Time Value of Money

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Contents

In this PDF unit, we will have a look at a couple of TVM applications, namely:

1. Funding a Future Obligation
2. Funding a Retirement Plan

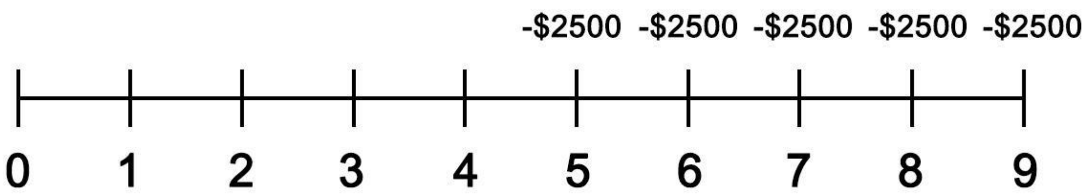
Funding a Future Obligation

When it comes to funding a future obligation, it becomes necessary to determine the size of the deposit that must be made over a specified period, in order to meet that future liability.

Nature of a future obligation may vary from person to person, such as saving for future college tuition fee, saving for emergency healthcare situations, saving for a down payment of your future home and so on.

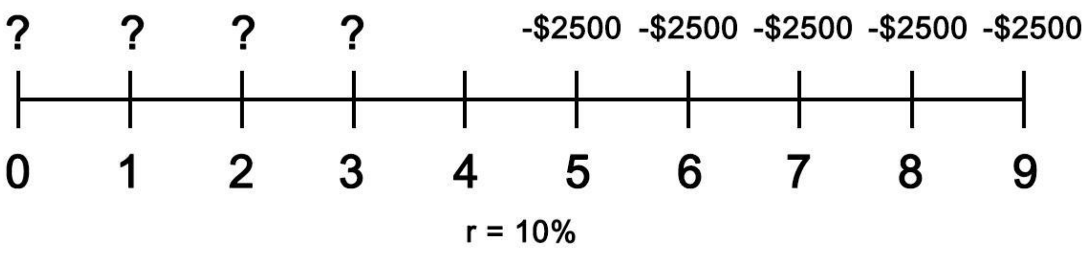
Let us have a look at one such example.

Let us assume that you have to make annual payments of say, $2,500 for five years, five years down the timeline.

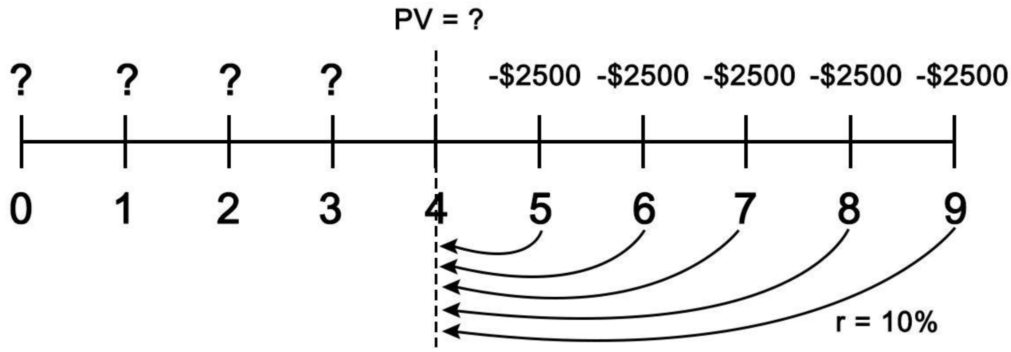


Hence, let us assume that you have 4 years to save for this expense, such that you have an adequate fund at the end of the 5th year to fulfil this obligation.

To accumulate this money, we will need to make these payments into an investment account. Assuming the rate of return to be 10%, what would be the amount of these 3 payments?



The first thing we will do is discount all the $2,500 annuity obligations back to the end of the year 4 with r = 10%.



