on various variables and parameters. There is a common misconception that knowing the greeks is a source of edge. This isn't true. Knowing the greeks won't give edge any more than knowing the price will. Nonetheless, the greeks do explain what is going on. Trading options without them is like driving a car with no instruments. It can be done, but it isn't optimal.

Delta

Delta is the amount an option's value changes if the underlying stock moves by one dollar. So, if a call has a delta of 0.5 (also referred to as "50 delta" or "50% delta") it will increase by 50c if the stock goes up by a dollar and drop by 50c if the stock moves down by a dollar. Calls have positive deltas (because they are bullish bets) and puts have negative deltas. Options that are a long way out-of-the-money have low deltas because they aren't worth much to start with and this continues to be the case even if the stock moves a bit. Options that are deep in the money have deltas of one for calls and negative one for puts, because they behave just like long or short stock positions respectively.

When we are using options to bet directionally, we are "trading delta".

But even when using options in the simplest possible way, we need to be aware of some things that might be counterintuitive.

It might seem that an at-the-money call would have a delta of 0.5, because it is halfway between the stock and being worthless. This is not true, and it doesn't matter whether we use the stock price or its forward price to choose the ATM strike. This is because of the way returns compound. Think of a stock that starts at $100, goes up 10% (to $110) then up another 10% (to $121). Compare this to what happens when the price drops. A $100 stock that has two consecutive down 10% moves will only drop to $81. The up moves have taken us further in dollar terms.

Delta is the option's exposure to stock moves, and the $100 call *does* have greater exposure to up moves, hence its delta will be greater than 0.5.

Because this effect is due to compounding, it is more pronounced for higher volatility stocks (to see this, replace the moves in our example with ones twice as big). Finally, as we saw last time, time has the same effects as volatility, so this effect is also magnified for options with long times to expiration.

This isn't just theoretical. A lot of people got confused when trading meme stock options. When GME was around $300, the volatility was over 1000%. The one-month ATM calls had a delta of 0.93. The 800 strike had a delta of 0.86. This makes it very difficult to get positive delta exposure from spreads, or to sell a delta-neutral strangle. But it isn't a mathematical oddity, and it isn't an example of the model breaking, it is an accurate reflection of how stocks and options change price.

All financial instruments have a price and delta, directional exposure, is also an easy concept. It is the other greeks that make options unusual.

Gamma

Gamma is the amount an option's delta changes if the underlying stock moves by one dollar. The fact that options have gamma is what makes them interesting instruments. Delta exposure can come from stocks, but gamma is the direct result of the non-linearity of the option payoff function. If an option is far out-of-the-money, it might have a delta of 5%, but if the same option becomes close to at-the-money its delta will be close to 50%. This is how options give the potential for explosive gains.

This optionality isn't free. Owning options gives gamma, but you pay for it with theta.

Theta

Theta is the amount an option's value changes as time passes. It is usually expressed in the units of dollars per day. As with all the other greeks, theta is not an edge (like all gangs, the "theta gang" is a dangerous organization to be a part of, and no matter the ideas of the leaders, the rank-and-file members usually end up dead or in trouble).

The major contribution of all option pricing models is that in a delta-hedged portfolio, the P/L from owning gamma is exactly cancelled by the theta cost. Alternatively, if you are collecting theta, on average you will lose the same amount to the expected stock moves.

So, if theta is never a true edge, what is?

Vega

Vega is the amount the option value changes when implied volatility changes. It is generally expressed in units of dollars per one point volatility move. No matter why you are trading options, vega is of great importance. It is closely related to the concept of *implied volatility*.

In our previous discussion of option pricing, I talked about how volatility was the important pricing variable. Specifically, I talked about the realized volatility of the stock, how much the stock moved around. But this is something we won't know when we price the option. We know how much the stock has moved, but we don't know how much it will move. The volatility input must be estimated. And the estimate that corresponds to the market price of the option is called the implied volatility. It is the option markets guess of what the stock volatility will be over the lifetime of an option. In a sport's gambling context, implied volatility would be the bookies line and realized volatility would be what happens when the game is played.

Vega quantifies exposure to implied volatility. If a trader is prepared to ignore mark-to-market PL, vega might not seem relevant. The bet gets places, she leaves the casino and watches the game to see what happens. But, even in this case, vega is a very useful measure: it quantifies how mispriced an option is. The expected profit (or loss) of an option is given by its vega times the difference between implied and realized volatility. Vega tells you how exposed you are to paying the wrong price.

And let's be very clear about this. While options have nice characteristics such as limited downside and positive convexity, if we pay too much for them, we will lose money in the long run. It is easy to get distracted by math and greek letters, but successful option trading is, like all trading, dependent on buying below fair value and selling above fair value.

IF we sell above fair value, we can expect to make money. But this isn't because of theta. It is because the market was incorrectly pricing implied volatility relative to realized volatility. The "theta gang" is really the "vega gang". And this is important. If you don't understand why you make money, you won't understand what is going on when you lose money.