The present value at the end of the year 4 is: PV = $9476.96. The calculation has been discussed below.

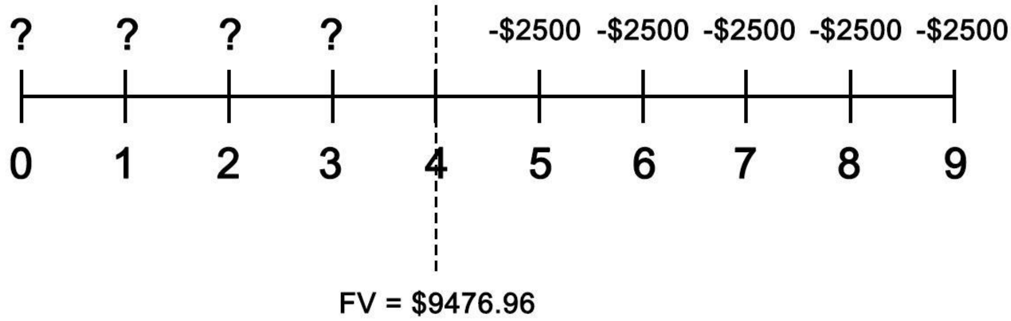
We will calculate these values in our iPython notebooks using the following formula.

$$PV = Annual\,Payments \times \frac{1-(1+r)^{-n}}{r}$$

where,
r is the interest rate, and
n is the number of periods

$$PV = 2500 \times \frac{1-(1+0.1)^{-5}}{0.1}$$
$$PV = \$9,476.97$$

However, we are actually standing at the year 0 (the beginning). Hence, the PV that we just calculated is actually our FV for the year 0.

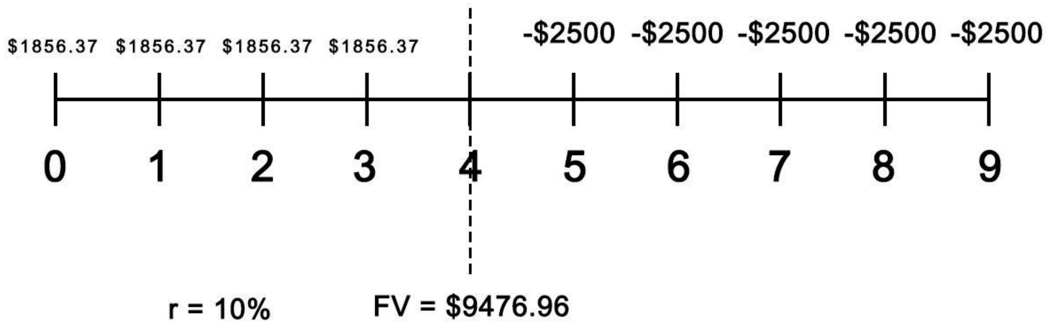


We now need to calculate the annual outflows that we will make for 4 years, so that we have a lump sum amount of $9,476.96 at the end of the 4th year or beginning of the 5th year.

$$Annual\ Payemnts = \frac{FV}{1+r} \times \frac{r}{(1+r)^n - 1}$$

where,
r is the interest rate, and
n is the number of periods

We will calculate these values in our IPython notebooks using the above formula.



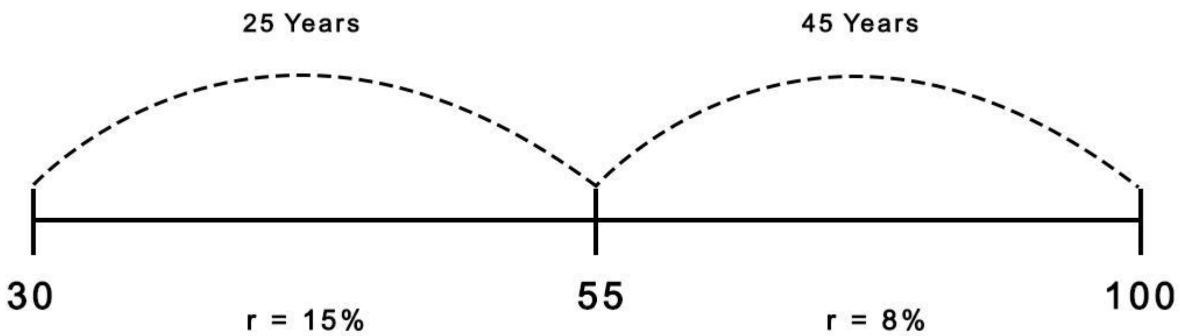
We will then invest this amount at r = 10% and take out $2,500 at the end of every year, such that at the end of the 9th year, the amount left with us will be $0.

Similarly, we can fund a retirement plan.

Funding a retirement plan

Assume that a 30 year old investor wants to retire in another 25 years at the age of 55. Over the long term, the investor expects to earn 15% on his investments prior to his retirement and 8% thereafter.

How much does the investor have to save at the end of every year for the first 25 years, such that he can withdraw $30,000 at the beginning of each year for the next 45 years till he is 100 years old?



Step 1:

FV = $0

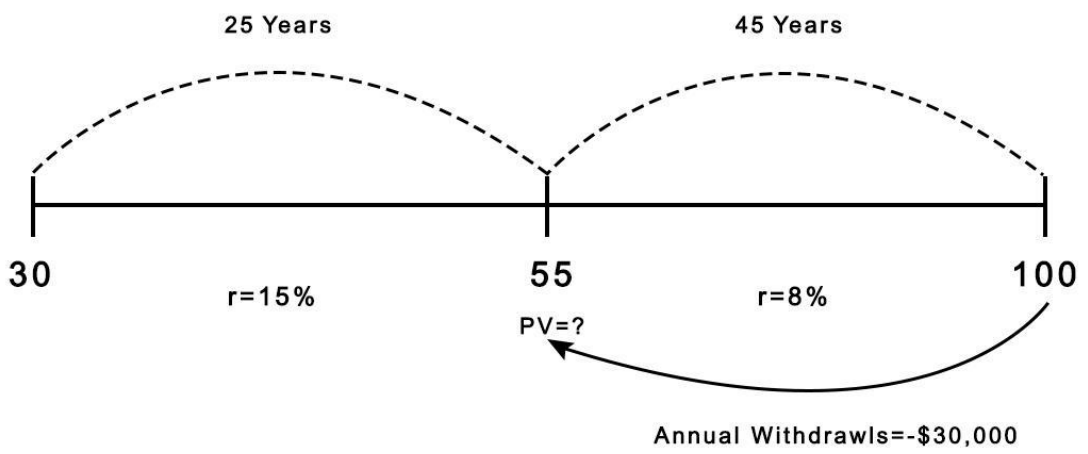
N = 45

Annual withdrawals = $30,000

R = 8%

PV =?

$$PV = Annual\,Payments \times \frac{1-(1+r)^{-n}}{r}$$
$$PV = \$30,000 \times \frac{1-(1+0.08)^{-45}}{0.08}$$
$$PV = \$363,252.05$$



Step 2:

FV = $363,252.05

R = 15%

N = 25

Annual saving amount =?

$$\boldsymbol{Annual\ Payemnts} = \frac{\boldsymbol{FV}}{\boldsymbol{1+r}} \times \frac{\boldsymbol{r}}{\boldsymbol{(1+r)^{n}-1}}$$

$$\begin{aligned}&where,\\&r\ is\ the\ interest\ rate,\ and\\&n\ is\ the\ number\ of\ periods\end{aligned}$$

Annual Payments = $1,484.41

Hence you have to save $1,484.41 at r = 15% to be able to withdraw $30,000 after 25 years, for the next 45 years at r = 8%.

These are some of the applications of the time value of money concepts. Stay tuned for more on TVM in our IPython notebooks.